ON SOME NEIGHBOURHOOD PRIME LABELING OF GRAPHS

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ABSTRACT: A labeling or numbering of a graph G with q edges is an assignment of labels to the vertices of G that induces for each edge uv a labeling depending on the vertex labels f(u) and f(v). A neighbourhood prime labeling of a graph is a variation of a prime labeling in which the vertices are assigned labels from 1 to |V(G)| such that the gcd of the labels in the neighbourhood of each non-degree 1 vertex is equal to 1. A graph which admits neighbourhood prime labeling is called a neighbourhood prime graph. In this paper, neighbourhood prime labelings for Z-P_n, Fan, Double Fan, Antiprism have been discussed.

KEYWORDS: Prime labeling, Neighbourhood prime labeling, Neighbourhood of a vertex.

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1.INTRODUCTION:

The field of graph theory plays a vital role in various fields. Graph labeling is one of the important area in graph theory. We follow Harary [3] for basic definitions and notations in graph theory. The notation of prime labeling for graphs originated with Roger Entringer and was introduces in the paper by Tout et al [1] in the early 1980's and since then it is an active field of research for many scholars. Motivated by the study of prime labeling, S.K.Patel and N.P.Shrimali in [4] introduced the notation of neighbourhood prime labeling in 2015, in which they have establish the sufficient condition for a graph to admit neighbourhood prime labeling and proves that paths, cycle, helm, closed helm and flower have neighbourhood prime labeling. Also C.Ananthavalli and K.Nagarajan [2] investigate neighbourhood prime labeling for special graphs like friendship graph, gear, ladder, triangular book and coconut tree. R.Senthil Amutha, N.Murugesan [5] proved some star related graphs, Petersen graph P(m,n), H-graphs, (m,n)-kite graphs, Jelly fish J(m,n) are neighbourhood prime labeling. In this paper, neighbourhood prime labelings for Z-P_n, Fan, Double Fan, Antiprism have been discussed.

Definition 1.1: Let $G = (V(G), E(G))$ be a graph with p vertices. A bijection $f: V(G) \rightarrow \{1,2,3, \dots, p\}$ is called a prime labeling if for each edge $e = uv$, $\gcd(f(u), f(v)\} = 1$. A graph which admits prime labeling is called prime graph.

Definition1.2: For a vertex v in G, the neighbourhood of v is the set of all vertex in G which are adjacent to v and is denoted by $N(v)$.

Definition1.3: Let $G = (V(G), E(G))$ be a graph with n vertices. A bijective function $f: V(G) \rightarrow \{1,2, \dots, n\}$ is said to be a neighbourhood prime labeling, if for every vertex $v \in V(G)$ with $\deg(v) > 1$, $\gcd\{f(u): u \in N(v)\} = 1$. A graph which admits neighbourhood prime labeling is called a neighbourhood prime graph.

Definition 1.4: Z-P_n is the graph obtained from two paths P_n and P_n' of same length n, by joining the i^{th} vertex of P_n to the $i-1^{th}$ vertex of P_n'. The resulting graph is denoted as Z-P_n.

Definition 1.5: The Graph P_n+K_1 is called a Fan and is denoted f_n.

Definition 1.6: The Graph P_n+2K_1 is called a Double fan.

Definition 1.7: The Antiprism graph A_n is the graph obtained by adding to these two cycles all edges of the form $v_i u_j$ and $v_i u_k$ such that u_j and u_k are adjacent.

2.Main Results:

THEOREM 2.1: The graph Z-P_n is neighbourhood prime graph.

PROOF: The graph Z-P_n has 2n vertices and 3(n-1) edges.

Define a bijection $f: V(G) \rightarrow \{1,2, \dots, 2n\}$ as follows

$f(u_i) = 2i - 1, \forall 1 \leq i \leq n$

$f(v_i) = 2i, \forall\ 1 \leq i\ \leq n$

In order to show that f has a neighbourhood prime labeling, we have to establish that if x is an arbitrary vertex of G, then gcd $\{f(p) \mid p \in N(x)\} = 1$.

The vertices with degree greater than 1 are $u_i, 2 \leq i \leq n$ and $v_i, 1 \leq i \leq n-1$.

$N(u_i) = \{u_{i-1}, u_{i+1}, v_{i-1}\}, 2 \leq i \leq n-1.$

$N(u_n) = \{u_{n-1}, v_{n-1}\}$

$N(v_1) = \{v_2, u_2\}$

$N(v_i) = \{ v_{i-1}, v_{i+1}, u_{i+1}\}, 2 \leq i \leq n-1$

Case (i) If $x = u_i, 2 \leq i \leq n-1$, then gcd $\{ f(p) | p \in N(x)\} = 1$, since $u_i, v_i \in N(x)$ such that

$f(u_i) = 2i - 1$ and $f(v_i) = 2i, \forall\ 2 \leq i \leq n$

Case (ii) If $x = u_n$, then gcd $\{ f(p) | p \in N(x)\} = 1$, since $u_{n-1}, v_{n-1} \in N(x)$, such that $f(u_{n-1}) = 2n - 3$.

