A STUDY ON SIMPLE HARMONIC MOTION & ITS APPLICATION

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Abstract

This paper tells us about simple harmonic motion and its applications. Simple harmonic motion are of constant amplitude in which acceleration is proportional and oppositely directed to the displacement of the body from a position of equilibrium. There are many applications such as clock, guitar, bungee jumping, rubber bands, earthquakes, diving boards, bungee jumping,etc.

Keywords: Acceleration, Amplitude, Frequency, Velocity.

Introduction

If a mass is attached to a spring and displaced after from its rest position and released ,it will oscillate around rest position in SHM. Mathematically, the restoring force F is given by

$$F= -K x$$

Where, F is the restoring elastic force exerted by the spring (in SI units: N), k is the spring constant (N·m−1), and x is the displacement from the equilibrium position (m).

Types of Simple Harmonic Motion:

There are different types of simple harmonic motion like:

[A] Damped Simple Harmonic Motion:

The oscillation in which amplitude decrease with the timrare damped simple harmonic motion. Simple pendulum is an example of Damped Simple Harmonic Motion.

Consider a block of mass m connected to elastic ring of spring constant k, if we push the block little and then release it, its angular frequency of oscillation is $\omega=\sqrt{k/m}$.

An external force will exert a damping force on the block which will reduce the mechanical energy of the block-string system. The energy lost will appear as the heat of the surrounding medium.

The damping force depends on nature of the surrounding medium. When a block is immersed in a liquid, the magnitude of damping increase and the dissipation energy becomes faster. Thus, the damping force is directly proportional to the velocity and acts opposite to the velocity. If damping force is F_d, we have,

$$F_d =-b\ v \qquad \text{…..(I)}$$

Where the constant b depends on the properties of the medium and size and shape of the block.

Let's assume O is the position where the block settles after releasing. Now, if the block is pushed little, the restoring force on the block is $F_s = -k\ x$, where x is the displacement of the mass position. Therefore, the force acting on the mass at any time t is,

$$F = -k\ x – b\ \upsilon$$

Now, if a (t) is the acceleration of mass *m* at time t, then by Newton's Law of Motion along the direction of motion, we have

$$ma(t) = -k\,x\,(t) - b\,\upsilon\,(t) \qquad \text{...... (II)}$$

here, we are not considering vector notation because we are only considering the one-dimensional motion. Therefore, using derivatives we have,

$$m\left(\frac{d^2x}{dt^2}\right) + b\left(\frac{dx}{dy}\right) + k\,x = 0 \qquad \text{...... (III)}$$

This equation describes the motion under the influence of a damping force. Therefore, the above expression is damped simple harmonic motion expression. The solution of the expression is of the form

$$x(t) = Ae^{\frac{-bt}{2m}}\cos(\omega' t + ø) \qquad \text{......(IV)}$$

where, A is the amplitude and ω′ is the angular frequency which is given by,

$$w' = \sqrt{\left[\frac{k}{m} - \frac{b^2}{4m^2}\right]} \qquad \text{...... (V)}$$

The function x (t) is not strictly periodic because of the factor $e^{-bt/2m}$ which decreases continuously with time. However, if the decrease is small in time period T, then the motion is approximately periodic. The amplitude is not constant in an damped oscillator.For small damping, the same expression can be used but by taking amplitude as $Ae^{\frac{-bt}{2m}}$

$$\therefore E(t) = \frac{1}{2}\,kA\,e^{\frac{-bt}{2m}} \qquad \text{...... (VI)}$$

This expression shows damping decreases with time. The dimensionless ratio (b/√km) is much less than 1 for small damping. Obviously, if b = 0, all equations of damped simple harmonic motion will turn into the equations of un-damped motion.

[B] Forced simple harmonic motion:

Whenever a pendulum is displaced from its equilibrium position, it moves in to and fro direction about its mean position.And then this motion dies out due to the opposing force present in medium.So when this pendulum is forced to oscillate ,this can be called as forced simple harmonic motion. ω is natural frequency .For example a child uses his legs to move the swing.So here external force is applied to maintain oscillations.

Here we have considered an external force i.e F(t) of amplitude F_0, that changes periodically with time. This force is applied to a damped oscillator. Therefore this can be represented as,

$$F(t) = F_0 \cos \omega_d t \qquad \text{......(I)}$$

So here, the that are forces acting on the oscillator are its restoring force, and the external force and a time-dependent driving force becomes,

$$ma(t) = -k\,x(t) - b\,\upsilon(t) + F_0 \cos \omega_d t \qquad \ldots\ldots(II)$$

We know that acceleration $=\frac{d^2x}{dt^2}$. Substituting this value of acceleration in equation II , we get,

$$m\left(\frac{d^2x}{dt^2}\right) + b\left(\frac{dx}{dy}\right) + k\,x = F_0 \cos \omega_d t \qquad \ldots\ldots(III)$$

In III equation

Its an equation of an oscillator of mass m on which a periodic force of frequency ω_d is applied. As we already know that the oscillator will first oscillates with its natural frequency. After applying the external periodic force, the oscillations dies out with the natural frequency. So the changed equation after the natural frequency dies out is given as:

$$x(t) = A \cos(\omega_d + ø) \qquad \ldots\ldots(IV)$$

where t is the time from when we started applying external force.

