ON THE CONSTRUCTION OF FOURIER MATRICES OF OREDR 2,3,5 AND 7

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Abstract: In this paper we forward methods of construction of Fourier matrices of order 2,3,5 and 7 by the suitable linear combination of adjacency matrices of suitable coherent configuration.

Keywords: **Coherent configuration, Complex Hadamard matrices.**

1.INTRODUCTION: Fourier matrices play very important role in modular data and modular data is an important factor of rational conformal field theory. The rational field theory has lots of application in Physics. The modular data are also used in the study of fusion rings and $C-$algebras. **(Vide [1, 2, 6, 7,10,and 17])**In this section we discuss some elementary definitions and some known methods of construction of complex Hadamard matrices.

1.1.HADAMARD MATRICES: An square matrix H with entries are $+1$ and -1, of order h is called Hadamard matrix if $HH^{'}=hI_h$.Where $H^{'}$=Transpose of H and I_h=Identity matrix of order h. **(vide [5])**

1.2.COMPLEX HADAMARD MATRICES:An square matrix C of order n with entries $+1,-1,+i or -i$ is called complex Hadamard if $CC^{\theta}=nI_n$ Where $i=\sqrt{-1}$ and C^{θ} is transposed conjugate of C.

The set of all complex Hadamard matrices of order n is denoted by H_n .

Example: Complex Hadamard matrix of order 1,2 and 3 are given below**:**

$$F_1=[1], F_2=\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix} \text{ and } F_3=\begin{bmatrix}1 & 1 & 1\\ 1 & \omega & \omega^2\\ 1 & \omega^2 & \omega\end{bmatrix}$$

(**Vide [14])**

1.3EQUIVALENT COMPLEX HADAMARD MATRICES:Two Complex Hadamard matrix $H_1 \text{ and } H_2$ are equivalent if H_1 can obtained from H_2 by a finite sequence of row permutations, column permutations or by multiplication of a row or column by the numbers 1, -1, i, and -i**.(Vide[16])**

1.4.DEPHASED COMPLEX HADAMAED MATRIX:

When the entries in the first row and first column of a complex Hadamard matrix H are all equal to unity then it is called dephasedcomplex Hadamard matrix or dephased form of complex Hadamard matrix.Two Hadamard matrices with the same dephased form are equivalent. Example of dephased complex Hadamard matrix: $F_4=\begin{bmatrix}1 & 1 & 1 & 1\\ 1 & i & -1 & -i\\ 1 & -1 & 1 & -1\\ 1 & -i & -1 & i\end{bmatrix}$.**(Vide [11])**

1.5.FOURIER MATRIX: An square matrix of order m is called Fourier matrix F such that $F_{jk}=e^{2\pi i(\frac{(j-1)(k-1)}{m})}=w_{jk}$,where j ,k=1,1,2,3,….,m. and $i=\sqrt{-1}$ .The general form of a Fourier matrix is given below:

$$F_m = \begin{bmatrix} 1 & 1 & 1 & \dots & \dots & 1 \\ 1 & \omega & \omega^2 & \dots & \dots & \omega^{m-1} \\ 1 & \omega^2 & \omega^4 & \dots & \dots & \omega^{2(m-1)} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 1 & \omega^{m-1} & \omega^{2(m-1)} & \dots & \dots & \omega^{(m-1)^2} \end{bmatrix}; \omega = e^{(\frac{2\pi i}{m})}$$

We note that Fourier matrices are well known complex Hadamard matrices.**(Vide[15]).** Following are the some are important theorems of Fourier matrices:

THEOREM:1.5.1.Let $F = F_{m_1} \otimes F_{m_2} \otimes F_{m_3} \otimes F_{m_4} \otimes .... \otimes F_{m_r}$ be a Kronecker product of Fourier matrices then F is equivalent to $F_{n_1} \otimes F_{n_2} \otimes F_{n_3} \otimes F_{n_4} \otimes ... \otimes F_{n_s}$ if and only if the sequence $(n_1, n_2, n_3, n_4, ..., n_r)$ is obtained from the sequence $(m_1, m_2, m_3, m_4, ..., m_s)$ using a series of operations from the list below

(i) permuting a sequence .

