t-NORM (δ,γ)-FUZZY BI-IDEAL OF A NEAR-RING

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Abstract
In this paper we introduce the notion of a t-norm (δ,γ)-Fuzzy bi-ideal of a near-ring and obtain the characterization of a bi-ideal in terms of a t-norm (δ,γ)-Fuzzy bi-ideal of a near-ring. We establish that every t-norm (δ,γ)-Fuzzy left (resp. right) N-subgroup or t-norm (δ,γ)-Fuzzy left (resp. right) ideal of a near-ring is a t-norm (δ,γ)-Fuzzy-fuzzy bi-ideal of a near-ring. But the converse is not neceassrily true. Further, we discuss the properties of t-norm (δ,γ)-Fuzzy bi-ideal of a near-ring.

Key words and phrases: t-norm (δ,γ)-Fuzzy sets, t-norm (δ,γ)-Fuzzy sublattices, t-norm (δ,γ)-Fuzzy subnear-ring, fuzzy two-sided N-subgroup. t-norm (δ,γ)-Fuzzy ideal, and t-norm (δ,γ)-Fuzzy bi-ideal.

1. Introduction

The notions of fuzzy ideals were introduced by S-Abou-Zaid in 1991[8,1]. The notion of fuzzy subgroup was introduced by A. Rosenfeld [5] in his pioneering paper. Subsequently the definition of fuzzy subgroup was generalized by Negoita and Ralescu [6]. Fuzzy ideals of a ring were first introduced by Liu [14]. T. Ali and A.K. Ray [7] studied the concepts of fuzzy sublattices and fuzzy ideals of a lattice. The notions of fuzzy subnear-ring, fuzzy ideal and fuzzy R-subgroup of a near-ring were introduced by Salah Abou-Zahid [8] and it has been studied by several authors [11,12,3,4]. We introduce the notion of a (δ,γ)-fuzzy bi-ideal of a near-ring and obtain the characterization of a bi-ideal in terms of a (δ,γ)-fuzzy bi-ideal of a near-ring. We establish that every (δ,γ)-fuzzy left (resp. right) N-subgroup or (δ,γ)-fuzzy left (resp. right) ideal of a near-ring is a (δ,γ)-fuzzy bi-ideal of a near-ring. But the converse is not neceassrily true. Further, we discuss the properties of (δ,γ)-fuzzy bi-ideal of a near-ring and also, we prove a correspondence theorem between the families of fuzzy ideals of two homomorphic lattices. This is an extension of the result of M. J. Rani [10] and T. Manikantan [9].

2. Preliminaries

In this section We recall some definitions and results that will be needed in the sequel. The interval $[0,1]$ is a lattice and this entity $([0,1],\leq)$ is denoted by I.

Definition 2.1 [16] A triangular norm, t-norm is a function $t:[0,1]\times[0,1]\to[0,1]$ satisfying, for each $a,b,c,d,\in[0,1]$, the following conditions:

(i) $t(0,0)=0, t(a,1)=a$; (ii) $t(a,b)\leq t(c,d)$, whenever $a\leq c, b\leq d$;
(iii) $t(a,b)=t(b,a)$; and (iv) $t(t(a,b),c)=t(a,t(b,c))$.

Definition 2.2 [10] A fuzzy subset μ of X is said to be a fuzzy sublattice of X if $\forall\, x,y\in X$, (i) $\mu(x\vee y)\geq\mu(x)\wedge\mu(y)$, (ii) $\mu(x\wedge y)\geq\mu(x)\wedge\mu(y)$.

Definition 2.3[10] Let $\mu \in I^X$, then μ is called a fuzzy ideal of X if $\forall x, y \in X$, $(I_1),\ \mu(x \vee y) \geq \mu(x) \wedge \mu(y),\quad (I_2),\ \mu(x \wedge y) \geq \mu(x) \wedge \mu(y).$
If I_2 holds, then $\mu(x \wedge y) \geq \mu(x) \wedge \mu(y)$. Thus by I_1 and $I_2, \mu \in FL(X)$, (i.e) a fuzzy ideal of X is fuzzy sublattice of X.

