VISCOUS DISSIPATION AND MASS TRANSFER EFFECTS ON UNSTEADY MHD FREE CONVECTIVE FLUID FLOW PAST AN INFINITE VERTICAL POROUS PLATE

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Abstract*— Numerical investigation of viscous dissipation and mass transfer effects on an unsteady MHD free convection flow of an incompressible and electrically conducting fluid past an infinite vertical porous plate under the influence of uniform magnetic field considered normal to the plate has been carried out. The Ritz finite element method has been applied to solve dimensionless governing equations of the flow. The effects of physical parameters such as Prandtl number, Schmidt number, Eckert number, magnetic field parameter, porosity parameter, ratio of mass transfer parameter and time parameter on the fluid velocity, fluid temperature and fluid concentration are presented through the graphs and then discussed.

Keywords*— MHD, Viscous dissipation, magnetic field, free convection, vertical plate.

I. INTRODUCTION

The study of simultaneous heat and mass transfer from different geometrics embedded in porous media has many engineering and geophysical applications such as geothermal reservoirs, drying of porous solids, thermal insulation and underground energy transport. The change in wall temperature causing the free convection flow could be a sudden or a periodic one, leading to a variation in the flow. In nuclear engineering, cooling of medium is more important safety point of view and during this cooling process the plate temperature starts oscillating about a non-zero constant mean temperature. Further, oscillatory flow has applications in industrial and aerospace engineering. Soundalgekar and Ganesan [1] have studied transient free convection with mass transfer on an isothermal vertical flat plate by using the finite difference method. Agrawal et. al [2] have presented the effects of Hall currents on hydro-magnetic free convection with mass transfer in a rotating fluid. The transient free convection with mass transfer from an isothermal vertical plate embedded in a porous medium was reported by Jang and Ni [3]. Jha [4] has studied MHD free convection and mass transfer flow through a porous medium. Soundalgekar and Wavre [5] have studied unsteady free convection flow past an infinite vertical plate with constant suction and mass transfer. Ganesan and Palani [6] investigated mass transfer effects on impulsively started semi-infinite inclined plate with constant heat flux. Ibrahim et. al [7] have studied unsteady magneto - hydrodynamic micro-polar fluid flow and heat transfer over a vertical porous plate through porous medium in the presence of thermal and mass diffusion with constant heat source. Chaudhary and Jain [8] analyzed the combined heat and mass transfer effects on MHD free convection flow past an oscillating plate embedded in porous medium. Chaudhary and Jain [9] hae studied heat and mass diffusion flow by natural convection past a surface embedded in a porous medium. Unsteady flow past an accelerated infinite vertical plate with variable temperature and uniform mass diffusion was studied by Muthucumaraswamy et. al [19].

In all the above mentioned investigations the effect of viscous dissipation is not considered. Viscous mechanical dissipation effects are important in geophysical flows and also in certain industrial operations and are usually characterized by the Eckert number. Gebhart [11] has shown the importance of viscous dissipative heat in free convection flow in the case of isothermal and constant heat flux at the plate. Gebhart and Mollendorf [12] have considered the effects of viscous dissipation for external natural convection flow over a surface. Viscous dissipation heat on the two-dimensional unsteady free convective flow past an infinite vertical porous plate when the temperature oscillates in time and there is constant suction at the plate was studied by Soundalgekar [13]. Gokhale and Samman [14] have presented the effects of mass transfer on the transient free convection flow of a dissipative fluid along a semi-infinite vertical plate with constant heat flux. Cookey et. al [15] have studied the influence of viscous dissipation and radiation on unsteady MHD free convection flow past an infinite heated vertical plate in a porous medium with time dependent suction.

The aim of the present work is to investigate the effects of viscous dissipation and mass transfer on unsteady MHD free convection fluid flow past an infinite vertical porous plate under the influence of uniform magnetic field applied normal to the plate. The problem is governed by coupled non-linear system of partial differential equations, whose exact solutions are difficult to obtain, if possible. So, Ritz finite element method has been adopted for its solution, which is more economical from computational point of view. The behavior of the fluid velocity, fluid temperature and fluid concentration discussed for variations in the governing parameters