(ii) replacing a subsequence m_x, m_y *by* $m_z = m_x m_y$ if m_x *and* m_y are relatively prime.

(iii) Replacing a sequence element m_z by a subsequence m_x, m_y if $m_z = m_x m_y$ and m_x, m_y are relatively prime**.(Vide[12 and 15])**

THEOREM:1.5.2. The Fourier matrix F_p is isolated amongst all $p \times p$ Complex Hadamard matrices for every prime P. **(Vide:[15])**

THEOREM:1.5.3. All Complex Hadamard matrices of orders $m = 1,2,3$ are equivalent to the Fourier matrix F_m. **(Vide[15])**

THEOREM:1.5.4. All complex Hadamard matrices of order 5 are equivalent to the Fourier matrix F_5. **(Vide[15])**

1.6. COHERENT CONFIGURATION(C.C.):

Let $C = \{C_1, C_2, C_2, ..., C_m\}$ be a class of binary relations defined on a finite set of X and $A_1, A_2, A_3, ..., A_m$ be adjacency matrices of the relations $C_1, C_2, C_3, ..., C_m$ respectively satisfying the following four conditions:

(i) $\{A_1, A_2, A_3, ..., A_m\}$ is a partition of all 1 matrix $J_{|X|}$ of order $|X|$;

(ii) There exists a subset of $\{A_1, A_2, A_3, ..., A_m\}$ which is a partition of unit matrix $I_{|X|}$;

(iii) The set $\{A_1, A_2, A_3, ..., A_m\}$ is closed under matrix transposition.

(iv) The product $A_i A_j$ for all $i, j \in \{1, 2, 3, ..., m\}$, is some linear combination of elements of the set $\{A_1, A_2, A_3, ..., A_m\}$ with non-negative integral coefficients.

Then the set $C = \{C_1, C_2, C_2, ..., C_m\}$ is called a Coherent Configuration.**(Vide[9]).**

2.SOME METHODS FOR CONSTRUCTION OF HADAMARD MATRICES:

2.1.BY ELEMENTARY MATRIX TRANSFORMATIONS:

If M is a dephased form of complexhadamard matrix then its transpose $M^{'}$,the conjugated matrix $\overline{M}$ and the Hermitian transpose H^{θ} all are complex Hadamard matrices in dephased form**.(Vide[11])**

2.2.DUPLICATION OF THE MATRIX SIZE METHOD:

If H_1 and H_2 belong to H_m then $H = \begin{bmatrix} H_1 & H_2 \\ H_1 & -H_2 \end{bmatrix} \in H_{2m}$ .Furthermore ,if H_1 and H_2 are taken to be in the dephased form ,so is H **.(Vide[11])**

2.3.QUADRUPLICATION OF THE MATRIX SIZE METHOD:

If $H_1, H_2, H_3, H_4 \in H_m$ are in the dephased form then $H = \begin{bmatrix} H_1 & H_2 & H_3 & H_4 \\ H_1 & -H_2 & H_3 & -H_4 \\ H_1 & H_2 & -H_3 & -H_4 \\ H_1 & -H_2 & -H_3 & H_4 \end{bmatrix} \in H_{4m}$ and it has the dephased form.This form is preserved,if the blocks H_2, H_3 and H_4 are multiplied by diagonal unitary matrices E_1, E_2 and E_3 respectively each containing unity and $(m-1)$ free phase.**(Vide[11]**

2.4 COMBINATION OF TWO COMPLEX HADAMARD MATRICES :

IF If $H_1 \in \mathrm{H}_m$ and $H_2 \in \mathrm{H}_n$, then kronecker product $H_1 \otimes H_2 \in \mathrm{H}_{mn}$**(Vide [11])**

Some more methods of construction of complex Hadamard matrices using amicable and some special Hadamard matrices are given in **(Vide [14]).**

3.MAIN WORKS:

In this section we forward some methods of construction of Fourier matrices of order 2,3,5,and 7 by the multiplication of suitable permutation matrix with suitable linear combination of adjacency matrices of suitable coherent configuration.

