Power Series Method for Solving Kolmogorov-Petrovsky-Piskunov Equation

[1]Pragneshkumar R Makwana, [2]Amit K. Parikh

[1]Research Scholar and Assistant professor, [2]Professor and Principal
[1] Department of Mathematics,
[1]C. U. Shah University, Wadhwan City, India.

Abstract: We propose a Power Series Method (PSM) to solve Kolmogorov-Petrovsky-Piskunov (KPP) equation with initial and boundary conditions and obtain exact solution in the form of known series. This shows that the propose method is very simple, efficient and reliable. The present paper is organized in four section. First section described introduction and literature review. In section 2, we present the basic idea of Power series method. In section 3, we obtained exact solution of Kolmogorov-Petrovsky-Piskunov (KPP) equation using Power Series Method (PSM). Finally, a conclusion is given in section 4.

Index Terms **- Power Series Method, Kolmogorov-Petrovsky-Piskunov equation, Boundary Value Problems.**

I. INTRODUCTION

Boundary value problems (BVPs) arise in many areas such as applied mathematics, sciences, engineering and several other fields. Recently many methods are available to solve boundary value problems, namely, Adomain Decomposition Method [4], Homotopy Perturbation Method [5], Variational Iteration Method [11], Differential Transform Method [6] and so on.

We consider Kolmogorov-Petrovsky-Piskunov equation

$$\frac{\partial z}{\partial t} = k\frac{\partial^2 z}{\partial x^2} + f(z)$$

where $k > 0$ is the diffusion coefficient and $f(z)$ is linear or nonlinear term. The KPP equation first appeared in the context of genetics model for the spread of an advantageous gene through a population. After, it has been applied to a number of biological, chemical and physics model. For some particular $f(z)$, we obtained exact solution of the KPP equation via symmetric method [8] and bi-linear method [12].

In this study, Power Series Method (PSM) is proposed to solve Kolmogorov-Petrovsky-Piskunov equation with initial and boundary conditions. Ameina S. Nuseir applied the PSM to solve Nonlinear Systems of Partial Differential Equations [1]. Moreover, the method was successfully applied to solve One-dimensional Time Dependent Reaction-Diffusion Equation [10], Non-linear Partial Differential Equations [7] and Nonlinear Diffusion Equations [9].

II. BASIC IDEA OF POWER SERIES METHOD (PSM)

Let $z(x,t)$ be an analytic function in the domain $G \subseteq R^2$. The function $z(x,t)$ is of the form

$$z(x,t) = \sum_{i=0}^{\infty}\sum_{j=0}^{\infty} c_{ij}\, x^i\, t^j \tag{1}$$

Differentiating both side of equation (1) partially with respect to x and t, we get series expansion of $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial t}$ are as follows:

$$\frac{\partial z}{\partial x} = \sum_{i=0}^{\infty}\sum_{j=0}^{\infty} (i+1)c_{(i+1)j}\, x^i\, t^j \tag{2}$$

$$\frac{\partial z}{\partial t} = \sum_{i=0}^{\infty}\sum_{j=0}^{\infty} (j+1)c_{i(j+1)}\, x^i\, t^j \tag{3}$$

Again differentiating both side of equation (2) partial with respect to x, we get series expansion of $\frac{\partial^2 z}{\partial x^2}$ as follows:

$$\frac{\partial^2 z}{\partial x^2} = \sum_{i=0}^{\infty}\sum_{j=0}^{\infty} (i+1)(i+2)c_{(i+2)j}\, x^i\, t^j \tag{4}$$

III. EXACT SOLUTION OF KOLMOGOROV-PETROVSKY-PISKUNOV (KPP) EQUATION USING PSM

Consider the following Kolmogorov-Petrovsky-Piskunov (KPP) equation [2, 3]:

$$\frac{\partial z}{\partial t} = \frac{\partial^2 z}{\partial x^2} - z, \quad (x,t) \in \Omega \subset R^2 \tag{5}$$

with initial and boundary conditions

$$z(x,0) = e^{-x} + x, \qquad x \in R \tag{6}$$

$$z(0,t) = 1 \tag{7}$$

$$\left(\frac{\partial z}{\partial t}\right)_{(1,t)} = e^{-t} - 1, \qquad t \in R \tag{8}$$

Let equation (1) be a series solution of the equation (5). Substituting equation (1), (3) and (4) into equation (5), to obtain the following recurrence relation

$$c_{i(j+1)} = \frac{1}{j+1}\left[(i+1)(i+2)c_{(i+2)j} - c_{ij}\right], \forall\, i,j \geq 0 \tag{9}$$