Definition 2.4 [8] A fuzzy sub set A of N is called a fuzzy subnear-ring of N if $\forall x, y \in N$, (i) $A(x-y) \geq min\{A(x), A(y)\}$, (ii) $A(xy) \geq min\{A(x), A(y)\}$.

Definition 2.5 [5] A fuzzy sub set A of a group $(G,+)$ is said to be a fuzzy subgroup of G if $\forall x, y \in G$, (i) $A(x+y) \geq min\{A(x), A(y)\}$, (ii) $A(-x) = A(x)$, or equivalently $A(x-y) \geq min\{A(x), A(y)\}$. If A is a fuzzy subgroupof a group G, then $A(0) \geq A(x \forall x \in G.)$

Definition 2.6 [8] A fuzzy sub set A of N is said to be a fuzzy two-sided N-subgroup of N if (i) A is a fuzzy subgroup of $(N,+)$, (ii) $A(xy) \geq A(x)\ \forall x, y \in N$, (iii) $A(xy) \geq A(y)\ \forall x, y \in N$. If A satisfies (i),(ii) then A is called a fuzzy right N-subgroup of N. If A satisfies (i) and (iii), then A is called a fuzzy left N-subgroup of N.

Definition 2.7 [8] A fuzzy sub set A of N is said to be a fuzzy ideal of N if

(i) A is a fuzzy subnear-ring of N, (ii) $A(y+x-y) = A(x)\ \forall x, y \in N$,

(iii) $A(xy) \geq A(y)\ \forall x, y \in N$. (iv) $A(a(b+i)-ab) \geq A(i)\ \forall a, b, i \in N$. A fuzzy subset with (i),(ii) and (iii) is called a fuzzy right ideal of N whereas a fuzzy subset with (i),(ii) and (iv) is called a fuzzy left ideal of N.

3. t-norm (δ, γ)-fuzzy bi-ideals of near-rings

Based on the notion of (λ, μ)-fuzzy ideals introduced by B. You [13]. In this section we introduce the notion of a t-norm (δ, γ)-fuzzy bi-ideal of a near-ring and obtain some of its characterizations and properties. In the following discussion, we always assume that $0 \leq \delta < \gamma \leq 1$.

Definition 3.1 A fuzzy sub set A of a group $(G,+)$ is said to be a t-norm (δ, γ)-fuzzy subgroup of G if $\forall x, y \in G$, (i) $A(x+y) \vee \delta \geq t(A(x), A(y), \gamma)$, (ii) $A(-x) \vee \delta = t(A(x), \gamma)$, or equivalently $A(x-y) \vee \delta \geq t(A(x), A(y), \gamma)$. If A is a fuzzy subgroupof a group G, then $A(0) \vee \delta \geq t(A(x), \gamma) \forall x \in G.$

Definition 3.2 A fuzzy sub set A of N is called a t-norm (δ, γ)-fuzzy subnear-ring of N if $\forall x, y \in N$, (i) $A(x-y) \vee \delta \geq t(A(x), A(y), \gamma)$, (ii) $A(xy) \vee \delta \geq t(A(x), A(y), \gamma)$.

Definition 3.3 A fuzzy sub set A of N is said to be a t-norm (δ, γ)-fuzzy two-sided N-subgroup of N if (i) A is a t-norm (δ, γ)-fuzzy subgroup of $(N,+)$, (ii) $A(xy) \vee \delta \geq t(A(x), \gamma)\ \forall x, y \in N$, (iii) $A(xy) \vee \delta \geq t(A(y), \gamma)\ \forall x, y \in N$. If A satisfies (i),(ii) then A is called a t-norm (δ, γ)-fuzzy right N-subgroup of N. If A satisfies (i) and (iii), then A is called a t-norm (δ, γ)-fuzzy left N-subgroup of N.