Case (iii) If $x = v_i, 2 \leq i \leq n-1$, then gcd $\{ f(p) | p \in N(x)\} = 1$,since $f(u_i) = 2i - 1, f(v_i) = 2i,\ \forall 2 \leq i \leq n-1$

Case (iv) If $x = v_1$,then gcd $\{f(p) | p \in N(x)\} = 1$,since $u_2, v_2 \in N(x)$, such that $f(u_2) = 3, f(v_2) = 3$

Hence the graph Z-P_n is a neighbourhood prime graph.

EXAMPLE 2.2:

Consider the graph Z-P_5 with $f(u_1) = 1, f(u_2) = 3, f(u_3) = 5, f(u_4) = 7, f(u_5) = 9$, and $f(v_1) = 2$,

$f(v_2) = 4, f(v_3) = 6, f(v_4) = 8, f(v_5) = 10$.

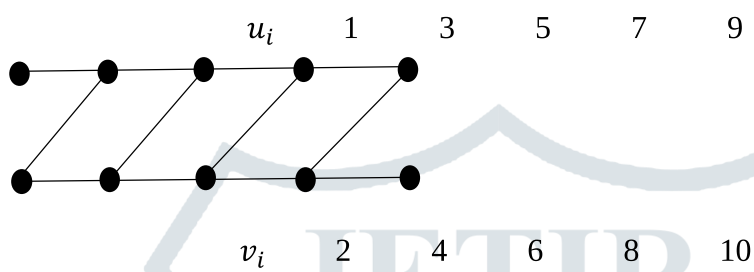

Figure 2.1 Z-P_5 is a neighbourhood prime labeling

Hence the graph Z-P_5 has neighbourhood prime labeling.

THEOREM 2.3: The Fan f_n is neighbourhood prime graph

Proof: The Fan has n+1 vertices and 2n-1 edges.

Define a bijection $f: V(G) \to \{1,2,\ldots,n+1\}$ as follows

$f(u_i) = i + 1, \forall\ 1 \le i \le n$

$f(u) = 1$

Here, all vertices have degree greater than 1.

$N(u) = \{u_i\}, 1 \le i \le n$

$N(u_i) = \{u_{i-1}, u, u_{i+1}\}, \forall\ 2 \le i \le n-1$

$N(u_1) = \{u, u_2\}$

$N(u_n) = \{u_{n-1}, u\}$

Case (i) If $x = u_i$, $1 \le i \le n$, then gcd $\{f(p) | p \in N(x)\} = 1$, since $u \in N(x)$ and $f(u) = 1$

Case (ii) If $x = u$, then gcd $\{f(p) | p \in N(x)\} = 1$ since$u_i \in N(x)$ such that $f(u_i) = i + 1, \forall\ 1 \le i \le n$.

Hence the fan f_n is a neighbourhood prime graph.

EXAMPLE 2.4:

Consider the fan f_n with $f(u) = 1, f(u_1) = 2, f(u_2) = 3, f(u_3) = 4, f(u_4) = 5, f(u_5) = 6$

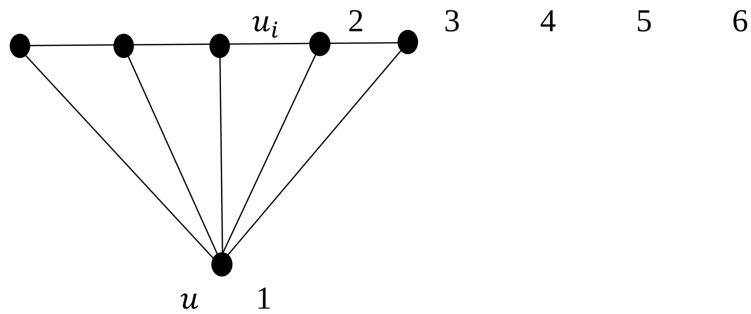

Figure 2.2 Fan f_5 with neighbourhood prime labeling

Hence the fan f_5 has neighbourhood prime labeling.

THEOREM 2.5: The Double fan, P_n+2K_1 has neighbourhood prime graph.

Proof: The Double fan, P_n+2K_1 has n+2 vertices and 3n-1 edges.

Define a bijection $f: V(G) \to \{1,2,\ldots,n+2\}$ as follows

$f(u_i) = i+1, \forall\ 1 \le i \ \le n$

$f(u) = 1$

$f(v) = n+2$

Here, all the vertices have degree greater than 1.

$N(u,v) = \{u_i\}$

$N(u_i) = \{u_{i-1}, u, v, u_{i+1}\}, \forall\ 2 \le i \le n-1$

$N(u_1) = \{u, v, u_2\}$

$N(u_n) = \{u_{n-1}, u, v\}$

Case (i) If $x = u_i, 1 \le i \ \le n$, then gcd $\{ f(p) |\ p \in N(x)\} = 1$, since $u \in N(x)$ such that $f(u) = 1$

Case (ii) If $x = u \ or \ v$, then gcd $\{ f(p) |\ p \in N(x)\} = 1$, since $u_i \in N(x)$ such that $f(u_i) = i+1$,

$\forall\ 1 \le i \le n$.