3.1. CONSTRUCTION OF FOURIER MATRIX OF ORDER 2:

Let us consider $X = \{i : i = 1,2\}$ and a partition $C = \{C_1, C_2, C_3, C_4\}$ of $X \times X$.Where $C_1 = \{(i,i) : i = 1\}$, $C_2 = \{(1,i) : i = 2\}, C_3 = \{(i,1) : i = 2\}, C_4 = \{(i,i) : i = 2\}$.Then adjacency matrices A_1, A_2, A_3 and A_4 of C_1, C_2, C_3 and C_4 are respectively given below:

$$A_1 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, A_2 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, A_3 = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, A_4 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

We see the following calculations:

$$(i).A_1^2 = A_1, A_1A_2 = A_2, A_1A_3 = 0, A_1A_4 = 0, A_2A_1 = 0, A_3A_1 = A_3, A_4A_1 = 0.$$

$$(ii).A_2^2 = 0, A_2A_3 = A_1, A_2A_4 = A_2, A_3A_2 = A_4, A_4A_2 = 0. \; (iii).A_3^2 = 0, A_3A_4 = 0, A_4A_3 = A_3. \; (iv).A_4^2 = A_4.$$

.Hence, product of any two adjacency matrices is some linear combinations of adjacency matrices $\{A_1, A_2, A_3, A_4\}$.Also we see that: $(i).A_1 + A_4 = I_2$. This implies that the set $\{A_1, A_2\}$ is a partition of diagonal relation I_2. $(ii).A_1 + A_2 + A_3 + A_4 = J_2$. This implies that the set $\{A_1, A_2, A_3, A_4\}$ is a partition of all 1-matrix J_2. of order 2.

$(iii).A_1' = A_1, A_2' = A_3, A_3' = A_2, A_4' = A_4$. This implies that the set $\{A_1, A_2, A_3, A_4\}$ is closed under matrix transpositions.Hence, product of any two adjacency matrices is some linear combinations of adjacency matrices $\{A_1, A_2, A_3, A_4\}$.Thus, the set $C = \{C_1, C_2, C_3, C_4\}$ is a Coherent Configuration.Consider the matrix

$F = 1.A_1 + 1.A_2 + 1.A_3 + \omega.A_4$. $F = \begin{bmatrix} 1 & 1 \\ 1 & \omega \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$, $\omega = \exp(2\pi i/2) = \cos\pi + i\sin\pi = -1$, *so* $\omega^2 = 1$.

F is equivalent todephsed form of aFourier matrix of order 2.

3.2. CONSTRUCTION OF FOURIER MATRIX OF ORDER 3:

Let us consider $X = \{i : i = 1,2,3\}$ and a partition $C = \{C_1, C_2, C_3, C_4, C_5\}$ of $X \times X$.Where $C_1 = \{(i,i) : i = 1\}$, $C_2 = \{(1,i) : i = 2,3\}, C_3 = \{(i,1) : i = 2,3\}, C_4 = \{(i,i) : i = 2,3\}, C_5 = \{(2,i) : i = 3\} \cup \{(3,i) : i = 2\}$, Then adjacency matrices A_1, A_2, A_3, A_4 and A_5 of C_1, C_2, C_3, C_4 and C_5 are respectively given below: $A_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$,

$A_2=\begin{bmatrix}0&1&1\\0&0&0\\0&0&0\end{bmatrix}$, $A_3=\begin{bmatrix}0&0&0\\1&0&0\\1&0&0\end{bmatrix}$, $A_4=\begin{bmatrix}0&0&0\\0&1&0\\0&0&1\end{bmatrix}$, $A_5=\begin{bmatrix}0&0&0\\0&0&1\\0&1&0\end{bmatrix}$, We see the following calculations:

$(i).A_1^2=A_1, A_1A_2=A_2, A_1A_3=0, A_1A_4=0, A_1A_5=0, A_2A_1=0, A_3A_1=A_3, A_4A_1=0, A_5A_1=0.$

$(ii).A_2^2=0, A_2A_3=2A_1, A_2A_4=A_2, A_2A_5=A_2, A_3A_2=A_4+A_5, A_4A_2=0, A_5A_2=0.$

$(iii).A_3^2=0, A_3A_4=0, A_3A_5=0, A_4A_3=A_3, A_5A_3=A_3.$ $(iv).A_4^2=A_4, A_4A_5=A_5, A_5A_4=A_5.$ $(v).A_5^2=A_4.$

Hence,product of any two adjacency matrices is some linear combination of adjacency matrices $\{A_1, A_2, A_3, A_4, A_5\}$. Also we see that: $(i).A_1+A_4=I_3$. This implies that $\{A_1, A_4\}$ is a partition of diagonal relation I_3

$(ii).A_1+A_2+A_3+A_4+A_5=J_3$. .This implies that $\{A_1, A_2, A_3, A_4, A_5\}$ is a partition of all 1 matrix J_3 oforder 3.

$(iii).A_1^{'}=A_1^{'}, A_2^{'}=A_3, A_3^{'}=A_2, A_4^{'}=A_4, A_5^{'}=A_5$ .This implies that the set $\{A_1, A_2, A_3, A_4, A_5\}$ is closed under matrix transposition.Thus, the set $C=\{C_1, C_2, C_3, C_4, C_5\}$ is a Coherent Configuration. Consider the matrix

$F=1.A_1+1.A_2+1.A_3+\omega.A_4+\omega^2A_5.$ $F=\begin{bmatrix}1&1&1\\1&\omega&\omega^2\\1&\omega^2&\omega\end{bmatrix}$, Where $\omega=\exp(2\pi i/3)$ so $\omega^3=1.$ F isequivalent to dephased form ofa Fourier matrix of order 3.

3.3. CONSTRUCTION OF FOURIER MATRIX OF ORDER 5:

Let us consider $X=\{i: i=2,3,4,5\}$ and a partition $C=\{C_1, C_2, C_3, C_4, C_5, C_6, C_7\}$ of $X\times X$. Where

$C_1=\{(i,i): i=1\}$, $C_2=\{(i,1): i=2,3,4,5\}$, $C_3=\{(i,1): i=2,3,4,5\}$, $C_4=\{(i,i): i=2,3,4,5\}$,

$C_5=\{\{(2,i): i=3\}\cup\{(3,i): i=2\}\cup\{(4,i): i=5\}\cup\{(5,i): i=4\}\}$,

$C_6=\{\{(2,i): i=4\}\cup\{(3,i): i=5\}\cup\{(4,i): i=2\}\cup\{(5,i): i=3\}\}$,

$C_7=\{\{(2,i): i=5\}\cup\{(3,i): i=4\}\cup\{(4,i): i=3\}\cup\{(5,i): i=2\}\}$. Then adjacency matrices

$A_1, A_2, A_3, A_4, A_5, A_6$ and A_7 of $C_1, C_2, C_3, C_4, C_5, C_6$ and C_7 are respectively given below:

$A_1=\begin{bmatrix}1&0&0&0&0\\0&0&0&0&0\\0&0&0&0&0\\0&0&0&0&0\\0&0&0&0&0\end{bmatrix}$, $A_2=\begin{bmatrix}0&1&1&1&1\\0&0&0&0&0\\0&0&0&0&0\\0&0&0&0&0\\0&0&0&0&0\end{bmatrix}$, $A_3=\begin{bmatrix}0&0&0&0&0\\1&0&0&0&0\\1&0&0&0&0\\1&0&0&0&0\\1&0&0&0&0\end{bmatrix}$, $A_4=\begin{bmatrix}0&0&0&0&0\\0&1&0&0&0\\0&0&1&0&0\\0&0&0&1&0\\0&0&0&0&1\end{bmatrix}$,

$A_5=\begin{bmatrix}0&0&0&0&0\\0&0&1&0&0\\0&1&0&0&0\\0&0&0&0&1\\0&0&0&1&0\end{bmatrix}$, $A_6=\begin{bmatrix}0&0&0&0&0\\0&0&0&1&0\\0&0&0&0&1\\0&1&0&0&0\\0&0&1&0&0\end{bmatrix}$, $A_7=\begin{bmatrix}0&0&0&0&0\\0&0&0&0&1\\0&0&0&1&0\\0&0&1&0&0\\0&1&0&0&0\end{bmatrix}$,