Equation Solving Methodology Headings

Simple Harmonic Equation Solving Method:
For solving simple harmonic motion equation there are three approaches.The first one uses the well-known result (of cosine and sin solutions of this form of a second order differential equation) which was derived from using methods of the full solution using *(e i ϕ)*, as in the second and third approaches.

1. Assume the sin and cosine results:

Cosine and Sine are both solutions of the above equation, so the solution of the two becomes $x=A\cos(\omega t) + B\sin(\omega t)$ which is equivalent to $x=A\cos t(\omega t + \phi)$, giving the common form.

2. The final approach is by solving the differential equation for ***z*** and just setting $\boldsymbol{x=\Re(z)}$.

Solving the equation for z gives,

$$x = A\, e\, i\, \omega\, t + B\, e{-}i\, \omega\, t$$

where , the constants are REAL. As this still reduces to,

$$z = (A+B)\cos(\omega t) + i(A-B)\sin(\omega t),$$

but this time for real A and B and z is considered as a complex number, we can see that this is equivalent to $z=Dei\phi ei\omega t$ where the complex amplitude component $ei\phi$ rotates the complex number $ei\omega t$ in the complex plane such that the ratio of the constants of the cosine and sin terms of z is correct and as, given to $z=e\ i(\omega\ t+\phi)$ yielding to the $z=(A+B)\cos(\omega\ t)+i(A-B)\sin(\omega\ t)$ form of z.

This reduces to $z=e\ i(\omega\ t+\phi)$ yielding $x=\Re(z)=D\cos(\omega\ t+\phi)$ as before.

The first sin and cos solution is just a shortcut using the known result derived from the more formal solution using $ei\phi$.
However the sin/cos form ONLY applies when the variable in the differential equation is REAL.
For example, it gives the correct solution for real x, but if had a complex z in there which I knew was meant to be complex, then assuming the sin/cos form would be wrong and would not give the complete solution/picture- it would only give the real component of z.

The difference between the second and third approaches seems to be that in one case we know we have a complex variable z.
In that case, we actually find that the constants can be real (although I think they may also be not real. Provided that $A\neq B*$, this still yields a complex z as the complex parts do not cancel out).
And in the second case we find that the constants must be real and that $A=B*$ such that we get a real x, as mandated by the boundary conditions.
So considering this we can't quite see how approaches 2 and 3 above would yield the same x form through these two different ways: using boundary conditions for a complex A and B so that complex parts cancel, or allowing z to be complex and thus $A\neq B*$ and making x the real part of this complex solution.

Implementations:

Example 1:

A sewing machine needle moves in a path 4 cm long and the frequency of its oscillations is 10 Hz. What is its displacement and acceleration 1/120 s after crossing the centre of its path?

Solution:

Given: Path length = 4 cm, amplitude = path length/2 = 4/2 = 2 cm, Frequency of oscillation = n = 10 Hz, Time elapsed = t = 1/120 s, particle passes through mean position, $\alpha = 0$.

To Find: Displacement = x =? acceleration = f =?

Angular velocity = $\omega = 2\pi n = 2\pi \times 10 = 20\pi$ rad/s

Displacement of a particle performing S.H.M. is given by

$x = a \sin(\omega t + \alpha)$

$\therefore\quad x = 2 \sin(20\pi \times 1/120 + 0)$

$\therefore\quad x = 2 \sin(\pi/6) = 2 \times 1/2 = 1$ cm

The magnitude of the acceleration of a particle performing S.H.M. is given by

$f = \omega 2x = (20\pi)^2 x\ 1 = (20 \times 3.142)^2 = 3944$ cm/s^2.

Ans: Displacement =1 cm and acceleration = 3944 cm/s2

Example 2

A block is on a piston which is moving vertically up and down with S.H.M. of period one second. At what amplitude of motion will the block and piston separate? At which point in the path of motion will the separation take place?

Solution: Given: Period = T = 1s

To Find: amplitude = a = ?

Angular velocity = $\omega = 2\pi/T = 2\pi/1 = 2\pi$ rad/s

At the topmost point, the block and piston will separate.