Hence the double fan P_n+2K_1 is a neighbourhood prime graph.

EXAMPLE 2.6:

Consider the double fan P_5+2K_1 with $f(u) = 1, f(u_1) = 2, f(u_2) = 3, f(u_3) = 4, f(u_4) = 5, f(u_5) = 6, f(v) = 7$.

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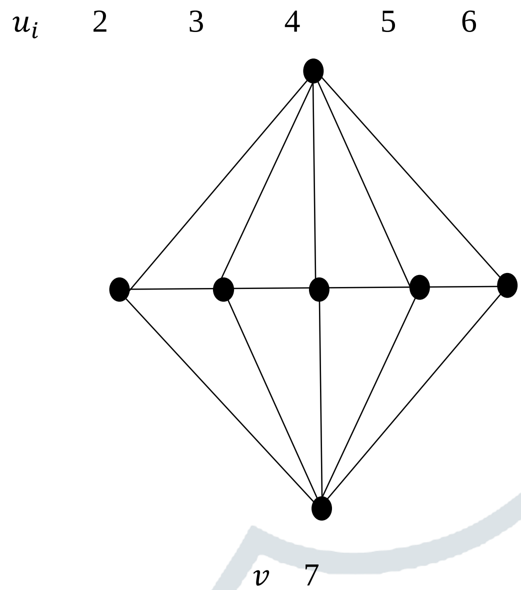

Figure 2.3 Double fan P_5+2K_1 with neighbourhood prime labeling

Hence the double fan P_5+2K_1 has neighbourhood prime labeling.

THEOREM 2.7: The Antiprism graph A_n is neighbourhood prime graph.

PROOF: The graph A_n has 2n vertices and 3n edges.

Define a bijection $f: V(G) \to \{1,2,\dots,2n\}$ as follows

$f(u_i) = 2i - 1, \forall 1 \le i \le n$

$f(v_i) = 2i, \forall\ 1 \le i \ \le n$

Here, all the vertices have degree greater than one.

$N(u_i) = \{u_{i-1}, u_{i+1}, v_i, v_{i-1}\}, 2 \le i \le n-1.$

$N(u_1) = \{u_2, u_n, v_1, v_n\}$

$N(u_n) = \{u_1, u_{n-1}, v_n, v_{n-1}\}$

$N(v_n) = \{u_1, u_n, v_1, v_{n-1}\}$

$N(v_1) = \{u_1, u_2, v_2, v_n\}$

$N(v_i) = \{\ u_i, u_{i+1}, v_{i-1}, v_{i+1}\}, 2 \le i \le n-1$

Case (i) If $x = u_i, 1 \le i \le n,$ then gcd $\{\ f(p) |\ p \in N(x)\} = 1,$ since $u_i, v_i \in N(x)$ such that $f(v_i) = 2i$ and $f(u_i) = 2i - 1, \forall\ 1 \le i \le n.$

Case (ii) If $x = v_i, 1 \le i \le n$, then gcd $\{\ f(p) |\ p \in N(x)\} = 1$,since $u_i \in N(x)$ such that $f(u_i) = 2i - 1, \forall\ 1 \le i \le n$

Hence the graph A_n is a neighbourhood prime graph.

EXAMPLE 2.8:

Consider the antiprism A_5 with $f(u_1) = 1, f(u_2) = 3, f(u_3) = 5, f(u_4) = 7,$ $f(u_5) = 9$ and $f(v_1) = 2, f(v_2) = 4, f(v_3) = 6, f(v_4) = 8, f(v_5) = 10, f(v_6) = 12.$

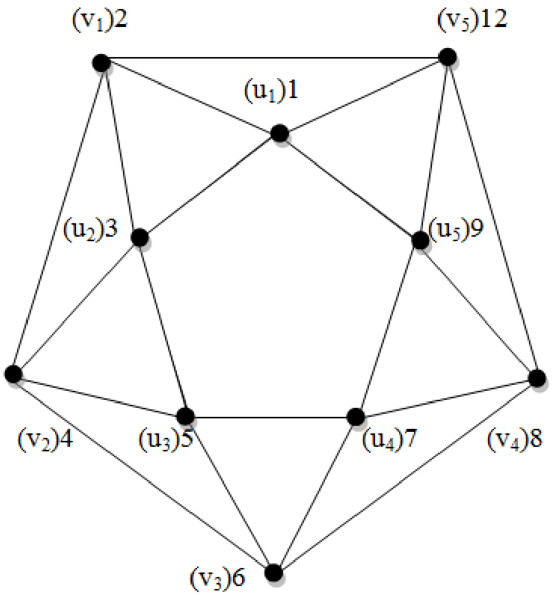

Figure 2.4 Anti prism graph A_5 with neighbourhood prime labeling

Hence the antiprism A_5 has neighbourhood prime labeling.

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