Definition 3.4 A fuzzy sub set A of N is said to be a t-norm (δ, γ)-fuzzy ideal of N if (i) A is a t-norm (δ, γ)-fuzzy subnear-ring of N, (ii) $A(y+x-y) \vee \delta = t(A(x), \gamma)\ \forall x, y \in N$, (iii) $A(xy) \vee \delta \geq t(A(y), \gamma)\ \forall x, y \in N$. (iv) $A(a(b+i)-ab) \vee \delta \geq t(A(y), \gamma)\ \forall a, b, i \in N$. A fuzzy subset with (i),(ii) and (iii) is called a t-norm (δ, γ)-fuzzy right ideal of N whereas a fuzzy subset with (i), (ii) and (iv) is called a t-norm (δ, γ)-fuzzy left ideal of N.

Definition 3.5 Let A and B be two fuzzy subsets of N. Wedefine a fuzzy subset $A*B$ of N by $((A*B)(x))\vee\delta=\begin{cases} sup_{x=a(b+i)-ab} min\{(A(a),A(b),B(i))\wedge\theta\} if x=a(b+i)-ab, a,b,i\in N \\ 0 \quad \text{otherwise}\end{cases}$

where $x\in N$. Note that if N is zero-symmetric and $A(o)\vee\delta\geq t(A(x),\gamma)\ \forall\ x\in N$, then $A\circ B\subseteq A*B.$

Definition 3.6 A fuzzy subgroup A of N is called a t-norm (δ,γ)-fuzzy bi-ideal of N if $((A\circ N\circ A)\cap((A\circ N)*A))\vee\delta\subseteq t(A,\gamma).$

Example 3.7 Let $N=0,a,b,c$ be the near-ring with $(N,+)$ as the Klein's four group and $(N,.)$ as defined below (Scheme 15: (0,13,0,13) See [1], p.408 [15]).

+	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

.	0	a	b	c
0	0	0	0	0
a	0	b	0	b
b	0	0	0	0
c	0	b	0	b

Define a t-norm (δ,γ)-fuzzy subset $A:N\rightarrow[0,1]$ by
$A(0)=0.8, A(a)=0.3, A(b)=0.6, A(c)=0.3$ and $\delta=0.1$ and $\gamma=0.9.$ Then
$(A\circ N\circ A)(0)=0.8, (A\circ N\circ A)(a)=0, (A\circ N\circ A)(b)=0,$
$(A\circ N\circ A)(c)=0, (A\circ N)*A)(0)=0.8, (A\circ N)*A)(a)=0,$
$(A\circ N)*A)(b)=0, (A\circ N)*A)(c)=0,$
and so A is a t-norm (δ,γ)-fuzzy bi-ideal of N.

Theorem 3.8 Let $\{A_i : i\in J\}$ be any family of t-norm (δ,γ)-fuzzy bi-ideals of N. Then $A=\cap_{i\in J}A_i$ is a t-norm (δ,γ)-fuzzy bi-ideal of N, Where J be an index set.

Proof. By theorem 3.4 of [9], A is a t-norm (δ,γ)-fuzzy subgroup of N. Now for all $x\in N$. since $A=\cap_{i\in J}A_i\subseteq A_i$, for every $i\in J$, we have

$$(((A\circ N\circ A)\cap((A\circ N)*A)(x)))\vee\delta\leq t((((A_i\circ N\circ A_i)\cap((A_i\circ N)*A_i)(x))),\gamma)$$

(Since A_i is a t-norm (δ,γ)-fuzzy bi-ideal of N)

$\leq t(A_i(x),\gamma)$ for every $i\in J$.

It follows that

$$(((A\circ N\circ A)\cap((A\circ N)*A)(x)))\vee\delta\leq inf\{t(A_i(x),\gamma): i\in J\}=t(\langle\cap_{i\in J}A_i\rangle(x),\gamma)=t(A(x),\gamma).$$

Thus $((A\circ N\circ A)\cap((A\circ N)*A))\vee\delta\subseteq t(A,\gamma)$. So A is a t-norm (δ,γ)-fuzzy bi-ideal of N.

