EFFECTS OF RADIATION ON MHD FREE CONVECTIVE FLOW OF VISCOUS INCOMPRESSIBLE FLUID PAST AN INFINITE VERTICAL POROUS PLATE WITH SORET AND DUFOUR EFFECTS

[1, 2] B. Prabhakar Reddy and [1] Paul Majani Matao
[1] Department of Mathematics, CNMS, The University of Dodoma, P. Box. No. 338, Dodoma, Tanzania
[2] Department of Mathematics, Geethanjali College of Engineering & Technology, Keesara, Medchal – 501301, Telangana, India

Abstract*— Numerical investigation for the effects of radiation on an unsteady MHD free convection flow of an incompressible, electrically conducting, viscous Newtonian fluid past an infinite vertical porous plate with Soret and Dufour effects taking Viscous and Darcy resistance terms into account and the constant permeability of the medium has been presented. The fluid is considered as a gray, absorbing-emitting but non-scattering medium. The Rosseland approximation in the energy equation is used to describe the radiative heat flux for optically thick fluid. The Galerkin finite element method has been applied to solve the dimensionless governing equations of the flow. The influence of the physical parameters involved in the problem under the investigation on the velocity, temperature and concentration fields within the boundary layer has been presented graphically and then discussed.

***Keywords*— MHD, free convection, Soret number, Dufour number, radiation parameter.**

I. INTRODUCTION

The study of heat and mass transfer free convection flows through a porous medium under the effect of magnetic field have been attracted the interest of a number of researchers because of their possible applications in many branches of science and technology, such as its applications in transportation of cooling of re-entry vehicles and rocket boosters, cross-hatching on ablative surfaces and film vaporization in combustion chambers. On the other hand, if the entire system involving the polymer extraction process is placed in a thermally controlled environment, then the thermal radiation effect is significant. Muthucumaraswamy and Janakiraman [1] studied the MHD and radiation effects on moving isothermal vertical plate with variable mass diffusion. Shanker and Gnaneshwar [2] analyzed the radiation effects on MHD flow past an impulsively started infinite vertical plate through a porous medium with variable temperature and mass diffusion. Ahmed and Sarmah [3] presented the thermal radiation effects on a transient MHD flow with mass transfer past an impulsively fixed infinite vertical plate. Mukhopadhyay [4] analyzed the effects of thermal radiation on unsteady mixed convection flow and heat transfer over a porous stretching surface in porous medium. Rao and Reddy [5] studied the heat and mass transfer of an unsteady MHD natural convection flow of a rotating fluid past a vertical porous plate in the presence of radiative heat transfer by finite element method. Muthucumaraswamy and Sivakaumar [6] studied the MHD flow past a parabolic flow past an infinite isothermal vertical plate in the presence of thermal radiation and chemical reaction.

In all the above investigations, the effects of Soret (thermal-diffusion) and Dufour (diffusion-thermo) are not considered. But separation of the fluids with very light molecular weight as well as the medium molecular weight, the effects of Soret and Dufour are important. Some of the investigators and reported results for these flows of whom are Eckert and Drake [7], Dursunkaya and Worek [8], Anghel et. al [9] and Postelnicu [10] are worth mentioning. Alam and Rahman [11] presented the Dufour and Soret effects on MHD free convective heat and mass transfer flow past a vertical flat plate embedded in a porous medium. Alam et. al [12] investigated the Dufour and Soret effects on unsteady MHD free convection and mass transfer flow past a vertical porous plate in a porous medium. Dufour and Soret effects on steady MHD combined free-forced convective and mass transfer flow past a semi-infinite vertical plate were analyzed by Alam et. al [13]. Mansour et. al [14] investigated the effects of chemical reaction and thermal stratification on MHD free convective heat and mass transfer over a vertical stretching surface embedded in a porous media by considering Soret and Dufour effects. Vempati and Narayana [15] analyzed the Soret and Dufour effects on unsteady MHD flow past an infinite vertical porous plate with thermal radiation and oscillatory suction velocity. The effects of thermal diffusion and viscous dissipation on unsteady MHD free convection flow past a vertical porous plate under oscillatory suction velocity was investigated by Reddy [18]. Venkateswarlu et. al [17] analyzed the thermal diffusion and radiation effects on unsteady MHD free convection heat and mass transfer flow past a linearly accelerated vertical porous plate with variable temperature and mass diffusion.