II. MATHEMATICAL ANALYSIS

A two dimensional unsteady free convection flow of an incompressible electrically conducting viscous dissipative fluid past an infinite vertical porous plate is considered. In the coordinate system, $x'-$ axis is chosen along the plate in the vertically upward direction and the $y'-$ axis is chosen normal to the plate. A uniform magnetic field of intensity H_0 is applied transversely to the plate. The induced magnetic field is neglected as the magnetic Reynolds number of the flow is taken to be very small. Initially, the temperature of the plate T' and the fluid T_∞' are assumed to be the same. The concentration of species at the plate C_w' and in the fluid throughout C_∞' is assumed to be the same. At time $t'>0,$ the plate temperature is changed to T_w', which is then maintained constant, causing convection currents to flow near the plate and mass is supplied at a constant rate to the plate. The plate is infinite in extant, so that the flow variables are functions of time y' and t' alone. The problem is governed by the following equations:

$$\frac{\partial u'}{\partial t'}=\nu\frac{\partial^2 u'}{\partial y'^2}+g\beta(T'-T_\infty')+g\beta^\bullet(C'-C_\infty')-\frac{\sigma\mu_e^2H_0^2u'}{\rho}-\frac{\nu u'}{K'} \tag{1}$$

$$\rho C_p\frac{\partial T'}{\partial y'}=k\frac{\partial^2 T'}{\partial y'^2}+\mu\left(\frac{\partial u'}{\partial y'}\right)^2 \tag{2}$$

$$\frac{\partial C'}{\partial t'}=D_M\frac{\partial^2 C'}{\partial y'^2} \tag{3}$$

where

$u',g,\beta,\beta^*,T',T_\infty',T_w',C',C_\infty',C_w',k,\nu,\rho,\sigma,C_p,D_M$ and t'

are, respectively, are fluid velocity components in the $x'-$ direction, acceleration due to gravity, volumetric coefficient of thermal expansion, volumetric coefficient of concentration expansion, temperature of the fluid, temperature of the fluid far away from the plate, temperature at the plate, species concentration in the fluid, concentration in the fluid far away from the plate, species concentration at the plate, thermal conductivity, kinematic viscosity, fluid density, electrical conductivity, specific heat at constant pressure, mass diffusion and time.

The corresponding initial and boundary conditions are:

$$\begin{aligned}&t'\le 0; u'=0, T'=T_\infty', C'=C_\infty' && \forall y'\\ &t'>0; u'=0, T'=T_w', C'=C_w' && \text{at } y'=0\\ &\quad u'=0, T'\to T_\infty', C'\to C_\infty' && \text{as } y'\to\infty\end{aligned} \tag{4}$$

Let us introduce the following non-dimensional quantities into the basic equations, initial and boundary conditions in order to make them dimensionless.

$$\begin{aligned}&U_0=(\nu g\beta\Delta T)^{\frac{1}{3}}, L=\left(\frac{g\beta\Delta T}{\nu^2}\right)^{-1/3}, T_R=\frac{(g\beta\Delta T)^{-2/3}}{\nu^{-1/3}},\\ &\Delta T=T_w'-T_\infty', t=\frac{t'}{T_R}, y=\frac{y'}{L}, u=\frac{u'}{U_0}, K=\frac{K'}{\nu T_R},\\ &\theta=\frac{T'-T_\infty'}{T_w'-T_\infty'}, \phi=\frac{C'-C_\infty'}{C_w'-C_\infty'}, P_r=\frac{\mu C_p}{k}, S_c=\frac{\nu}{D_M},\\ &E_c=\frac{U_0^2}{C_p\Delta T}, N=\frac{\beta^\bullet(C_w'-C_\infty')}{\beta(T_w'-T_\infty')}, M=\frac{\sigma\mu_0^2H_0^2T_R}{\rho}.\end{aligned} \tag{5}$$

On substitution of equation (5) into equations $(1)-(4)$, the following governing equations in dimensionless form are obtained:

$$\frac{\partial u}{\partial t}=\theta+\frac{\partial^2 u}{\partial y^2}+N\phi-Mu-\frac{1}{K}u \tag{6}$$

$$\frac{\partial \theta}{\partial t}=\frac{1}{P_r}\frac{\partial^2 \theta}{\partial y^2}+E_c\left(\frac{\partial u}{\partial y}\right)^2 \tag{7}$$

$$\frac{\partial \phi}{\partial t}=\frac{1}{S_c}\frac{\partial^2 \phi}{\partial y^2} \tag{8}$$

where

N,M,K,P_r,E_c and S_c are, respectively, ratio of mass transfer parameter, magnetic parameter, porosity parameter, Prandtl number, Eckert number and Schmidt number.