Theorem 3.9 Let A be a t-norm (δ,γ)-fuzzy subgroup of N. If $(A\circ N\circ A)\vee\delta\subseteq t(A,\gamma),$ then A is a t-norm (δ,γ)-fuzzy bi-ideal of N.

Proof. Assume that A is a t-norm (δ,γ)-fuzzy subgroup of N such that $(A\circ N\circ A)\vee\delta\subseteq t(A,\gamma).$ For all $x\in N$, we have

$$(((A\circ N\circ A)\cap((A\circ N)*A)(x)))\vee\delta=t((A\circ N\circ A)(x),(A\circ N)*A)(x),\gamma)\leq t((A\circ N\circ A)(x),\gamma)\leq t((A(x)),\gamma)$$

Therefore $((A\circ N\circ A)\cap((A\circ N)*A))\vee\delta\subseteq t(A,\gamma)$. Hence A is a t-norm (δ,γ)-fuzzy bi-ideal of N.

Theorem 3.10 Let N be a zero-symmetric near-ring. If A is a t-norm (δ,γ)-fuzzy bi-ideal of N, then $(A\circ N\circ A)\vee\delta\subseteq t(A,\gamma)$.

Proof. Assume that A is a t-norm (δ,γ)-fuzzy bi-ideal of N. Then , we have $((A\circ N\circ A)\cap((A\circ N)*A))\vee\delta\subseteq t(A,\gamma)$. Since A is a fuzzy subgroup of N, $A(0)\vee\delta\geq t(A(x),\gamma)\ \forall\ x\in N.$ and since $N(x)=1\ \forall\ x\in N,$ we have $((A\circ N)(0))\vee\delta\geq t((A\circ N)(x),\gamma)\ \forall\ x\in N$. since N is zero-symmetric, $((A\circ N)\circ A)\vee\delta\subseteq t(((A\circ N)*A),\gamma)$. Then it is clear that $((A\circ N\circ A)\cap((A\circ N)*A))\vee\delta\subseteq t(A,\gamma)$.

Theorem 3.11 Let I be a non-empty subset of N and K_1 be a t-norm (δ,γ)-fuzzy subset of N. Then the following conditions are equivalent:

(i) I is a t-norm (δ,γ)-fuzzy bi-ideal of N,

(ii) K_1 is a t-norm (δ,γ)-fuzzy bi-ideal of N.

Proof. First assume that I is a t-norm (δ,γ)-fuzzy bi-ideal of N. Let a be any element of N. If $a\in I$, then $K_1(a)\vee\delta\geq((K_1\circ N\circ K_1)\cap((K_1\circ N)*K_1))(a)\vee\delta\leq t(1,\gamma)$. If $a\notin I$, then $K_1(a)\vee\delta=t(0,\gamma)$ on the otherhand assume that

$((K_1\circ N\circ K_1)\cap((K_1\circ N)*K_1))(a)\vee\delta=t(1,\gamma)$,

that is, $min\{(K_1\circ N\circ K_1)(a)((K_1\circ N)*K_1))(a)\}\vee\delta=t(1,\gamma)$.Then

$(K_1\circ N\circ K_1)(a)\vee\delta=((K_1\circ N)\circ K_1)(a)\vee\gamma=t(sup_{a=pq}min\{(K_1\circ N),K_1(q)\},\gamma)$

$=t(sup_{a=pq}min\left\langle sup_{p=p_1p_2}min\{K_1(p_1),N(p_2)\},K_1(q)\right\rangle,\gamma)$

(since $N(x)=1,\forall x\in N$) $=t(sup_{a=pq}min\left\langle sup_{p=p_1p_2}min\{K_1(p_1),K_1(q)\}\right\rangle,\gamma)=1\rightarrow(1)$ and