The aim of the present work is to study the effects of radiation on unsteady MHD free convection flow past a vertical porous plate taking Viscous and Darcy resistance terms into account and constant permeability of the medium with Soret and Dufour effects. The dimensionless governing of the flow has been solved numerically by applying Galerkin finite element method, which is more economical from computational point of view. The effects of the physical parameters on the velocity, temperature and concentration fields are presented through the graphs and then discussed.

II. BASIC GOVERNING EQUATIONS

The unsteady MHD flow of an incompressible, electrically conducting, viscous Newtonian fluid past an infinite vertical porous plate taking Viscous and Darcy's resistance terms into account and constant permeability of the medium is considered. In the coordinate system, the $x'-$axis is taken along the plate in the upward direction and $y'-$axis is taken normal to the plate. The fluid is considered as a gray absorbing-emitting but non-scattering medium. A magnetic field of strength B_0 applied transversely to the direction of the flow. The magnetic Reynolds number is assumed to be very small so that induced magnetic field is neglected. The suction velocity normal to the plate is assumed to be a function of time i.e., $v' = -U_0$, where the minus sign indicates the suction directed towards the plate. Initially $(t' = 0)$, the plate and fluid are at same temperature $T_\infty^{'}$ and concentration $C_\infty^{'}$ at all points. Subsequently, $t' > 0$, the plate temperature rises to $T_w^{'}$ and the concentration level at the plate rises to $C_w^{'}$. Under the above assumptions the physical variables are the functions on y' and t' only. Assuming Boussinesq approximation and boundary layer approximation hold, the basic equations which govern the problem, are given by

$$\frac{\partial v'}{\partial y'} = 0 \tag{1}$$

$$\frac{\partial u'}{\partial t'} + v'\frac{\partial u'}{\partial y'} = g\beta(T' - T_\infty^{'}) + g\beta^*(C' - C_\infty^{'}) + \upsilon\frac{\partial^2 u'}{\partial y'^2} - \frac{\upsilon u'}{K'} - \frac{\sigma B_0^2 u'}{\rho} \tag{2}$$

$$\frac{\partial T'}{\partial t'} + v'\frac{\partial T'}{\partial y'} = \frac{k}{\rho c_p}\frac{\partial^2 T'}{\partial y'^2} - \frac{\partial q_r^*}{\partial y'} + \frac{D_m k_T}{C_s c_p}\frac{\partial^2 C'}{\partial y'^2} \tag{3}$$

$$\frac{\partial C'}{\partial t'} + v'\frac{\partial C'}{\partial y'} = D_M\frac{\partial^2 C'}{\partial y'^2} + \frac{D_M k_T}{T_m}\frac{\partial^2 T'}{\partial y'^2} \tag{4}$$

with the initial and boundary conditions:

$$\begin{aligned} &t' \le 0; u' = 0, T' = T_\infty^{'}, C' = C_\infty^{'} && \forall y' \\ &t' > 0; u' = 0, T' = T_w^{'}, C' = C_w^{'} && \text{at } y' = 0 \\ &\quad u' = 0, T' = T_\infty^{'}, C' = C_\infty^{'} && \text{as } y' \to \infty \end{aligned} \tag{5}$$

where

$u', v', T', C', \nu, g, \rho, T_\infty^{'}, T_w^{'}, C_\infty^{'}, C_w^{'}, c_p, k, D_M, T_m, \beta, \beta^*, q_r^*, B_0$ and t' are, respectively, fluid velocity in $x'-$direction, fluid velocity in $y'-$ direction, fluid temperature, fluid concentration, kinematic viscosity, acceleration due to gravity, fluid density, free stream temperature, surface temperature, free stream concentration, surface concentration, specific heat at constant pressure, thermal conductivity of the fluid, chemical molecular diffusivity, mean fluid temperature, volumetric coefficient of thermal expansion, volumetric coefficient of concentration expansion, radiative heat flux, uniform magnetic field and time.

