IFP-IDEALS IN NEAR SUBTRACTION SEMIGROUPS

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Abstract:

In this paper we introduced the concept of IFP-ideals in near subtraction semigroup.ie) X is said to fulfil the insertion-of- factors property (IFP) provided that for all a,b,x ∈ X: ab = 0,implies axb = 0.Such a near subtraction semigroups are called IFP-near subtraction semigroups. An ideal P of a near subtraction semigroup X is called an IFP-ideal if it is provided that ab ∈ P ∀ a,b ∈ X implies axb ∈ P, ∀ x ∈ X.

Introduction

B.M. Schein **[5]** considered systems of the form (X; ∘, −), where X is a set of functions closed under the composition "∘" of functions (and hence (X;∘) is a function semigroup) and the set theoretic subtraction "−" (and hence (X;−)is a subtraction algebra in the sense of **[1]**). He proved that every subtraction semigroup is isomorphic to a difference semigroup of invertible functions. B.Zelinka **[6]** discussed a problem proposed by B.M.Schein concerning the structure of multiplication in a subtraction semigroup. He solved the problem for subtraction algebras of a special type, called the atomic subtraction algebras. For basic definition one may refer to Pilz **[4]**. Motivated by the study of IFP-ideal in near rings by Akin Osman Atagun. We introduced the new concept IFP-ideals in near subtraction semigroups.

2. Preliminary definitions and results

Throughout X will denote a right near subtraction semigroup. It is assumed that the reader is familiar with the basic definitions of right near subtraction semigroup , zero – symmetric near subtraction semigroup , and ideal.

Definition 2.1:

A non-empty set X together with binary operation "-" and is said to be subtraction algebra if satisfies the following:

1) $x-(y-x)=x.$
2) $x-(x-y)=y-(y-x).$
3) $(x-y)-z=(x-z)-y \; \forall \, x, y.z \in X.$

Definition 2.2:

A non-empty set X together with two binary operations "-" and "•" is said to be a *subtraction semigroup* if it satisfies the following:

1) (X,-) is a subtraction algebra.
2) (X,•) is a semigroup.
3) $x(y-z) = xy - xz$ and

$$(x-y)z = xz - yz, \forall\, x, y, z \in X$$

Definition 2.3:

A non-empty set X together with two binary operations "-"and "•"is said to be a *near subtraction semigroup* (right) if it satisfies the following:

1) (X,-) is a subtraction algebra.
2) (X,•) is a semigroup.
3) $(x-y)z = xz - yz, \forall\, x, y, z \in X.$

Definition 2.4 :

If X is said to fulfil the insertion-of-property (IFP) provided that for all a, b, n ∈ X: ab =0 implies anb = 0. Such near subtraction semigroup are called *IFP-near subtraction semigroups* .

Definition 2.5 :

If an ideal P of a near subtraction semigroup X is called *an IFP-ideal* if it is provided that ab $\in P$ $\forall\, a, b \in$ X implies anb ∈ P,∀ n∈X.

Definition 2.6 :

The ideal P of X is called a *0-prime ideal* if for every A, B ⊲ X, AB ⊆ P implies A ⊆ P or B ⊆ P.

Definition 2.7:

If P ⊲ X is called a *3-prime (3-semiprime) ideal* if for a,b ∈ X, aXb ⊆ P (aXa ⊆ P) implies a∈P or b∈P (a ∈ P).

Definition 2.8:

If for a ,b ∈ X, ab ∈P (a∈P) implies a∈P or b∈P (a∈P), then P ⊲ X is called a *completely prime (completely semi prime) ideal*.

Definition 2.9:

If the zero ideal of X is v-prime (v = 0, 3, completely), then X is called a *v-prime near subtraction semigroups*.

Lemma 2.10:

We have shown that a reduced near subtraction semigroup then X is weakly regular iff every ideal is completely semiprime and X\P is left weakly regular for all prime ideal P of X.

Lemma 2.11:

Let X be a finite near subtraction semigroup

1) If X is a zero symmetric right permutable near subtraction semigroup, then X is a near-field iff X is an equiprime near subtraction semigroup.
2) If X is a left permutable near subtraction semigroup, then X is a near-field iff X is an equiprime near subtraction semigroup.

It easily seen that any completely prime near subtraction semigroup has no non-zero nilpotent elements. If for all a, b, c, d ∈ X, (resp. abc = bac, abcd = acbd), then X is called a right permutable (resp. left permutable,medial)near subtraction semigroup . If abc = abac(resp. abc = acbc), then

X is called a left self distributive (resp. right self distributive)near subtraction semigroup.

For some definitions of strongly regular near subtraction semigroup, reduced subtraction semigroup, Boolean near subtraction semigroup and p-near subtraction semigroup , left weakly regular near subtraction semigroup , left (right)strongly regular near subtraction semigroups.

3. Prime ideals and IFP ideals

Proposition 3.1:

If P is an IFP-ideal and a 3-(semi) prime ideal of X, then P is a completely (semi) prime ideal.

Proof :

Let $ab \in P$ for $a, b \in X$. Since P is an IFP-ideal, then $aXb \subseteq P$. Hence $a \in P$ or $b \in P$, since P is a 3-prime ideal. Therefore P is a completely prime ideal. To prove the semiprime case, it is enough to take a=b.

From now on, all near subtraction semigroup will be zero-symmetric in this section.

Proposition 3.2:

Let P be a completely semiprime ideal of X. Then P is an IFP-ideal.