$((K_1\circ N)*K_1)(a)\vee\delta=t(sup_{a=n(m+j)-nm}min\{(K_1\circ N)(n),(K_1\circ N)(m),(K_1(j)\},\gamma)$

$=t(sup_{a=n(m+j)-nm}min\{sup_{a=n_1n_2}K_1(n_1),sup_{m=m_1m_2}K_1(m_1),k_1(j)\},\gamma)=1\rightarrow(2)$

This implies that there exist elements $b,c,b_1,b_2,x,y,i,x_1,x_2,y_1,y_2$ in N with $a=bc=x(y+i)-xy,b=b_1,b_2,x=x_1x_2$ and $y=y_1y_2$ such that $K_1(b_1)=K_1(c)=K_1(x_1)=K_1(y_1)=K_1(i)=1.$ So $b_1,c,x_1,y_1,i\in I.$ Therefore $a=bc=(b_1b_2)c\in INI$ and $a=x(y+i)-xy=(x_1x_2)((y_1y_2)+i)-(x_1x_2)(y_1y_2)\in(IN)*I$ and so $a\in(INI\cap(IN)*I)\vee\delta\subseteq t(I,\gamma)$ which contradicts $a\notin I$. Thus $K_1(a)\vee\delta=((K_1\circ N\circ K_1)\cap((K_1\circ N)*K_1))(a)\vee\delta=t(0,\gamma)$ This shows that $((K_1\circ N\circ K_1)\cap((K_1\circ N)*K_1))\vee\delta=K_1\vee\delta$. By theorem 3.8 of [9], K_1 is a t-norm (δ,γ)-fuzzy subgroup of N. Hence K_1 is a t-norm (δ,γ)-fuzzy bi-ideal of N.

Conversely, assume that K_1 is a t-norm (δ,γ)-fuzzy bi-ideal of N. Let a be any element of $INI\cap(IN)*I$. Then there exist elements b,b_2,x,y,x_2,y_2 of N and elements c,b_1,x_1,y_1,i of I such that $a=bc=x(y+i)-xy,b=b_1,b_2,\ x=x_1x_2$ and $y=y_1y_2$. Now

$(K_1\circ N\circ K_1)(a)\vee\delta=t(sup_{a=pq}min\left\langle sup_{p=p_1p_2}min\{K_1(p_1),K_1(q)\}\right\rangle,\gamma)\geq t(min\{K_1(b_1),K_1(c)\},\gamma)=t\{1,1\}=1$

and so $(K_1\circ N\circ K_1)(a)\vee\delta=1$. By (2), we have

$((K_1\circ N)*K_1)(a)\vee\delta=t(sup_{a=n(m+j)-nm}min\{sup_{a=n_1n_2}K_1(n_1),sup_{m=m_1m_2}K_1(m_1),k_1(j)\},\delta)$

$\geq t\{K_1(x_1),K_1(y_1),K_1(i)\}=t\{1,1,1\}=1$

and so $(K_1\circ N\circ K_1)(a)\vee\delta=1$.

Therefore

$t(K_1(a)\vee\delta\geq((K_1\circ N\circ K_1)\cap((K_1\circ N)*K_1))(a),\gamma)=t(min\{(K_1\circ N\circ K_1)(a),((K_1\circ N)*K_1)(a)\},\gamma)=1$

Thus $a \in I$. So $INI \cap (IN)*I \subseteq I$. By theorem 3.8 of [9], I is a subgroup of N. This shows that I is a t-norm (δ,γ)-fuzzy bi-ideal of N.

Theorem 3.12 Every t-norm (δ,γ)-fuzzy right N-subgroup of N is a t-norm (δ,γ)-fuzzy bi-ideal of N.