The radiative heat flux q_r^* under the Rosseland approximation has the form

$$q_r^* = -\frac{4\sigma^*}{3k^*}\frac{\partial T'^4}{\partial y'} \tag{6}$$

where σ^* is the Stefan-Boltzmann constant and k^* is the mean absorption coefficient. It is assumed that the temperature differences within the flow are sufficiently small such that the term T'^4 is expressed as the linear function of temperature. Thus expanding T'^4 about $T_\infty^{'}$ using the Taylor series and neglecting higher order terms, one obtain

$$T'^4 \cong T'^4_\infty + 4T'^3_\infty(T' - T_\infty^{'}) \cong 4T'^3_\infty T' - 3T'^4_\infty$$

Eq. (6) gives

$$q_r^* = -\frac{16\sigma^* T'^3_\infty}{3k^*}\frac{\partial T'}{\partial y'} \tag{7}$$

From Eqs. (7) and (2), we arrive at the modified energy equation:

$$\frac{\partial T'}{\partial t'} + v'\frac{\partial T'}{\partial y'} = \frac{k}{\rho c_p}\frac{\partial^2 T'}{\partial y'^2} + \frac{16\sigma^* T'^3_\infty}{3k^*}\frac{\partial^2 T'}{\partial y'^2} + \frac{D_m k_T}{C_s c_p}\frac{\partial^2 C'}{\partial y'^2} \tag{8}$$

Let us introduce the following non-dimensional quantities.

$$\begin{aligned} &u = \frac{u'}{U_0^{'}}, y = \frac{y' U_0^{'}}{\nu}, t = \frac{t' U_0'^2}{\nu}, P_r = \frac{\mu c_p}{k}, S_c = \frac{\nu}{D_M}, \\ &M = \frac{\sigma B_0^2 \upsilon}{\rho U_0'^2}, K = \frac{U_0^2 K'}{\upsilon^2}, H = M + \frac{1}{K}, \\ &N = \frac{\beta^*(C_w^{'} - C_\infty^{'})}{\beta(T_w^{'} - T_\infty^{'})}, \theta = \frac{(T^{'} - T_\infty^{'})}{(T_w^{'} - T_\infty^{'})}, \phi = \frac{(C^{'} - C_\infty^{'})}{(C_w^{'} - C_\infty^{'})}, \\ &R = \frac{16\sigma^* T_\infty^{'3}}{3k^*}, F^* = \frac{P_r}{1+R}, D_u = \frac{D_m k_T (C_w^{'} - C_\infty^{'})}{C_s c_p \nu (T_w^{'} - T_\infty^{'})}, \\ &S_r = \frac{D_M k_T (T_w^{'} - T_\infty^{'})}{T_m \nu (C_w^{'} - C_\infty^{'})}, G_r = \frac{g\beta\nu(T_w^{'} - T_\infty^{'})}{U_0^3}. \end{aligned} \tag{9}$$

where $P_r, S_c, M, K, N, D_u, R, S_r$ and G_r are the Prandtl number, Schmidt number, magnetic parameter, permeability parameter, buoyancy ratio, Dufour number, radiation parameter, Soret number and Grashof number and the other symbols have their usual meaning.

