Power Decomposition under Non-sinusoidal Conditions - A Case Study

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Abstract: The revolution and wide usage of power electronics devices in modern power system leads to non-sinusoidal voltage and current waveforms due to addition of harmonic signals to the fundamental signals. Defining the total apparent power under non-sinusoidal condition by obtaining different power components for a balanced three phase system is presented in this paper. The apparent power is decomposed in the average and oscillatory power components. The power decomposition proposed in this paper can be used in the power balance equations in load flow analysis under non-sinusoidal conditions as well as in compensator design. To verify the equations derived, a Case study of a distorted system is considered. The power components and total apparent power calculated are compared with the power decomposition given in IEEE Standard 1459-2010 . It is found that the values are satisfactorily matching. Thus accurateness of the proposed method is demonstrated.

IndexTerms: Apparent power, balanced systems, non-sinusoidal conditions, power decomposition

I. INTRODUCTION

The use of modern equipment as nonlinear loads is increasing day by day results in high harmonic penetration in the power system due to which utilities are finding hard to have a perfect sinusoidal voltage at the point of common coupling(PCC). The nonlinear loads like arc furnaces, variable frequency drives (VFDs), computers, commercial lighting systems etc. draw non-sinusoidal current, causing non-sinusoidal voltage drops across transmission lines and transformer impedances [1]. This results in poor power quality of grid which further affects the performance of various equipment in power system. The distorted system voltage or current is a mixture of periodic fundamental frequency component as well as additional components such as aperiodic components like constant dc offsets and periodic harmonic components withfrequencies which are multiples of fundamental frequency [2]. Each harmonic component can be fully described by four parameters as order, magnitude, phase angle and sequence of harmonics.As harmonics are additional signals which increases the r.m.s. value of the system current and voltage, causes increase in total apparent power and decrease in power factor due to additional distortion power. The classical definitions of active, reactive and apparent power as well as power factor are inadequate under non-sinusoidal conditions [3].

From pioneer work by Budeanu [4], many researchers have explained the decomposition of apparent power under non-sinusoidal operation conditions with either frequency domain based approach or time domain based approach.Budeanuwho is owner of three dimensional powerhas decomposed apparent power into active, reactive and distortion powers as three orthogonal power components.The non-active powers are defined into two power components as Budeanu's reactive power as a sum of all individual harmonic reactive powers and a new power as distortion power [2]. Czarnecki criticizes Budeanu's reactive power and has identified five mutually orthogonal power components as active, reactive, distortion scattered, unbalanced and generated power for three phase unbalanced networks after developing the theory for single phase networks with non-sinusoidal voltage source[5],[6]. He further extended his research work for three phasefour-wire systems[7]. The physical meaning of non-active power is found in literature published in 2005 by A. E. Emanuel and the physical mechanism of power flow in 2010 by A. E. Emanuel. [8]. Hanoch Lev-Ari has suggested current decomposition in terms of frequency dependent weighted mean[4]. The three phase power in harmonic environment is first explained by Hirofumi Akagi in terms of average powers and oscillating powers using instantaneous power theory[9]. For balanced and unbalanced three phase systems with non-sinusoidal waveform situation is explained in IEEE standard 1459-2000 and further revised in IEEE standard 1459-2010 which is the most recommended definition[10][11].The revised IEEE standard 1459-2010 provides definitions for active, reactive and apparent powers as well as power factor for balanced and unbalanced three phase systems under harmonic content of signal data frame is provided.

The decomposition of total apparent power and the power flows in harmonic environment is necessary for optimal power flow analysis and the harmonic mitigation techniques. This paper describes an approximate method which gives power components of total three phase apparent power in non-sinusoidal voltage and current situation under balanced load condition.

This paper is organized as follows:
The apparent power decomposition issummarized in section II. The mathematical formulation of proposed power decomposition method is described in section III. In section IV, the method is implemented on a Case study and results are compared with IEEE Standard 1459-2010 calculation results. Finally, section V summarizes the conclusions of the paper.