We see the following calculations:

$(i.).A_1^2=A_1, A_1A_2=A_2, A_1A_3=0, A_1A_4=0, A_1A_5=0,$ $(ii.).A_2^2=0, A_2A_3=4A_1, A_2A_4=A_2, A_2A_5=A_2,$

$A_1A_6=0, A_1A_7=0, A_2A_1=0, A_3A_1=0, A_4A_1=0,$ $A_2A_6=A_2, A_2A_7=A_2, A_3A_2=A_4+A_5+A_6+A_7,$

$A_5A_1=0, A_6A_1=0, A_7A_1=0.$ $A_4A_2=0, A_5A_2=0, A_6A_2=0, A_7A_2=0.$

$(iii.).A_3^2=0, A_3A_4=0, A_3A_5=0, A_3A_6=0, A_3A_7=0,$ $(iv.).A_4^2=A_4, A_4A_5=A_5, A_4A_6=A_6, A_4A_7=A_7,$

$A_4A_3=A_3, A_5A_3=A_3, A_6A_3=A_3, A_7A_3=A_3.$ $A_5A_4=A_5, A_6A_4=A_6, A_7A_4=A_7.$

$(v.).A_5^2 = A_4, A_5A_6 = A_7, A_5A_7 = A_6, A_6A_5 = A_7, A_7A_5 = A_6.$ $(vi.).A_6^2 = A_4, A_6A_7 = A_4, A_7A_6 = A_7.$ $(vii.).A_7^2 = A_4.$ We also see that $(i).A_1 + A_4 = I_5.$ This implies that $\{A_1, A_4\}$ is a partition of diagonal relation I_5. $(ii).A_1 + A_2 + A_3 + A_4 + A_5 + A_6 + A_7 = J_5.$ This implies that $\{A_1, A_2, A_3, A_4, A_5, A_6, A_7\}$ is a partition of all 1 matrix J_5 of order 5. $(iii).A_1' = A_1, A_2' = A_3, A_3' = A_2, A_4' = A_4, A_5' = A_5, A_6' = A_6, A_7' = A_7.$ This implies that the set $\{A_1, A_2, A_3, A_4, A_5, A_6, A_7\}$ is closed under matrix transposition.Hence, product of any two adjacency matrices is some linear combinations of adjacency matrices $\{A_1, A_2, A_3, A_4, A_5, A_6, A_7\}$. Thus, the set $C = \{C_1, C_2, C_3, C_4, C_5, C_6, C_7\}$ is a Coherent Configuration. Consider the matrix. $F = 1.A_1 + 1.A_2 + 1.A_3 + \omega.A_4 + \omega^2.A_5 + \omega^3 A_6 + \omega^4 A_7.$

$$F = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & \omega & \omega^2 & \omega^3 & \omega^4 \\ 1 & \omega^2 & \omega & \omega^4 & \omega^3 \\ 1 & \omega^3 & \omega^4 & \omega & \omega^2 \\ 1 & \omega^4 & \omega^3 & \omega^2 & \omega \end{bmatrix},$$ Where $\omega = \exp(2\pi i/5)$ so $\omega^5 = 1.$ F isequivalent to dephased form of a Fourier matrix of order five.

3.4. CONSTRUCTION OF FOURIER MATRIX OF ORDER 7:

Let us consider $X = \{i : i = 1,2,3,4,5,6,7\}$ and a partition $C = \{C_1, C_2, C_3, C_4, C_5, C_6, C_7, C_8, C_9\}$ of $X \times X$. Where:

$C_1 = \{(i,i) : i = 1\}, C_2 = \{(1,i) : i = 2,3,4,5,6,7\}, C_3 = \{(i,1) : i = 2,3,4,,5,6,7\}, C_4 = \{(i,i) : i = 2,3,4,5,6,7\}$