Proof. Let A be a t-norm (δ,γ)-fuzzy right N-subgroup of N. Choose $a,b,c,x,y,i,b_1,b_2,x_1,x_2,y_1,y_2$ in N such that $a = bc = x(y+i) - xy, b = b_1,b_2, x = x_1x_2$ and $y = y_1y_2$. Then

$$(((A\circ N\circ A)\cap((A\circ N)*A))(a))\vee\delta$$
$$= t\{((A\circ N)\circ A)(a),((A\circ N)*A)(a),\gamma\} = t(min\{sup_{a=bc}min\{(A\circ N)(b),A(c)\},$$
$$((A\circ N)*A)(x(y+i)-xy)\},\gamma) = t(min\{sup_{a=bc}min\left[sup_{b=b_1b_2}min\{A(b_1),N(b_2)\},A(c)\right],$$
$$((A\circ N)*A)(x(y+i)-xy)\},\gamma).$$

(Since

$N(z) = 1\,\forall\, z \in N.) = t(min\{sup_{a=bc}min\left[sup_{b=b_1b_2}A(b_1),A(c)\right],((A\circ N)*A)(x(y+i)-xy)\},\gamma).$

(Since A is a t-norm (δ,γ)-fuzzy right N-subgroup of N, we have $A(bc)\vee\delta = A((b_1b_2)c)\vee\delta = A(b_1(b_2c))\vee\delta \ge t(A(b_1),\gamma)$

$$\le t(min\{sup_{a=bc}min\{A(bc),N(c)\},N(x(y+i)-xy)\},\gamma)\,\mathrm{mod}*3.8cm$$
$$= t(min\{A(bc),N(x(y+i)-xy)\},\gamma)\,\mathrm{mod}*3.8cm = t(A(bc),\gamma) = t(A(a),\gamma).$$

Thus $((A\circ N\circ A)\cap((A\circ N)*A))\vee\lambda \subseteq t(A,\gamma)$.

Hence A is a t-norm (δ,γ)-fuzzy bi- ideal of N.

Theorem 3.13 Every t-norm (δ,γ)-fuzzy left N-subgroup of N is a t-norm (δ,γ)-fuzzy bi-ideal of N.

Proof. Let A be a t-norm (δ,γ)-fuzzy left N-subgroup of N. Choose $a,b,c,x,y,i,c_1,c_2,x_1,x_2,y_1,y_2$ in N such that $a = bc = x(y+i) - xy, c = c_1,c_2, x = x_1x_2$ and $y = y_1y_2$. Then

$$(((A\circ N\circ A)\cap((A\circ N)*A))(a))\vee\delta$$
$$= min\{((A\circ N)\circ A)(a),((A\circ N)*A)(a),\gamma\}$$
$$= t(min\{sup_{a=bc}min\{A(b),(N\circ A)(c)\},((A\circ N)*A)(x(y+i)-xy)\},\gamma)$$
$$= t(min\{sup_{a=bc}min\left[A(b),sup_{c=c_1c_2}min\{N(c_1),A(c_2)\}\right],((A\circ N)*A)(x(y+i)-xy)\},\gamma)$$
$$= t(min\{sup_{a=bc}min\left[A(b),sup_{c=c_1c_2}A(c_2)\right],((A\circ N)*A)(x(y+i)-xy)\},\gamma)$$

(Since A is a t-norm (δ,γ)-fuzzy left N-subgroup of N, we have $A(bc)\vee\delta = A(b(c_1c_2))\vee\delta = A((bc_1)c_2)\vee\delta \ge t(A(c_2),\gamma)$

$$\le t(min\{sup_{a=bc}min\{N(b),A(bc)\},N(x(y+i)-xy)\},\gamma)\,\mathrm{mod}*3.8cm = t(A(bc),\gamma) = t(A(a),\gamma).$$

Thus $((A\circ N\circ A)\cap((A\circ N)*A))\vee\delta \subseteq t(A,\gamma)$.

Hence A is a t-norm (δ,γ)-fuzzy bi-ideal of N.

Theorem 3.14 Every t-norm (δ,γ)-fuzzy two sided N-subgroup of N is a t-norm (δ,γ)-fuzzy bi-ideal of N.

Proof. The proof is straightforward from Theorem (3.12) and Theorem (3.13).