II. APPARENT POWER DECOMPOSITION

This section describes, the decomposition of apparent power as per IEEE Standard $1459-2010$[11]. The total $r.m.s.$ voltage and current is described as a squared term with addition of squared fundamental components, dc components and harmonic components. Considering balanced system and per phase calculations, the total apparent power$[S]$can be written in terms of $r.m.s.$voltage $[V_{rms}]$and current$[I_{rms}]$ as follows:

$$S = V_{rms}.I_{rms} \quad (1)$$

Now,

$$S^2 = (V_{rms}.I_{rms})^2 = {V_{rms}}^2.{I_{rms}}^2 \quad (2)$$

$${V_{rms}}^2 = {V_1}^2 + {V_0}^2 + \sum_{h=2}^{hmax} {V_h}^2 \quad (3)$$

$${I_{rms}}^2 = {I_1}^2 + {I_0}^2 + \sum_{h=2}^{hmax} {I_h}^2 \quad (4)$$

Also,

$${V_H}^2 = {V_0}^2 + \sum_{h=2}^{hmax} {V_h}^2 \quad (5)$$

$${I_H}^2 = {I_0}^2 + \sum_{h=2}^{hmax} {I_h}^2 \quad (6)$$

where,V_0, V_1, I_0 and I_1 are dc components and $r.m.s.$ values of fundamental components of voltage and current. V_h and I_h are h^{th}harmonic voltage and current respectively and h_{max} is the highest order of harmonics under consideration.

$$S^2 = \left({V_1}^2 + \sum_{h=2}^{hmax} {V_H}^2\right).\left({I_1}^2 + \sum_{h=2}^{hmax} {I_H}^2\right) \quad (7)$$

$$S^2 = {V_1}^2.{I_1}^2 + {V_1}^2.\sum_{h=2}^{hmax} {I_H}^2 + {I_1}^2.\sum_{h=2}^{hmax} {V_H}^2 + \sum_{h=2}^{hmax} {V_H}^2.\sum_{h=2}^{hmax} {I_H}^2 \quad (8)$$

$$S^2 = {S_{11}}^2 + S_N^2 \quad (9)$$

where the fundamental apparent power, $S_{11}$is given as,

$${S_{11}}^2 = (V_1.I_1)^2 = {P_{11}}^2 + {Q_{11}}^2 \quad (10)$$

Where P_{11} and Q_{11} are fundamental real and reactive powers respectively given as,

$$P_{11} = V_1.I_1.cos\phi_1 \quad (11)$$

$$Q_{11} = V_1.I_1.sin\phi_1 \quad (12)$$

where ϕ_1= angle between fundamental voltage and current
S_N is non-fundamental apparent power given as,

$$S_N^2 = {D_I}^2 + {D_V}^2 + {S_H}^2$$

The current distortion power, D_I (var)due to products of magnitudes of fundamental voltage and harmonic currents is given as,

$${D_I}^2 = {V_1}^2.\sum_{h=2}^{hmax} {I_H}^2 \quad (13)$$

The voltage distortion power, D_V (var)due to products of magnitudes of fundamental current and harmonic voltages is given as,

$${D_V}^2 = {I_1}^2.\sum_{h=2}^{hmax} {V_H}^2 \quad (14)$$

The harmonic apparent power, S_H (VA)due to products of similar order harmonic voltages and currents magnitudes is given as,

$${S_H}^2 = (\sum_{h=2}^{hmax} {V_H}^2).(\sum_{h=2}^{hmax} {I_H}^2) \quad (15)$$

$${S_H}^2 = {P_H}^2 + D_H^2$$

where $P_H$as harmonic active power including dc power is given as,

$$P_H = V_0.I_0 + \sum_{h=2}^{hmax} V_h.I_h.cos\phi_h \quad (16)$$

where ϕ_h=angle between h^{th}harmonic voltage and current.
An additional power called Harmonic distortion power, $D_H(var)$ is given as,

$$D_H = \sqrt{(S_H^2 - P_H^2)} \quad (17)$$

The total non-active power, $N(VA)$ is given as,

$$N = \sqrt{(S^2 - P^2)} \quad (18)$$

where$P$ is total active power including harmonics. The total non-active power, $N = Q_1$, only when waveforms are sinusoidal.

III. PROPOSED METHOD

According to guidelines by IEEE Standard$1459 - 2010$, the apparent power is decomposed in seven powers as described in previous section.As per $p - q$ theory, for a three phasesystem with nonlinear load, the total power can be visualized as a composition of real power,p imaginary power,q and zero sequence power, p_0. The real power, p is decomposedin two parts as average power,$\bar{p}$ and oscillating power, $\tilde{p}$. On the similar basis, the imaginary powers and zero sequence powers can be decomposed. The average powers can be obtained using the similar order positive and negative sequence harmonic voltages and currents. The oscillating powers can be obtained from the dissimilar order positive and negative sequence harmonic voltages and currents.Neglecting zero sequence powers, the squared value of three phase real poweris given as,

$$P^2 = \bar{P}^2 + \tilde{P}^2 \quad (19)$$

Where,

$\bar{P}$ = Average value of three phase real power

The squared value of $\bar{P}$ can be written as,

$$\bar{P}^2 = \sum_{h=1}^{hmax} (\sqrt{3}.V_h.I_h.cos\phi_h)^2 \quad (20)$$

$\tilde{P}$ = The value of oscillating part of three phase real power contributing to total apparent power.