At topmost point acceleration is maximum. Hence force is maximum

Maximum force on the block = weight of the block m,

f max = mg

∴ f max = g

∴ $\omega^2 a = g$

∴ $a = g / \omega^2 = 980/ (2 \times 3.142)^2 = 24.82$ c

Ans: At amplitude = 24.82 cm block w will separate at topmost point of the path

Example 3

A cord is 20 m long unstretched if its elasticity is 50% and the spring constant is 20 N/m . What is the maximum force which it can hold?

Solution: We know that,

F = k×x is calculated by taking 50% of the unstretched cord

Therefore X = 10m, simply plug the values into formula

F= (20 N/m) (10m)

F= 200 N

Result: Max force the cord can held is 200 N.

Literature Review

To study the concept of simple harmonic motion we have referred several books. We also referred online websites to get the details. The study was drawing our interest thoroughly to the concepts of physics which are hard to study and understand. Simple harmonic motion is the topic which exists in physics including some part of mathematics

Lab Experiment

Aim: To find the relation between the spring constant and initial length of rope

Theory: What is Bungee Jumping?

Bungee jumping is a activity which involves jumping from a certain given height connected by a rope which is tied to the persons leg. .This activity is full of thrill and enthusiasm .The rope will keep on to oscillating up and down until all kinetic energy is disappeared.

Requirement:

Table, Table stand, Measuring tape, Bungee cord, Eggs, Hooks

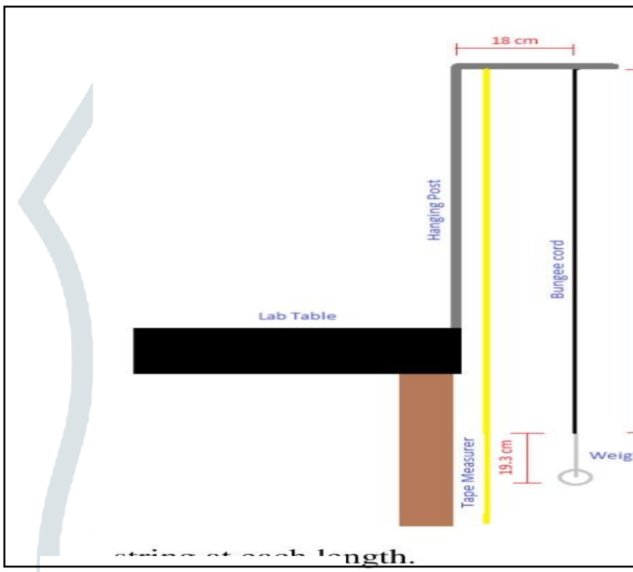


Fig. 1: Depiction of our Bungee Model at the largest displacement of the jump

Procedure:

- We set up a hanging post on the edge of our lab table, where the length from top of the post to the floor was 244 cm. and then attached measuring tape, to calculate displacement.
- The bungee cord was tied to a knob 18 cm away from the vertical post in order to leave room for the bungee weight to bounce without interference.
- For testing the k value, we chose three different initial lengths of the rope: 20 cm, 40 cm, and 60 cm. To test how the k value varies with added mass, we attached a 19.3 cm long hanging mass to the bungee cord that weighed either 100, 110, 120, 130, 150, 170 grams.
- To attach the hanging mass, we looped the knot three times around the top of the hanging mass and secured the weights with tape. For each mass and length, we dropped the hanging mass from the hanging post, and recorded three jumps each.
- We took the readings from the bottom of the weight, and therefore, had to subtract the height of the weight (19.3 cm) from each reading, in order to isolate the bungee cord.

Results :

Different observations on various length of above mentioned FIGURE

A] Data table for the 20 cm bungee cord

Mass m	Length of total fall h	Standard deviation for h	Displacement x	Standard deviation for x	K value (N/m)
(g,± l g)	(cm,±1cm)		(cm,±1cm)		± 1 (N/m)
100	78	1	39	0	46
110	82	1	43	1	46
120	85	1	46	1	47
130	89	1	50	1	47
150	97	1	58	1	48
170	108	1	69	1	46

Average k value: 47(N/m)