The corresponding initial and boundary conditions are:

$$\begin{aligned}&t\le 0; u=0,\theta=0,\phi=0 && \forall y\\ &t>0; u=0,\theta=1,\phi=1 && \text{at } y=0\\ &\quad u=0,\theta\to 0,\phi\to 0 && \text{as } y\to\infty\end{aligned} \tag{9}$$

III. SOLUTION OF THE PROBLEM

Equations $(6)-(8)$ are coupled non-linear system of partial differential equations to be solved under the initial and boundary conditions (9). However, whose exact or approximate solutions are not possible to solve the set of these equations. Hence, these equations are solved by Ritz finite element method. The algorithm for Ritz finite element method can be summarized by the following steps.

Step 1: Division of the whole domain into smaller elements of finite dimensions called "finite elements".
Step 2: Generation of the element equations using variational formulations.
Step 3: Assembly of element equations as obtained in step (2).
Step 4: Imposition of boundary conditions to the equations obtained in step (3).
Step 5: Solution of the assembled algebraic equations.

The final assembled equations can be solved by any of the numerical technique viz. Gauss-Seidal iteration method. The numerical solutions for the velocity profiles u, temperature profiles θ and concentration profiles ϕ are computed by using $C-$ program. To prove the convergence and stability of the Ritz finite element method, the same program was run making with small changes in t and $y-$ directions. For these slightly changed values, no significant change was observed in the values of velocity, temperature and concentration profiles. Hence, we conclude that the Ritz finite element method is convergent and stable.

Skin friction (τ) at the plate is given by $\tau=\left(\frac{\partial u}{\partial y}\right)_{y=0}$

IV. RESULTS AND DISCUSSION

The problem of unsteady free convection mass transfer flow of an incompressible electrically conducting viscous fluid past an infinite vertical porous plate with viscous dissipation is addressed in this study. Numerical calculations have been carried out for non-dimensional velocity u, temperature θ and concentration ϕ for various values of the material parameters encountered into the problem under the investigation. The numerical calculations of these results are presented through graphs 1-11.

Figure 1 depicts the effects of Prandtl number P_r on the temperature filed for $P_r = 0.71$ as corresponds to air, $P_r = 1.0$ as corresponds to electrolytic solution, $P_r = 7.0$ as corresponds to water and $P_r = 11.4$ as corresponds to water at 4^0C respectively. The numerical results show that an increasing value of Prandtl number decreases the temperature field. The effects of viscous dissipation parameter i.e. Eckert number E_c and time parameter t on the temperature field are shown in figure 2. It is observed that an increase in the Eckert number and time parameter increases the temperature field. Figure 3 depicts the effects of Schmidt number S_c and time parameter t on the species concentration for $S_c = 0.22, 0.60$ and 0.78 as would corresponds to hydrogen, water-vapour and ammonia respectively, and observed that an increase in the Schmidt number decreases the species concentration whereas an increase in the time parameter increases the species concentration.

Figure 4 shows the velocity profiles in the boundary layer for various values of Prandtl number P_r. It is observed that an increase in the Prandtl number decreases the velocity field. The effects of magnetic parameter M on the velocity field are presented in the figure 5. It is seen that an increase in the magnetic parameter decreases the velocity field. This result qualitatively agrees with the expectations. Since the magnetic field a retarding force on the free convective flow. Magnetic field controls the flow characteristics. Figure 6 depicts the effects of porosity parameter K on the velocity field. It can be seen that an increase in the porosity parameter increases the velocity field. The effects of ratio of mass transfer parameter N on the velocity field are presented in figure 7.Iit is clear that an increase in N leads to increase the velocity field. Figures 8 to 11 shows the effects of Eckert number E_c and time parameter t on the velocity field for a fixed value of Schmidt number $S_c = 0.22, 0.60.0.78$ and 2.62, respectively. From these figures, it can be clearly seen that an increase in the Eckert number and time parameter increases the velocity field. Also, observe that an increase in the Schmidt number decreases the velocity field.