The squared value of $\tilde{P}$ can be written as,

$$\tilde{P}^2 \cong \sum_{\substack{m=1,n=1\\ m\neq n}}^{hmax} \left(\sqrt{3}.V_m.I_n.\cos(\phi_m - \phi_n)\right)^2 \quad (21)$$

Similarly,

The three phase imaginary poweris given as,

$$Q^2 = \bar{Q}^2 + \tilde{Q}^2 \quad (22)$$

Where,

$\bar{Q}$ = Total three phase imaginary power

The squared value of $\bar{Q}$ can be written as,

$$\bar{Q}^2 = \sum_{h=1}^{hmax} (\sqrt{3}.V_h.I_h.sin\phi_h)^2 \quad (23)$$

$\tilde{Q}$ = The value of three phase oscillating power contributing to total apparent power.

The squared value of $\tilde{Q}$can be written as,

$$\tilde{Q}^2 \cong \sum_{\substack{m=1,n=1\\ m\neq n}}^{hmax} \left(\sqrt{3}.V_m.I_n.\sin(\phi_m - \phi_n)\right)^2 \quad (24)$$

Thus, total squared apparent power can be written in terms of squares of only the four components as given below,

$$S^2 = \bar{P}^2 + \tilde{P}^2 + \bar{Q}^2 + \tilde{Q}^2 (25)$$

The value of S gives the total three phase apparent power in harmonic environment that is with voltage and current harmonics under balanced load condition neglecting the dc components as well as zero sequence harmonics.

Looking at equations (9)and(25), it is clear that the apparent power is calculated by two different methods. Total fundamental and harmonic power defined as per IEEE Standard$1459 - 2010$, given by vector addition of equations $(10)$and (15) must be equal to the total average power calculated from proposed method given by equations $(20)$and (23). Similarly, the total distortion power given by vector addition of $(13), (14)$and (18) must be equal to the total oscillatory power from the proposed method given by equations $(21)$and (24).

IV.CASE STUDY

To validate the proposed equations, a Case study of a steel industry is considered. The industry has $33KV$ incomer bus with $33KV/850V$ transformer with secondary star/delta to which $12 -$pulse convertor is connected to supply arc furnaces. Both 12 pulse converter and arc furnace are nonlinear loads. As $12 -$pulse convertor is fed from star/delta transformer with 30° phase shift between their secondary, the 5^{th} and 7^{th}harmonics are eliminated. The typical order of harmonics in 12 pulse converter is $11^{th}, 13^{th}, 23^{th}and25^{th}$. The nature of current in arc furnace changes with cycle to cycle. During early melt, the arc is extremely unstable and contains even order harmonics like2^{nd}and4^{th}. With stable arc operation, small amount of 3^{rd} and 5^{th} harmonics. But does not appear at the primary side of the transformer.

The harmonic measurement is done at both high tension (H.T.) and low tension (L.T.) side. To study the effect of harmonics generated due to nonlinear load at L.T. side on H.T. side, the readings of H.T. side are considered here. As the furnace load is balanced load, harmonic readings of any of the phase can be considered, hence focused on phase B readings only.

The harmonic voltage magnitudes and phase angles up-to 15^{th} order are noted as shown in Table I.

The $r.m.s.$value of phase voltage and current for B-phase is $17709 Volts$ and $44.040A$ respectively. Also, $\%V_{thd}$and $\%I_{thd}$ values observed as 2.39% and 5.08% respectively.

The harmonic active powers are calculated using equation (16) as shown in Table II.

From Table II, it is observed that active powers calculated for $6^{th}, 8^{th}, 9^{th}, 11^{th}, 12^{th}$and 13^{th}harmonics are negative and that for the other harmonics are positive. This indicates that the above mentioned harmonic active powers are flowing from load side to the source side and confirms that the load is source of these harmonics. The other harmonics like $3^{rd}, 5^{th}$and 10^{th} are produced at the utility side and propagating towards the load.

A) Apparent Power calculations as per Proposed Method

From equations (20),(21),(23)and(24), the four power components are calculated. To get the total apparent power, the vector addition of all the powers is done as given in Table III.

