Antimagic Labeling on Z_p, Z_q

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*Abstract:*Let G = (V, E) be a graph. The Antimagic labeling on Z_p, Z_q is defined on the group of integers modulo p, q where p and q are the vertices and edges in the graph. In Antimagic labeling on Z_p, Z_q, either the weight of all vertices or the weight of all edges in the graph are distinct positive integers. In this paper we introduce the concept of Edge Antimagic Vertex labeling on Z_p, Z_q, Vertex Antimagic Edge labeling on Z_p, Z_q and discuss these labelings for friendship graphs, Complete graphs, Wheel graphs, Stars and Bistars. Also we find the bounds for the weights of vertices as well as the edges in the graph.

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Introduction

Graph labeling is an assignment of labels to the vertices or edges or to both the vertices and edges of a graph subject to certain conditions. The concept of Antimagic labeling was introduced by Hartsfield and Ringel in 1990. In these type of labelings either the weight of all vertices in a graph or the weight of all edges in a graph are distinct positive integers. In this paper we define Edge Antimagic Vertex labeling and Vertex Antimagic Edge labeling in relation with the vertices and edges of a graph. In an Edge Antimagic Vertex labeling, labels are assigned only to the vertices such that the edge weights are all distinct positive integers. Similarly, in a Vertex Antimagic Edge labeling labels are assigned only to the edges such that the vertex weights are distinct positive integers. In this paper we discuss these labelings on the group of integers modulo p as well as modulo q and investigate some special classes of graphs satisfying this labeling and also find out graphs that do not satisfy this labeling in connection with the vertices and edges of a graph.

Edge Antimagic Vertex Labeling on $\boldsymbol{Z_p}, \boldsymbol{Z_q}$

Definition 2.1. Let $G(p, q)$ be a graph with p vertices and q edges. An Edge Antimagic Vertex labeling on $Z_p$is defined as a one-one function $f: V(G) \to Z_p$such that the induced edge weights defined by $w(uv) = f(u) + f(v)$ where $uv \in E(G)$ are all distinct positive integers. A graph G which admits such a labeling is called an Edge antimagic vertex graph onZ_p. An Edge Antimagic Vertex labeling on Z_q is defined as a one-one function $f: V(G) \to Z_q$such that the induced edge weights defined by $w(uv) = f(u) + f(v)$ where $uv \in E(G)$ are all distinct positive integers. A graph G which admits such a labeling is called an Edge antimagic vertex graph onZ_q.

Definition 2.2. In an Edge Antimagic Vertex labeling on Z_p if the edge weights form an Arithmetic Progression with the first term as 'a' and Common difference as 'd' then the labeling is called as an (a, d)Edge Antimagic Vertex labeling on Z_p. If such a labeling exists in a graph then the graph is called an (a, d)Edge Antimagic Vertex graph. An Edge Antimagic Vertex labeling on Z_p is said to be a Super Edge Antimagic Vertex labeling on $Z_p$if the set of vertex labels consists of consecutive integers. A graph G which admits such a labeling is called a Super Edge Antimagic Vertex graph.Similar ideas also hold good for Edge Antimagic Vertex labeling on Z_q.

Theorem 2.3. All Friendship graphs got by the one point union of t copies of the Cycle C_3 admit Edge AntimagicVertex labeling on Z_q.

Proof. Let $G = {C_3}^{(t)}$ be a friendship graph with the one point union of t copies of the cycle C_3. Let p, q denote the number of vertices and edges in the graph where $p = 2t + 1, q = 3t$. Let v_0 denote the central vertex of $G$and v_1,v_2,…,v_{2t} be the other vertices of the graph in the clockwise direction.

Define $f: V(G) \to Z_q$ as follows:

$$f(v_0) = 0$$

$f(v_i) = i for\ 1 \leq i \leq 2t - 1, i is odd$

$f(v_2) = q - 1$

$f(v_i) = f(v_{i-2}) - 1\ for\ 4 \leq i \leq 2t$, i is even

By the above labeling all the edge weights are distinct. Here the set of all weights for the odd vertices form an arithmetic progression with first term as 1 and common difference as 2 and the set of all weights for the even vertices form an arithmetic progression with the first term as $q - 1$ and the common difference as 1. Hence all Friendship graphs got by the one point union of t copies of the Cycle C_3 admit Edge Antimagic Vertex labeling on Z_q .