V. FIGURES

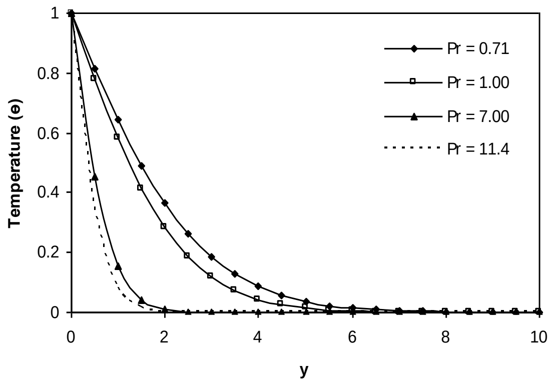

Fig. 1: Effect of P_r on the temperature field.

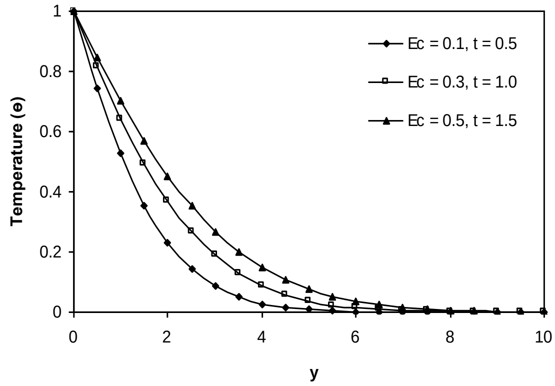

Fig. 2: Effect of E_c and t on the temperature field.

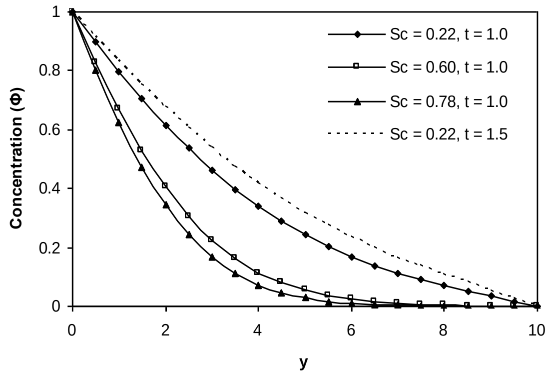

Fig. 3: Effect of S_c and t on the concentration field.

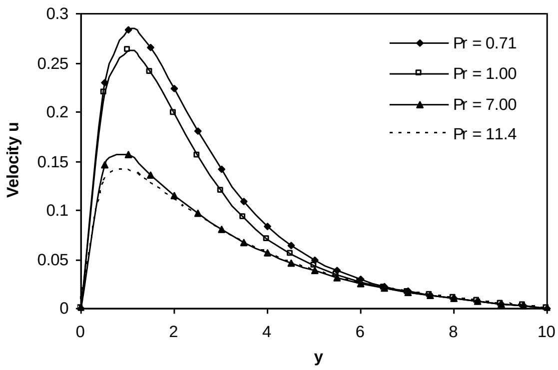

Fig. 4: Effect of P_r on the velocity field

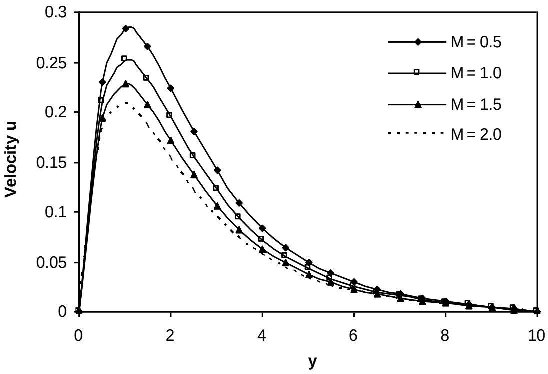

Fig. 5: Effect of M on the velocity field.

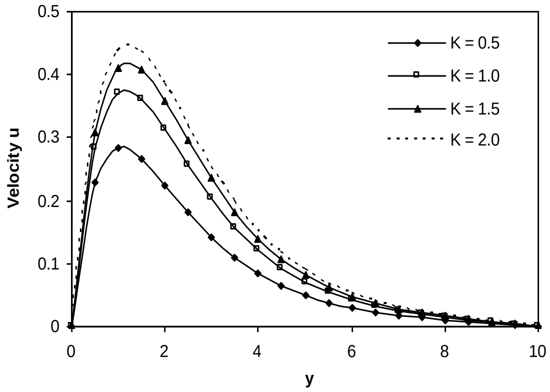

Fig. 6: Effect of K on the velocity field.