Theorem 3.15 Every t-norm (δ,γ)-fuzzy left ideal of N is a t-norm (δ,γ)-fuzzy bi-ideal of N.

Proof. Let A be a t-norm (δ,γ)-fuzzy left ideal of N. Choose $a,b,c,x,y,i,b_1,b_2,x_1,x_2,y_1,y_2$ in N such that $a = bc = x(y+i)-xy, b = b_1,b_2, x = x_1x_2$ and $y = y_1y_2$. Then

$$(((A\circ N\circ A)\cap((A\circ N)*A))(a))\vee\delta = min\{((A\circ N)\circ A)(a),((A\circ N)*A)(a),\gamma\}$$
$$= t(min\{sup_{a=bc}min\{(A\circ N)(b),A(c)\},((A\circ N)*A)(x(y+i)-xy)\},\gamma)$$
$$= t(min\{sup_{a=bc}min\{(A\circ N)(b_1b_2),A(c)\},sup_{a=x(y+i)-xy}min\{(A\circ N)(x),(A\circ N)(y),A(i)\}\},\gamma)$$

(Since $A\circ N\subseteq N$ and since A is a (δ,γ)-fuzzy left ideal of $N, A(x(y+i)-xy)\vee\delta\geq t(A(i),\gamma))$

$$t(\leq min\{sup_{a=bc}min\{(N)(b_1b_2),N(c)\},sup_{a=x(y+i)-xy}min\{(N)(x),(N)(y),A(x(y+i)-xy)\}\},\gamma)$$
$$= t(A(x(y+i)-xy)\}\},\gamma)\bmod{*}2.5cm = A(a)\wedge\gamma.$$

Therefore $((A\circ N\circ A)\cap((A\circ N)*A))\vee\delta\subseteq t(A,\gamma)$. Hence A is a t-norm (δ,γ)-fuzzy bi-ideal of N.

Theorem 3.16 Every t-norm (δ,γ)-fuzzy right ideal of N is a t-norm (δ,γ)-fuzzy bi-ideal of N. **Proof.** The proof is similar to that of Theorem (3.12).

Theorem 3.17 Every t-norm (δ,γ)-fuzzy ideal of N is a t-norm (δ,γ)-fuzzy bi-ideal of N.
Proof. The proof is straightforward from Theorem (3.15) and Theorem (3.16).

Example 3.18 Let $N = 0,a,b,c$ be the near-ring with $(N,+)$ as the Klein's four group and $(N,.)$ as defined below (Scheme 18: (7,7,1,1) See [1], p.408[15]).

+	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

.	0	a	b	c
0	0	0	0	0
a	a	a	a	a
b	0	0	b	b
c	a	a	c	c

Define a t-norm (δ,γ)-fuzzy subset $A: N\to[0,1]$ by
$A(0) = 0.9, A(a) = 0.4, A(b) = 0.4, A(c) = 0.7$ and $\delta = 0.1$ and $\gamma = 0.95$. Then
$(A\circ N\circ A)(0) = 0.9, (A\circ N\circ A)(a) = 0.7,$
$(A\circ N\circ A)(b) = 0.4, (A\circ N\circ A)(c) = 0.7, (A\circ N)*A)(0) = 0.9,$
$(A\circ N)*A)(a) = 0, (A\circ N)*A)(b) = 0.7, (A\circ N)*A)(c) = 0.$
Therefore A is a t-norm (δ,γ)-fuzzy bi-ideal of N. Since $A(a)\vee\delta = A(ca)\vee\delta < t(A(c),\gamma)$ and $A(a)\vee\delta = A(a0)\vee\delta < t(A(0),\gamma), A$ is not a t-norm (δ,γ)-fuzzy two-sided N-subgroup of N. Since $A(a)\vee\delta = A(c0)\vee\delta < t\{A(c),A(0),\gamma\}, A$ is not a t-norm (δ,γ)-fuzzy subnear-ring of N and so A is not a t-norm (δ,γ)-fuzzy ideal of N.

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