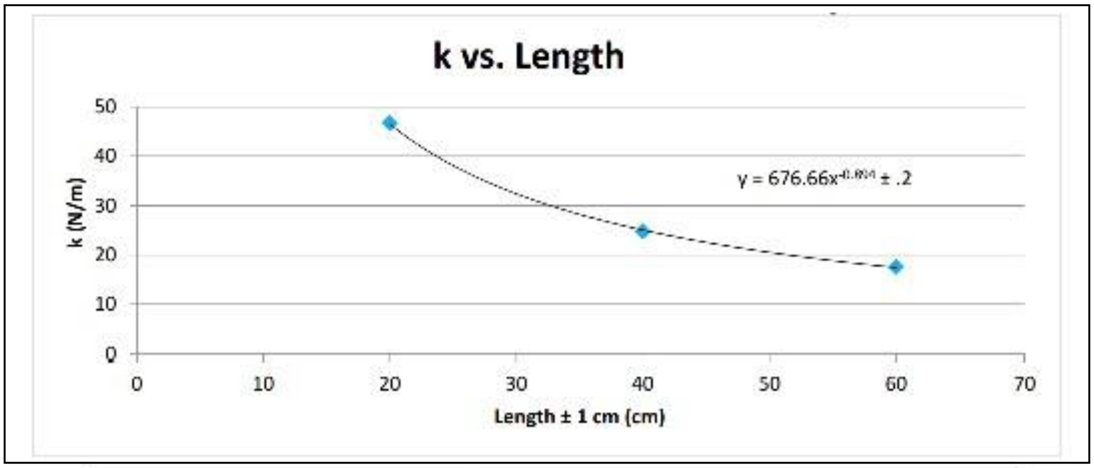
B] Data table for the 40 cm bungee cord

MASS	Length of total fall h	Standard deviation for h	Displacement x	Standard deviation for x	K value (N/m)
(g,± l g)	(cm,±1cm)		(cm,±1cm)		± 1(N/m)
100	147	1	87	1	25
110	159	2	99	2	24
120	163	3	104	3	25
130	173	1	114	1	25
150	191	2	132	2	25
170	212	1	152	1	24

Average k value : 25(N/m)

Results:

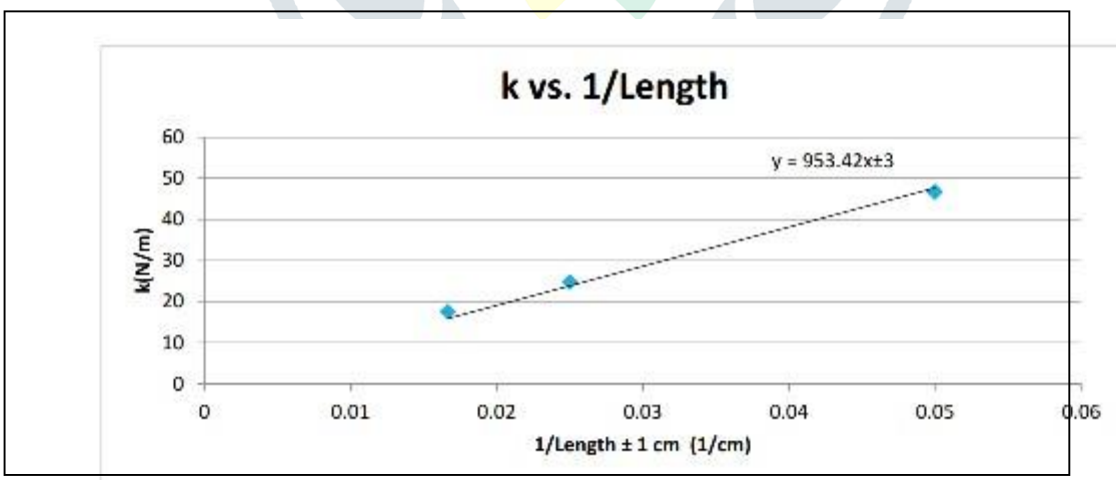
In analyzing our data, we were able to show that k of the bungee cord had an inverse relationship to the length of the cord.



: Graph of the k vs. length of the bungee cord
Graph showing the k value on the y-axis and the length of the bungee cord on the x-axis. The power function was fitted to the trendline.

Graph 1 : Based on data Table 1

The data listed shows the data for the bungee cord at the length of 60 cm. We did not have any mass measurements for this data table because a jump with heavier masses would hit the floor. We averaged the heights and displacements from our three trials, and calculated the standard deviations and the k values. The k values at this length were the lowest.



gure 7: Graph of the k vs. 1/length of the bungee cord
Graph showing the k value on the y-axis and the inverse of the length of the bungee cord on the x-axis. Shows a linear relationship.

Graph Based on Table 2

We graphed the average k values for each length and the lengths in order to get a better understanding between them. The graph shows an inverse relationship between the k and the length .

Hence, Our experiment proved that the spring constant and initial length are inversely proportional. Using this relationship, we can predict the final stretch of a bungee cord by knowing the mass, initial height, and the length.

Survey Data Collection

Bungee jumping is a free fall jump, but one can wonder how the "winners" will be decided , the judges will judge the jumps according to the time they take to leave the ledge, distance forward of the leap , consistency of body moves during the jump and style of jump.

SR NO	HEIGHT(in m)	PLACE
1	206	Rio Grand Bridge, New Mexico
2	216	Bloukrans Bridge , South Africa
3	220	Verzasca Dam, Switzerland
4	233	Macau Tower , China

CONCLUSION

In the above case study simple harmonic motion and its applications. Different problems related to the application are solved analytically with exact equation of simple harmonic motion. We can calculate the periodic time value of oscillating a object from origin by this methods..

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