Using (9) into Eqs. (2),(4),(5) and (8), we obtain the following governing equations in dimensionless form:

$$\frac{\partial u}{\partial t} - \frac{\partial u}{\partial y} = G_r(\theta + N\phi) + \frac{\partial^2 u}{\partial y^2} - Hu \tag{10}$$

$$\frac{\partial \theta}{\partial t} - \frac{\partial \theta}{\partial y} = \frac{1}{F^*}\frac{\partial^2 \theta}{\partial y^2} + D_u\left(\frac{\partial^2 \phi}{\partial y^2}\right) \tag{11}$$

$$\frac{\partial \phi}{\partial t} - \frac{\partial \phi}{\partial y} = \frac{1}{S_c}\frac{\partial^2 \phi}{\partial y^2} + S_r\left(\frac{\partial^2 \theta}{\partial y^2}\right) \tag{12}$$

The corresponding initial and boundary conditions:

$$\begin{aligned} &t \le 0; u = 0, \theta = 0, \phi = 0 && \forall y \\ &t > 0; u = 0, \theta = 1, \phi = 1 && \text{at } y = 0 \\ &\quad u = 0, \theta = 0, \phi = 0 && \text{as } y \to \infty \end{aligned} \tag{13}$$

.

III. SOLUTION OF THE PROBLEM

The dimensionless governing system of coupled non-linear partial differential equations of the flow $(10) - (12)$ are solved subject to the boundary conditions given in equation (13). However, whose exact or approximate solutions are difficult to obtain, if possible. So that, the Galerkin finite element method has been adopted for its solution, which is more economical from a computational point of view. The algorithm for Galerkin finite element method can be summarized by the following steps:

Step 1: Division of the whole domain into smaller elements of finite dimensions called "finite elements".

Step 2: Generation of the element equations using variational formulations.

Step 3: Assembly of element equations as obtained in step (2).

Step 4: Imposition of boundary conditions to the equations obtained in step (3).

Step 5: Solution of the assembled algebraic equations.

The assembled equations can be solved by any of the numerical technique viz. Gauss-Seidal iteration method. Here, the boundary condition $y \to \infty$ is approximated by $y_{\max} = 5$, which is sufficiently large for the velocity to approach convergence criterion. Numerical solutions for the velocity u, temperature θ and concentration ϕ are computed by using $C-$ program. To judge the convergence of method, computations are carried out by making small changes in time t and $y-$directions; no significant change was observed in the velocity, temperature and concentration. Hence, we conclude that the Galerkin finite element method is convergent and stable.

IV. RESULTS AND DISCUSSION

In order to get the physical insight flow, numerical results computed to study the effects of the physical parameters of interest on the velocity field, temperature field and concentration field. The obtained numerical results have been presented through the graphs 1 -14. During the numerical computations, the values of the Prandtl number are chosen $P_r = 0.71, 1.00$ and 7.00, which corresponds to air, electrolytic solution and water at $20^0 C$ and one atmosphere pressure and values of the Schmidt number are taken $S_c = 0.22, 0.60$ and 0.78 which corresponds to hydrogen, water-vapour and ammonia, respectively. The other physical parameters are considered as: $K = 1.0$, $R = 0.5$, $D_u = 0.03$, $S_r = 0.5$ at time $t = 0.5$. These values are kept as common in the entire investigation except variations in respective figures.

Figure 1 illustrates the effect of the Prandtl number P_r on the velocity field. It is observed that an increase in the Prandtl number decreases the fluid velocity due to large P_r has high viscosity and small thermal conductivity, which makes the fluid thick and causes a decrease in the fluid velocity. The effects of the Schmidt number S_c on the velocity field are presented in Fig. 2. It is clearly seen that the velocity of the fluid decreases with increasing values of S_c. This is due to the fact that increase of S_c leads to decrease of molecular diffusivity, which results in a decrease in the concentration and velocity boundary layer thickness. Figure 3 presents the influence of buoyancy ratio N on the velocity field. It can be clearly seen that the fluid velocity increases with increasing values of the buoyancy ratio in the boundary layer. Figure 4 illustrates the effect of the Dufour number D_u on the velocity field. It is noticed that an increase in the Dufour number from 0.03 to 0.5 and then 0.9, increases the fluid velocity. The effects of the magnetic parameter M on the velocity field are shown in Fig.5. It is observed that an increase in the magnetic field parameter tends to decrease the fluid velocity. Magnetic field may control the flow characteristics. Figure 6 shows the variation of the velocity profiles with dimensionless permeability parameter K. It is seen that the fluid velocity increases with increasing values of permeability parameter. Figure 7 demonstrates the effect of the radiation parameter R on the velocity field. It is noticed that the fluid velocity increases with increasing radiation parameter. The effects of Soret number S_r on the velocity field are shown in Fig.8. It is observed that the fluid velocity increases with increasing values of Soret number. The variation of the velocity field with thermal Grashof number G_r is presented in Fig. 9. It is observed that the fluid velocity increases with increasing thermal Grashof number.