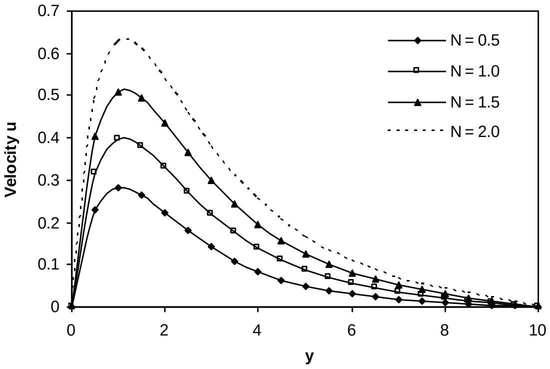

Fig. 7: Effect of N on the velocity field.

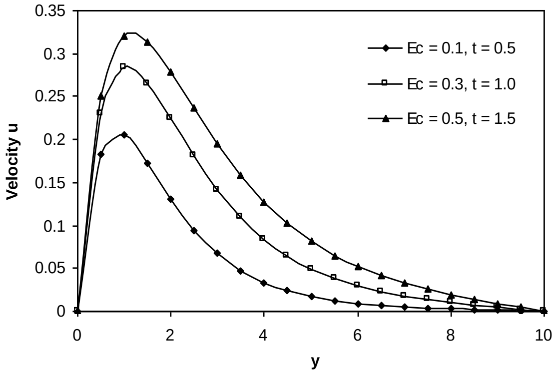

Fig. 8: Effects of E_c and t on the velocity field.

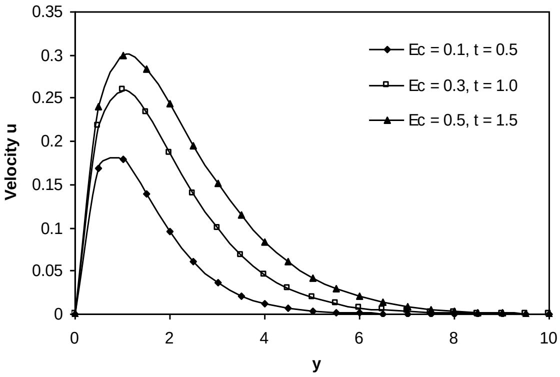

Fig. 9: Effects of E_c and t on the velocity field.

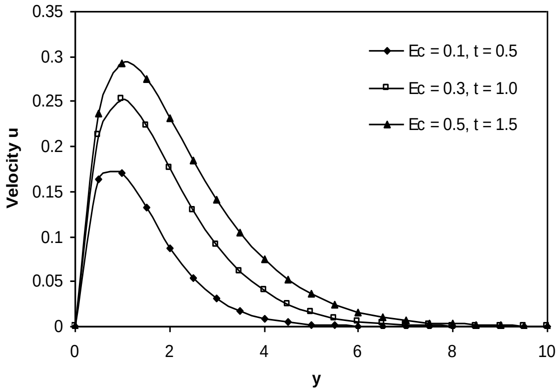

Fig. 10: Effects of E_c and t on the velocity field.

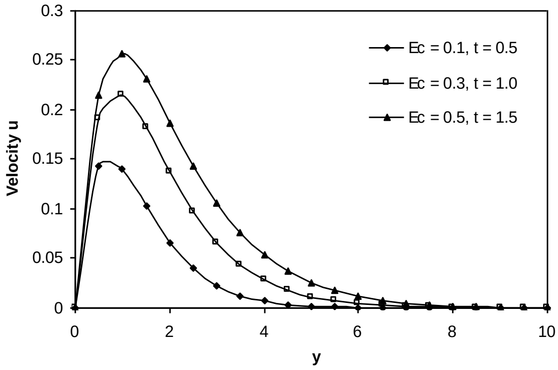

Fig. 11: Effects of E_c and t on the velocity field.

CONCLUSION

In this study we have been examined the governing equations for viscous dissipation and mass transfer effects on an unsteady MHD free convection fluid flow past an infinite vertical porous plate under the influence of magnetic field is provided. The leading equations for this investigation have been solved numerically by using the Ritz finite element method. The present results to illustrate the flow characteristics for the velocity, temperature and concentration and show how the flow fields are influenced by the material parameters of the flow problem. We can conclude that an increase in Prandtl number tends to decrease velocity and temperature fields. As increase in the Schmidt- number decreases the velocity and concentration of the fluid. Also, an increase in the Eckert number and time parameter increases the velocity and temperature of the fluid.

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