Table.1 Harmonic Voltages and Currents at B-phase

Sr. No.	Harmonic Order	Harmonic Phase Voltages		Harmonic Phase Currents	
		Magnitude (Volts)	Angle (deg.)	Magnitude (Volts)	Angle (deg.)
1	1	17700	195	44.002	181
2	2	10.62	177	0.12	117
3	3	77.88	140	0.23	149
4	4	19.47	119	0.04	135
5	5	196.47	80	0.13	52
6	6	10.62	121	0.03	217
7	7	40.71	360	0.03	8
8	8	17.7	102	0.04	287
9	9	21.44	169	0.1	19
10	10	35.4	151	0.27	303
11	11	131	71	1.79	287
12	12	44.25	187	0.33	95
13	13	60.18	43	0.64	280
14	14	3.54	156	0.008	164
15	15	46.02	125	0.1	47

Table.2 3-Phase harmonic active powers obtained

Sr. No.	Harmonic Order	3-phase Harmonic Active Power
1	1	P1=2339.846W
2	2	P2=1.91149W
3	3	P3=53.0755W
4	4	P4=2.24577W
5	5	P5=67.6542W
6	6	P6=0.0999W
7	7	P7=3.6282W
8	8	P8=-2.1158W
9	9	P9=-5.5702W
10	10	P10=-25.3173W
11	11	P11= -569.1191W
12	12	P12= -1.5288W
13	13	P13= -62.930W
14	14	P14= 0.00.0841W
15	15	P15= 2.8703W

B. Apparent Power calculations as per IEEE Standard 1459-2010

From equations(10),(13),(14),(15)and(18), all components of apparent power as per IEEE Standard1459-2010are calculated which are orthogonal to each other. Therefore the vector addition of them gives the total apparent power. The comparison of values obtained by both the methods is given in Table III.

Table.3 Comparison of Power COMPONENTS obtained by different methods

Sr. No.	Proposed Method	IEEE Standard 1459-2010	Derived by cross products
1	$\bar{P}^2$=5.139744354x10^{12}	${S_{11}}^2$=5.459253069x10^{12}	${S_{11}}^2$=5.459253069x10^{12}
		$S_{11} = 2336.504KVA$	$S_{11} = 2336.504KVA$
2	$\tilde{P}^2$=4.6551x10^8	${D_I}^2$=11.01x10^9 ${D_V}^2$=12.8643x10^8	${D_I}^2$=11.01x10^9 ${D_V}^2$=12.8643x10^8
		$D_I = 104.928KVAR$ $D_V = 35.866KVAR$	$D_I = 104.928KVAR$ $D_V = 35.866KVAR$
3	$\bar{Q}^2$=3.195092352x10^{11}	$S_H = 2.8367967KVA$	$S_H = 2.8367967KVA$
4	$\tilde{Q}^2$=11.8329x10^9	$S_N = 110.92476KVA$	${D_C}^2$=20.7444x10^{5}VA
5	S=2339.134883KVA	S=2339.135576KVA	S=2339.134903KVA

From Table III, it is observed that the apparent powers calculated by both the methods are approximately same. Also, one important observation is that, the squared total oscillatory power calculated by the proposed method is given by equation(26),

$$\tilde{P}^2 + \tilde{Q}^2 = 1.229841x10^{10}VA^2 \qquad (26)$$

The squared total distortion power that includes current distortion power, voltage distortion power and harmonic cross product power is given by equation(27),

$${D_I}^2 + {D_V}^2 + {S_H}^2 = {S_N}^2 = 1.229850x10^{10}VA^2 \qquad (27)$$

From equations (26)and(27), it is clear that these two values are almost equal. Thus, the total three phase oscillating power balances the total three phase distortion power. The actual value of three phase apparent power is calculated using $r.m.s.$ voltages and currents as,

$$S = 3xV_{rms}xI_{rms} = 2339.711KVA \qquad (28)$$

The small difference between the actual value and the calculated value is due to consideration of harmonics up-to 15^{th} order only and also neglecting the presence of small amount of even harmonics and inter-harmonics present in the system.

By the proposed method, the apparent power is first decomposed in three phase real power and three phase imaginary power and then both are further decomposed into average powers and oscillating powers. Due to this, it is possible to separate the average and oscillatory powersfor the steady state analysis.

V.CONCLUSIONS

The equations for the three phase power components for a balanced three phase distorted system are developed. The decomposition of total three phase apparent power is done as average part and oscillating part of power. To validate the equations, a case study of distorted system is considered and the total apparent three phase power is calculated after obtaining all power components. Also, the power components are calculated as per IEEE Standard1459-2010. It is observed that the apparent powers calculated by both the methods are equal and also matching with the actual value calculated from the r.m.s. voltages and currents. Another important observation is that, the total three phase oscillating power is equal to the total three phase distortion power. By the method proposed, the separation of real and imaginary powers as average and oscillatory parts is possible. The equations obtained can be further used in power balance equations of load flow analysis under steady state conditionin harmonic environment and also for compensation techniques for harmonic mitigation

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