Figure 10 shows the effect of the Prandtl number P_r on the temperature field. It can be seen that an increases in the Prandtl number tends to decreases the fluid temperature due to increase of P_r decreases the thermal conductivity of the fluid which results a decrease in the thermal boundary layer thickness. Figure 11 depicts the effect of radiation parameter R on the temperature field. It is observed that the temperature increases as the radiation parameter R increases due to the large R correspond to increased dominance of conduction over radiation thereby increasing the thickness of the thermal boundary layer. The effects of Dufour number D_u on the temperature field is shown in Fig.12. It can be is clearly seen that temperature increases with increasing values of Dufour number. Figure 13 displays the effect of the Schmidt number S_c on the concentration field. It is evident from this figure that with increasing value of the Schmidt number decreases in the concentration due to increase in the Schmidt number tends to a decrease of molecular diffusivity results a decrease of concentration boundary layer. Hence, the concentrations of the species is higher for small values of S_c and lower for large values of S_c. Figure 14 presents the effect of the Soret number S_r on the concentration field. It is noticed that there is a marked effect of increasing values of Soret number S_r on the concentration distribution in the boundary layer. This implies that increasing values of S_r tends to increase fluid concentration.

V. FIGURES

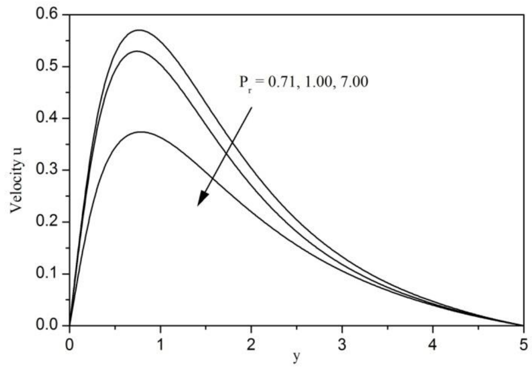

Fig. 1: Variation of the velocity field with P_r.

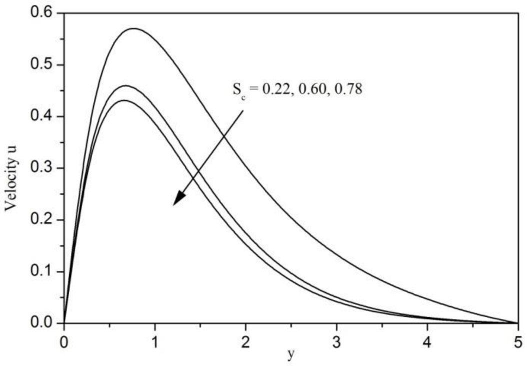

Fig. 2: Variation of the velocity field with P_r.

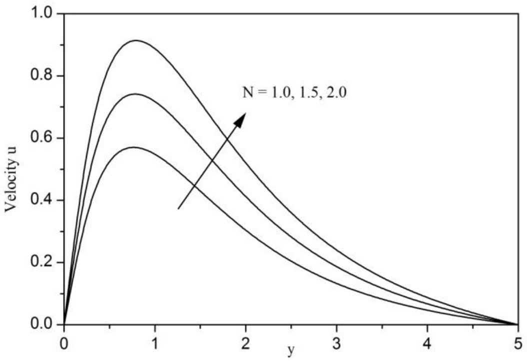

Fig. 3: Variation of the velocity field with N.