$C_5 = \{\{(2,i) : i = 5\} \cup \{(3,i) : i = 2\} \cup \{(4,i) : i = 6\} \cup \{(5,i) : i = 3\} \cup \{(6,i) : i = 7\} \cup \{(7,i) : i = 4\}\},$

$C_6 = \{\{(2,i) : i = 6\} \cup \{(3,i) : i = 4\} \cup \{(4,i) : i = 2\} \cup \{(5,i) : i = 6\} \cup \{(6,i) : i = 5\} \cup \{(7,i) : i = 3\}\},$

$C_8 = \{\{(2,i) : i = 4\} \cup \{(3,i) : i = 7\} \cup \{(4,i) : i = 3\} \cup \{(5,i) : i = 6\} \cup (6,i) : i = 2\} \cup \{(7,i) : i = 5\}\},$

$C_9 = \{\{(2,i) : i = 7\} \cup \{(3,i) : i = 6\} \cup \{(4,i) : i = 5\} \cup \{(5,i) : i = 4\} \cup (6,i) : i = 3\} \cup \{(7,i) : i = 2\}\}.$ Then adjacency matrices $A_1, A_2, A_3, A_4, A_5, A_6, A_7, A_8$ and A_9 of $C_1, C_2, C_3, C_4, C_5, C_6, C_7, C_8,$ amd C_9 are respectively given below:

$$F_1 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} F_2 = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} F_3 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$A_1 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} A_2 = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} A_3 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$A_4=\begin{bmatrix}0&0&0&0&0&0&0\\0&1&0&0&0&0&0\\0&0&1&0&0&0&0\\0&0&0&1&0&0&0\\0&0&0&0&1&0&0\\0&0&0&0&0&1&0\\0&0&0&0&0&0&1\end{bmatrix}\quad A_5=\begin{bmatrix}0&0&0&0&0&0&0\\0&0&0&0&1&0&0\\0&1&0&0&0&0&0\\0&0&0&0&0&1&0\\0&0&1&0&0&0&0\\0&0&0&0&0&0&1\\0&0&0&1&0&0&0\end{bmatrix}\quad A_6=\begin{bmatrix}0&0&0&0&0&0&0\\0&0&0&0&0&1&0\\0&0&0&1&0&0&0\\0&1&0&0&0&0&0\\0&0&0&0&0&0&1\\0&0&0&0&1&0&0\\0&0&1&0&0&0&0\end{bmatrix}$$

$$A_7=\begin{bmatrix}0&0&0&0&0&0&0\\0&0&1&0&0&0&0\\0&0&0&0&1&0&0\\0&0&0&0&0&0&1\\0&1&0&0&0&0&0\\0&0&0&1&0&0&0\\0&0&0&0&0&1&0\end{bmatrix}\quad A_8=\begin{bmatrix}0&0&0&0&0&0&0\\0&0&0&1&0&0&0\\0&0&0&0&0&0&1\\0&0&1&0&0&0&0\\0&0&0&0&0&1&0\\0&1&0&0&0&0&0\\0&0&0&0&1&0&0\end{bmatrix}\quad A_9=\begin{bmatrix}0&0&0&0&0&0&0\\0&0&0&0&0&0&1\\0&0&0&0&0&1&0\\0&0&0&0&1&0&0\\0&0&0&1&0&0&0\\0&0&1&0&0&0&0\\0&1&0&0&0&0&0\end{bmatrix}$$

We see the following calculations:

$(i.).A_1^2=A_1, A_1A_2=A_2, A_1A_3=0, A_1A_4=0, A_1A_5=0, A_1A_6=0, A_1A_7=0, A_1A_8=0, A_1A_9=0, A_2A_1=0,$

$(ii.).A_2^2=0, A_2A_3=3A_1, A_2A_4=A_2, A_2A_5=A_2, A_2A_6=A_2, A_2A_7=A_2, A_2A_8=A_2, A_2A_9=A_2,$

$A_3A_1=A_3, A_4A_1=0, A_5A_1=0, A_6A_1=0, A_7A_1=0, A_8A_1=0, A_9A_1=0.$

$A_3A_2=A_4+A_5+A_6+A_7+A_8+A_9, A_4A_2=0, A_5A_2=0, A_6A_2=0, A_7A_2=0, A_8A_2=0, A_9A_2=0.$

$(iii.).A_3^2=0, A_3A_4=0, A_3A_5=0, A_3A_6=0, A_3A_7=0, A_3A_8=0, A_3A_9=0, A_4A_3=A_3, A_5A_3=A_3, A_6A_3=A_3, A_7A_3=A_3, A_8A_3=A_3, A_9A_3=A_3.$