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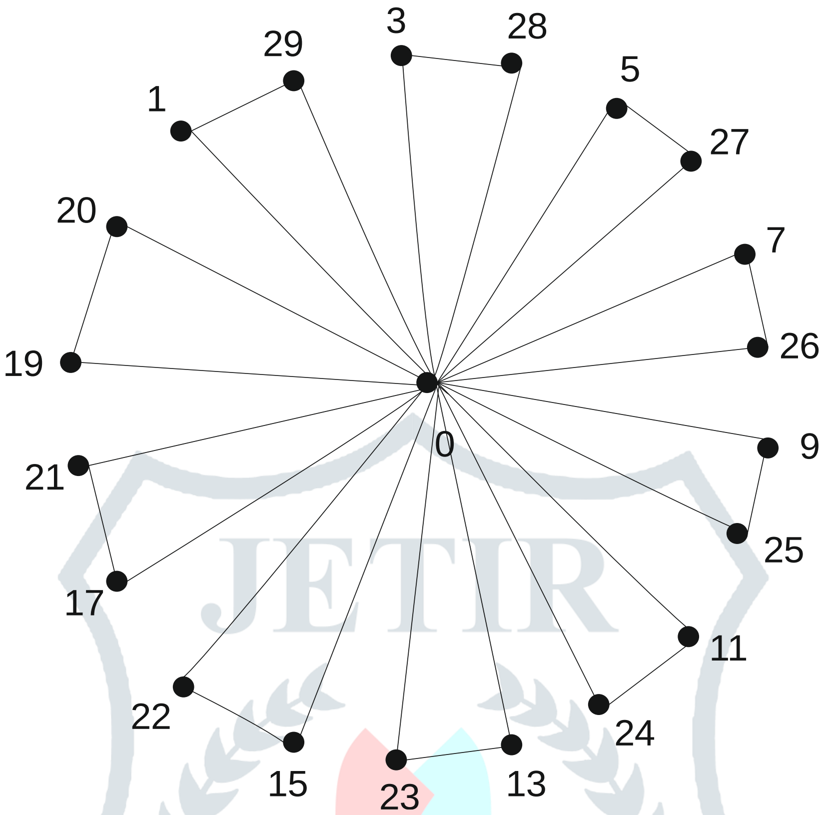
Example:



Edge Antimagic Vertex labeling of $C_3^{(10)}$ on Z_q

Theorem 2.4. All Complete graphs K_p,p≥4, are not Edge Antimagic Vertex graphs on Z_p.

Proof. Let G = K_p be a Complete graph with $p \geq 4$ vertices. Let $v_1, v_2, \dots v_p$ be the vertices of K_p.Suppose K_p admits edge antimagic vertex labeling on Z_p. If possible let $f: V(G) \to Z_p$be the EAV labeling defined on K_p . Now the vertices of K_p are labeled with $0,1, \dots, p-1$. Let the vertices be labeled in any order. Without loss of generality assume that the vertex v_1 is labeled with 0. Since deg $v_1 = p-1$ there are $p-1$ vertices adjacent to v_1. Now the weight of the edges incident with v_1 are $1,2,, \dots, p-1$ in any order . Consider any vertex labeled with 1. Again there are $p-1$ vertices adjacent to the vertex labeled with 1. The corresponding edge weights for these $p-1$ edges are $1,3,4, \dots, p$ in any order. Hence it is not possible to get distinct edge weights which is a contradiction. Also in any complete graph the number of edges $q = \binom{p}{2}$. Clearly $\binom{p}{2} = \frac{p(p-1)}{2} > p$for $p \geq 4$. As the number of edges exceed the maximum range of the labels by atleast two it is not possible to get distinct edge weights by assigning the labels $0,1, \dots, p-1$ to the vertices of the graph. Hence all Complete graphs $K_p,\ p \geq 4$ do not satisfy the condition of edge antimagic vertex labeling on Z_p. Therefore $K_p,\ p \geq 4$, are not Edge AntimagicVertex graphs on Z_p.

Remark 2.5:. All Friendship graphs got by the one point union of t copies of the Cycle C_3 have Edge Antimagic Vertex labeling on Z_p only when t = 2 under the mapping $f: V(G) \to Z_p$ defined by $f(v_0) = 1, f(v_1) = 0, f(v_i) = i for\ 2 \leq i \leq 4$ where v_0 is the central vertex. All Wheel graphs $W_n, n \geq 3$, admit Edge Antimagic Vertex labeling on Z_q for all odd n and for even n only when n = 4.