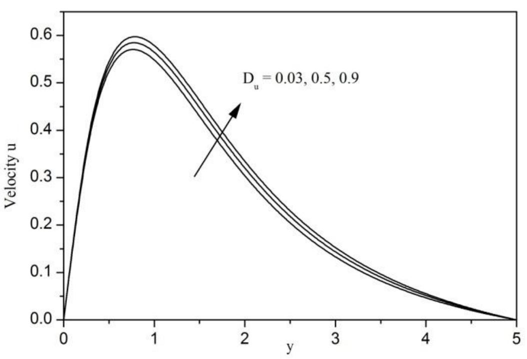

Fig. 4: Variation of the velocity field with D_u.

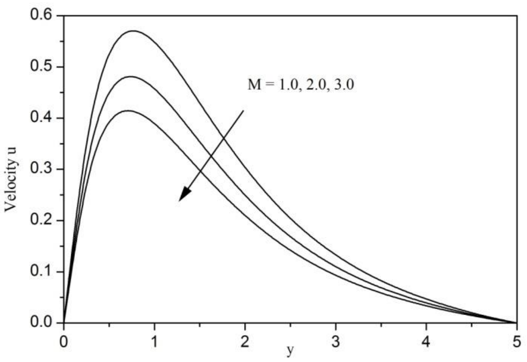

Fig. 5: Variation of the velocity field with M.

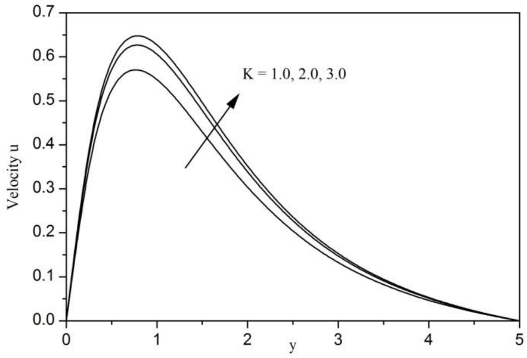

Fig. 6: Variation of the velocity field with K.

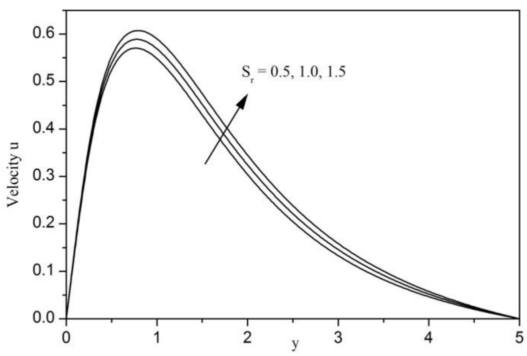

Fig. 7: Variation of the velocity field with S_c.

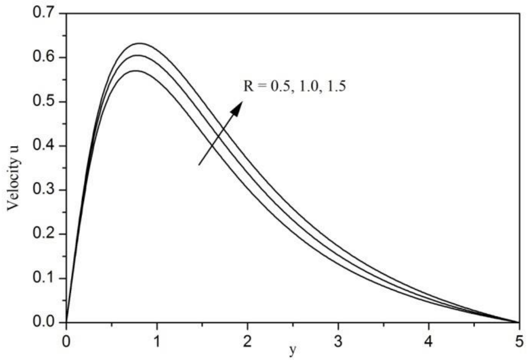

Fig. 8: Variation of the velocity field with R.

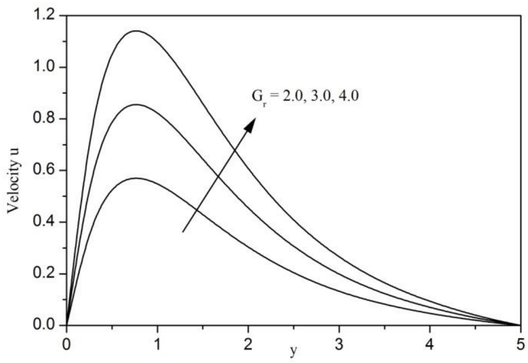

Fig. 9: Variation of the velocity field with G_r.