$(iv.).A_2^4=A_4, A_4A_5=A_5, A_4A_6=A_6, A_4A_7=A_7, A_4A_8=A_8, A_4A_9=A_9, A_5A_4=A_5, A_6A_4=A_6, A_7A_4=A_7, A_8A_4=A_8, A_9A_4=A_9.$

$(v.).A_5^2=A_7, A_5A_6=A_9, A_5A_7=A_4, A_5A_8=A_6, A_5A_9=A_8,$

$(vi.).A_6^2=A_5, A_6A_7=A_8, A_6A_8=A_4, A_6A_9=A_7, A_6A_5=A_9, A_7A_5=A_4, A_8A_5=A_6, A_9A_5=A_8.$

$A_7A_6=A_8, A_8A_6=A_4, A_9A_6=A_7.$

$(vii..).A_7^2=A_5, A_7A_8=A_9, A_7A_9=A_6, A_8A_7=A_9, A_9A_7=A_6.$ $(viii.).A_8^2=A_7, A_8A_9=A_5, A_9A_8=A_5.$

$(ix.).A_9^2=A_4.$ We see that $(i).A_1+A_2+A_3+A_4+A_5+A_6+A_7+A_8+A_9=J_7.$ This implies that $\{A_1,A_2,A_3,A_4,A_5,A_6,A_7,A_8,A_9\}$ is a partition of all 1 matrix J_7 of order 7. $(ii).A_1+A_4=I_7$ This implies that $\{A_1,A_4\}$ is a partition of diagonal relation I_7.

$(iii).A_1^{'}=A_1, A_2^{'}=A_3, A_3^{'}=A_2, A_4^{'}=A_4, A_5^{'}=A_7, A_6^{'}=A_8, A_7^{'}=A_5, A_8^{'}=A_6, A_9^{'}=A_9.$ This implies that the set $\{A_1,A_2,A_3,A_4,A_5,A_6,A_7,A_8,A_9\}$ is closed under matrix transposition.Hence, product of any two adjacency matrices is some linear combinations of adjacency matrices $\{A_1,A_2,A_3,A_4,A_5,A_6,A_7,A_8,A_9\}$. Thus, the set $C=\{C_1,C_2,C_3,C_4,C_5,C_6,C_7\}$ is a Coherent Configuration. Consider the matrix.

$$F=1.A_1+1.A_2+1.A_3+\omega.A_4+\omega^2.A_5+\omega^3.A_6+\omega^4A_7+\omega^5A_8+\omega^6A_9$$

$$F=\begin{bmatrix}1&1&1&1&1&1&1\\1&\omega&\omega^4&\omega^5&\omega^2&\omega^3&\omega^6\\1&\omega^2&\omega&\omega^3&\omega^4&\omega^6&\omega^5\\1&\omega^3&\omega^5&\omega&\omega^6&\omega^2&\omega^4\\1&\omega^4&\omega^2&\omega^6&\omega&\omega^5&\omega^3\\1&\omega^5&\omega^6&\omega^4&\omega^3&\omega&\omega^2\\1&\omega^6&\omega^3&\omega^2&\omega^5&\omega^4&\omega\end{bmatrix}$$

Where $\omega = \exp(2\pi i/7)$ *so* $\omega^7 = 1$. F is equivalent to dephased form of a Fourier matrix of order 7.On the basis of above result we state the following conjecture:

3.5.Conjecture: there exist a coherent configuration C defined on a finite set X of size p where p is a prime number so that the suitable linear combination of adjacency matrices of the coherent configuration C form a matrix equivalent to Fourier matrix of order p.

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