Proof:

Assume P is a completely semiprime ideal of X and $ab \in P$ for $a, b \in X$. It is easily seen that $XP \subseteq P$, since X is zero-symmetric. Then $(ba)^2 = baba \in XPX \subseteq P$ and then $ba \in P$ since P is completely semiprime. Hence $(anb)^2 = anbanb \in XPX \subseteq P$ for all $n \in X$, whence $anb \in P$, since P is completely semiprime. Therefore P is an IFP-ideal .

Corollary 3.3:

Let P be a completely prime ideal of X. Then P is an IFP-ideal.

Proof :

If P is a completely prime ideal of X, then it is completely semiprime. Hence the result follows proposition 3.5.

4. IFP ideals occurring naturally in some near subtraction semigroups

The class consisting of zero-symmetric near subtraction semigroups (resp., consisting of the near subtraction semigroups with identity) will be denoted by R_0 (resp.,R_1).

Proposition 4.1:

Let $X \in R_0$ be a Boolean near subtraction semigroup and $P \lhd X$.Then P is an IFP-ideal.

Proof:

Assume $ab \in P$ for $a,b \in X$.Since $ba=(ba)^2 =baba \in XPX \subseteq P$, then $anb =(anb)^2 = anbanb \in XPX \subseteq P$ for all $n \in X$. Therefore P is an IFP-ideal of X.

Proposition 4.2:

Let $X \in R_0$ be a p-near subtraction semigroup and $P \lhd X$. Then P is an IFP-ideal.

Proof:

If p=2, the result follows Proposition 4.1. Assume p>2 and that $ab \in P$ for $a,b \in X$.Since $ba=(ba)^p \in XPX \subseteq P$, then $anb=(anb)^p \in XPX \in P$ for all $n \in X$. Therefore P is an IFP-ideal of X.

.We have the following :

Proposition 4.3:

Let X be an IFP near subtraction semigroup . Then for all $x \in$ X , (0:x) is an IFP-ideal of X.

Proof :

Clearly (0:x) ⊲ X for all $x \in$ X when X is an IFP near subtraction semigroup. Let ab ∈ (0:x) for all a, b ∈ X. Then ab$x$ = 0. Since X is an IFP near subtraction semigroup, then an(bx) = 0 for all n ∈ X . Hence anb ∈ (0:x) for all n ∈ X, i.e (0:x) is an IFP-ideal of X.

Proposition4.5:

If P is an IFP-ideal of a near subtraction semigroup X, then (P:P) is an ideal of X. Furthermore (P:P) is also an IFP-ideal.

Proof:

To prove (P:P) ⊲ X, it is enough to show that (P:P)X ⊆ (P:P). Let y ∈ (P:P)X.Then there exist an a ∈ (P:P) and an n ∈ X such that y=an. Since a ∈ (P:P), then ap ∈ P for all p ∈ P. Since P is an IFP-ideal ,then anp ∈ P for all n ∈ X.Then yp ∈ P for all p ∈ P. Hence y ∈ (P:P). Now, we show that (P:P) is an IFP-ideal. Assume xy ∈ (P:P) for x, y ∈ X . Then xyp ∈ P for all p ∈ P. Since P is an IFP-ideal, xnyp ∈ P for all n ∈ X and for all p ∈ P. Therefore, xny ∈ (P:P), which completes the proof

Proposition 4.6 :

Let X ∈ $R_0 \cap R_1$ be a reduced left weakly regular near subtraction semigroup and let P ⊲ X .Then P is an IFP-ideal of X.

Proof :

The result follows from Lemma 2.10 and Proposition 3.5.

Proposition 4.7:

Let P ⊲ X. Then,

a) If X is right permutable, then P is an IFP-ideal

b) If X is left permutable , then P is an IFP-ideal.

c) If X is right self distributive, then P is an IFP-ideal.

d) If X ∈ R_0 is left self distributive, then P is an IFP-ideal.

Proof:

For a, b ∈ X, assume ab ∈ P.Then for all n ∈ X,

a) anb=abn ∈ PX ⊆ P.

b) anb=nab ∈ XP ⊆ P, since X ∈ R_0 [Lemma 2.11]

c) anb=abnb ∈ PX ⊆ P.

d) anb=anab ∈ XP ⊆ P, since X ∈ R_0 [Lemma 2.11]

Proposition 4.8 :

Let X be a medial near subtraction semigroup and P ⊲ X.Then,

a) If X is regular, then P is an IFP-ideal.

b) If X is right strongly regular, then P is an IFP-ideal.

c) If X ∈ R_0 is left strongly regular, then P is an IFP-ideal.

Proof :

For x, y ∈ X, assume xy ∈ P.

a) Since X is regular, then there exist a ,b ∈ X such that $x = xax$ and $y = yby$. Then for all n ∈X, $xny = xaxnyby = x(axn)y(by)$. Since X is medial, then $xny = xy(axn)by$ ∈ PN ⊆ P.

b) Since X is right strongly regular, then there exist a,b ∈ X such that $x = x^2a$ and $y = y^2b$. Then for all n ∈ X, $xny = xxanyyb = x(xan)y(yb)$. Since X is medial, then $xny = xy(xan)(yb)$ ∈ PX ⊆ P.

c) Since X ∈ R_0 is left strongly regular, then there exist a, b ∈ X such that $x = ax^2$ and $y = by^2$. Then for all n ∈ X, $xny = axxnbyy = (ax)(xnb)y(y)$. Since X is medial, then $xny = (ax)y(xnb)y = a(xy)xnby$ ∈ XPX ⊆ P.

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