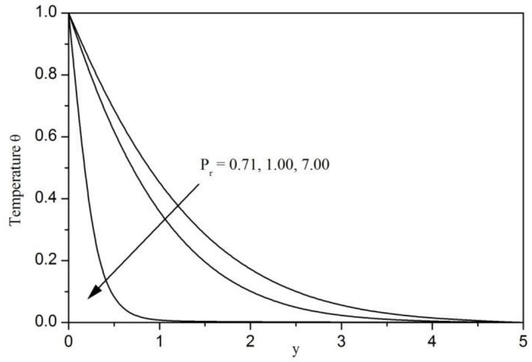

Fig. 10: Variation of the temperature field with P_r.

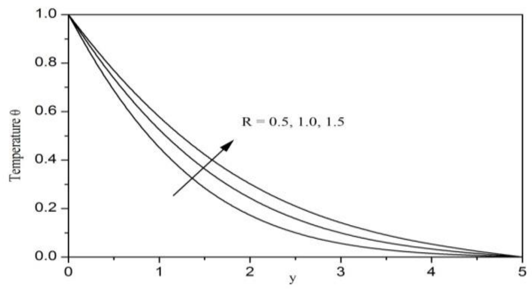

Fig. 11: Variation of the temperature field with R.

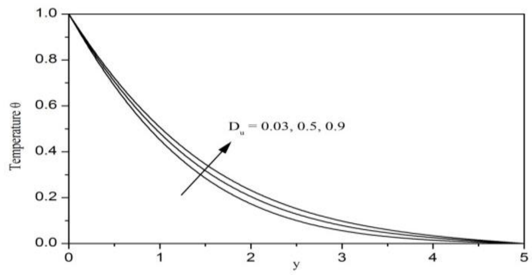

Fig. 12: Variation of the temperature field with D_u.

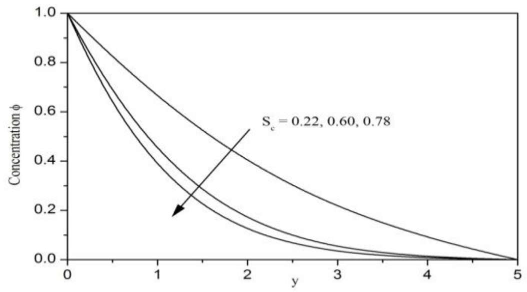

Fig. 13: Variation of the concentration field with S_c.

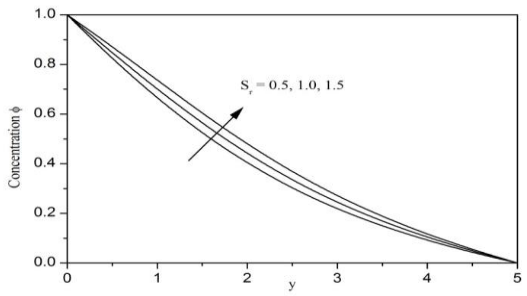

Fig. 14: Variation of the concentration field with S_r.

REFERENCES

[1] R. Muthucumaraswamy and Janakiraman, "MHD and radiation effects on moving isothermal vertical plate with varaiable mass diffusion", Theoret. Appl. Mech, vol.33, pp.17 – 29, 2006.

[2] R. T. Shanker and R. M. Gnaneshwar, "Radiation effects on MHD flow past an impulsively started infinite vertical plate through a porous medium with variable temperature and mass diffusion", J. Pure and Appl. Phys, vol.19, No. 3, pp.191 – 200, 2007.

[3] N. Ahmed and H. K. Sarmah, "Thermal radiation effects on a transient MHD flow with mass transfer past an impulsively fixed infinite vertical plate", Int. J. Appl. Math. and Mech., vol.5, No.5, pp.87 – 98, 2009.