Remark 2.6. For any EdgeAntimagic Vertex labeling on Z_p, the edge weights range from $1 \leq w(uv) \leq 2p-3$ and for any Edge Antimagic Vertex labeling on Z_q, the edge weights range from $1 \leq w(uv) \leq 2q-3$ where $uv \in E(G)$.

Vertex antimagic edge labeling on Z_p, Z_q

Definition 3.1. Let $G(p,q)$ be a graph with p vertices and q edges. A Vertex Antimagic Edge labeling on Z_p is defined as a one-one function $f: E(G) \to Z_p$such that the induced vertex weights defined by $w(v) = \sum_{u \in N(v)} f(vu)$ are all distinct positive integers. Here $N(v)$ is the neighbourhood of the vertex v. A graph G which admits such a labeling is called a Vertex antimagic edge graph onZ_p. A Vertex Antimagic Edge labeling on Z_q is a one-one function $f: E(G) \to Z_q$such that the induced vertex weights defined by $w(v) = \sum_{u \in N(v)} f(vu)$ are all distinct positive integers. A graph G which admits such a labeling is called a Vertex antimagic edge graph onZ_q.

Remark 3.2. An (a, d)Vertex Antimagic edge labeling on Z_p, Z_q and Super Vertex Antimagic Edge labeling on Z_p , Z_q are defined in relation with the vertex weights and vertex labels as in definition 2.2.

Theorem 3.3. All Stars, Bistarsadmit Vertex Antimagic Edge labeling on Z_p and they are Super Vertex Antimagic Edge graphs.

Proof. Let $G = K_{1,n}$ be a Star with v_o as the root vertex, .v_i, i = 1 to n, the pendent vertices and e_i, i = 1 to n the edges.

By defining $f: E(G) \to Z_p by f(e_i)= i for\ 1 \leq i \leq n$ the weights of the vertices v_1,v_2,..,v_n are 1,2,3,. . , n and the weight of the root vertex $v_o = \frac{n(n+1)}{2}$. Hence the vertex weights are all distinct positive integers. As the vertex labels are consecutive integers all stars are Super Vertex AntimagicEdge graphson Z_p.

Let $G = B_{m,n}$ be a Bistar with V$(B_{m,n})$ = $\{u, v, u_i , v_{j,}\ 1 \leq i \leq m, 1 \leq j \leq n\}$ and E$(B_{m,n})$= $\{uu_{i\ ,} vv_j\ , uv , 1 \leq i \leq m , 1 \leq j \leq n\}$. Therefore $|V(G)|$ = m+n+2 $|E(G)|$= m+n+1

By defining $f: E(G) \to Z_p by$f$(uv) = 0$, f(e_i)= $for\ 1 \leq i \leq n + m$,

the weights of the vertices u_1,u_2,…,u_m are 1,2,. . , m and the weights of the vertices v_1,v_2,..,v_n are $m + 1, m + 2, \dots , m + n$. Now, the weight of the vertex $u = \frac{m(m+1)}{2}$ and the weight of the vertex $v = (m + 1) + (m + 2) + \cdots + (m + n)$.

Hence the vertex weights are distinct and the edge labels of the bistar are all consecutive integers. Therefore all bistars admit Vertex Antimagic Edge labeling on Z_p.and they are also Super Vertex Antimagic Edge graphs on Z_p. Here, as the pendent vertices have degree 1 their weights form an arithmetic progression of the sequences of natural numbers.

Remark 3.4. For any VertexAntimagic Edge labeling on Z_p, the vertex weights range from $1 \leq w(v) \leq (p - 1) + \sum_{i=1}^{\Delta-1}(p - 1) - i$.

For any Vertex Antimagic Edge labeling onZ_q, the vertex weights range from 1≤ $w(v) \leq (q - 1) + \sum_{i=1}^{\Delta-1}(q - 1) - i$where $w(v)$ is the weight of the vertex v.

Conclusion

When the number of vertices exceed the number of edges in a graph it fails to attain Edge Antimagic Vertex labeling defined on Z_q. Similarly when the number of edges in a graph exceed the number of vertices then the graph fails to attain the Vertex Antimagic Edge labeling on Z_p. In these cases the pigeonhole principle or box principle can be restated in such a way that if m items are put into n containers with n >m there is atleast one container left out without an item which leads to a situation that there is atleast one vertex or edge left without a label, thus violating the labeling condition. Hence it is interesting to find out new Antimagic graphs in relation with the vertices and edges of the graph.

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