[4] S. Mukhopadhyay, "The effects of thermal radiation on unsteady mixed convection flow and heat transfer over a porous stretching surface in porous medium", Int. J. Heat and Mass Transfer, vol.52, pp.3261 – 3265, 2009.

[5] J. Ananad Rao and B. Prabhakar Reddy, "Finite element analysis of heat and mass transfer of an unsteady MHD natural convection flow of a rotating fluid past a vertical porous plate in the presence of radiative heat trasnfer", J. Energy, Heat and Mass Transfer, vol. 32, pp. 223 – 241, 2010.

[6] R. Muthucumaraswamy and P. Sivakaumar, "MHD flow past a parabolic flow past an infinite isothermal vertical plate in the presence of thermal radiation and chemical reaction", Int. J. Appl. Mech. and Engg, vol.21, No.1, 95 – 105, 2016.

[7] F. R. C. Eckert and R. M. Drake, "Analysis of heat and mass transfer", McGraw-Hill, New York, 1972.

[8] Z. Dursunkaya and W. M. Worek,"Diffusion-thermo and thermal-diffusion effects on transient and steady natural convection from vertical surface", Int. J. Heat and Mass Transfer, vol.35, No. 8, pp. 2060-2065, 1992.

[9] M. Anghel, H, S, Takhar, I. Pop, "Dufour and Soret effects on free convection boundary layer over a vertical surface embedded in a porous medium", Studia Universitatis Babes-Balyai, Mathematica, Vol.XLV, No.4, pp.11-21, 2000.

[10] A. Postelnicu, "Influence of a magnetic field on heat and mass transfer by natural convection from vertical surface in porous media by considering Soret and Dufour effects", Int. J. Heat and Mass Transfer, vol. 47, No.6-7, pp. 1467-1472, 2004.

[11] M. S. Alam, M. M. Rahman, "Dufour and Soret effects on MHD free convective heat and mass transfer flow past a vertical flat plate embedded in a porous medium", J. Naval Architecture and Marine Engineering, vol.2, No.1, pp. 55-65, 2005.

[12] M. S. Alam, M. M. Rahman and M. A. Samad, "Dufour and Soret effects on unsteady MHD free convection and mass trasnfer flow past a vertical porous plate in a porous medium", Nonlinear Analysis, Modeling and Control, vol. 11, No.3, pp. 217-226, 2006.

[13] M. S. Alam, M. M. Rahman, A. Maleque and M. Ferdows, "Dufour and Soret effects on steady MHD combined free-force convective and mass transfer flow past a semi-infinite vertical plate", Thammasat Int. J. Science and Technology, vol.11, No.2, pp. 1-12, 2006.

[14] M. A. Mansour, N. F. El-Anssary and A. M. Aly, "Effects of chemical reaction and thermal stratification on MHD free convective heat and mass transfer over a vertical stretching surface embedded in a porous media considering Soret and Dufour numbers", J. Chemical engineering, vol.145, No.2, 340-345, 2006.

[15] S. R. Vempati and A. B. Laxmi Narayana Gari, "Soret and Dufour effects on unsteady MHD flow past an infinite vertical porous plate with thermal radiation", Appl. Math. Mech- Engl. Ed, vol.31, No.12, pp. 1481-1496, 2010.

[16] B. Prabhakar Reddy, "Effects of thermal diffusion and viscous dissipation on unsteady NHD free convection flow past a vertical porous plate under oscillatory suction velocity", Int. J. Appl. Mech. and Eng, vol. 19, No.2,303-320, 2014.

[17] M. Venkateswarlu, G. V. Ramana Reddy and D. V. Laxmi, "Thermal diffusion and radiation effects on unsteady MHD free convection heat and mass transfer flow past a linearly accelerated vertical porous plate with variable temperature and mass diffusion", The Korea Society for Industrial and Applied Mathematics, vol. 183, pp.257-268, 2014.

[18] J. N. Reddy, "An introduction to the finite element method", 3rd Ed., Mac-Graw Hill, New York, 2005.