The initial condition (6) should be transformed as follows:

$$c_{i0} = \begin{cases} 1, & if\ i = 0 \\ 0, & if\ i = 1 \\ \dfrac{(-1)^i}{i!}, & if\ i \geq 2 \end{cases} \tag{10}$$

Also the boundary conditions (7) and (8) should be transformed as follows:

$$c_{0j} = \begin{cases} 1, & if\ j = 0 \\ 0, & if\ j \geq 1 \end{cases} \tag{11}$$

and

$$c_{1j} = \begin{cases} 1, & if\ j = 0 \\ 0, & if\ j = 1 \\ \dfrac{(-1)^j}{j!}, & if\ j \geq 2 \end{cases} \tag{12}$$

Substituting equations (10), (11) and (12) into equation (9), and by recursive method, to get all other c_{ij} are zero. Substituting all values of c_{ij} in equation (1) to obtain the following approximate solution

$$z(x,t) = 1 - xt + \frac{1}{2}xt^2 - \frac{1}{6}xt^3 + \frac{1}{24}xt^4 - \frac{1}{120}xt^5 + \cdots + \frac{1}{2}x^2 - \frac{1}{6}x^3 + \frac{1}{24}x^4 - \frac{1}{120}x^5 + \cdots \tag{13}$$

Rewrite approximate solution (13), we get the exact solution

$$z(x,t) = xe^{-t} + e^{-x} \tag{14}$$

IV. Conclusion

In this paper, we have shown that the Power Series Method (PSM) is suitable for solving Kolmogorov-Petrovsky-Piskunov equation. The present study has confirmed that the PSM offers great advantage of applying initial condition as well as boundary conditions. Hence, we can conclude that the PSM is powerful and effective method in finding approximate exact solution of initial value problems as well as boundary value problems.

V. Acknowledgment

I thankful to the Department of Mathematics, C. U. Shah University for providing research facilities.

References

[1] Nuseir, A. and Al-Hasoon, A. 2012. Power Series Solutions for Nonlinear Systems of Partial Differential Equations. Applied Mathematical Sciences, 6(104), 5147-5159.

[2] Yildirim, A. 2009. Application of He's homotopy perturbation method for solving the Cauchy reaction diffusion problem. Computers and Mathematics with Applications, 57(4), 612-618.

[3] Lesnic, D. 2007. The decomposition method for Cauchy reaction–diffusion problems. Applied Mathematics Letters, 20(04), 412-418.

[4] Emad, H., Abdelhalim E. and Randolph, R. 2012. Advances in the Adomian decomposition method for solving two-point nonlinear boundary value problems with Neumann boundary conditions. Computers and Mathematics with Applications, 63(06), 1056-1065.

[5] He, J. 2006. Homotopy perturbation method for solving boundary value problems. Physics Letters A, 350(1-2), 87-88.

[6] Xi, L., Zhou, C. and Xu, S. 2016. A new algorithm based on differential transform method for solving multi-point boundary value problems. International Journal of Computer Mathematics, 93(06), 981-994.

[7] Roopashree, N. and Nargund, L. 2016. Power Series Solution of Non-linear Partial Differential Equations. International Journal of Mathematics and Computer Research, 4(6), 1514-1520.

[8] Clarksonz, P. and Mansfield, E. 1994. Symmetry reductions and exact solutions of a class of nonlinear heat equations. Physica D: Nonlinear Phenomena, 70(3), 250-288.

[9] Makwana, P. and Parikh, A. 2017. Approximate Solution of Nonlinear Diffusion Equation Using Power Series Method (PSM). International Journal of Current Advanced Research, 6(10), 6374-6377.

[10] Makwana, P. and Parikh, A. 2018. Solution of One-dimensional Time Dependent Reaction-Diffusion Equation by Power Series Method (PSM). The Proceedings of 3rd International Conference on Emerging Global Trends in Humanities, Science, Technology, Commerce and Management Education (ICEGTE-2018), 183-186 [ISBN: 978-93-86608-48-2].

[11] Momani, S., Abuasad, S. and Odibat, Z. 2006. Variational iteration method for solving nonlinear boundary value problems. Applied Mathematics and Computation, 183(2), 1351-1358.

[12] Kawahara, T. and Tanaka, M. 2006. Interactions of traveling fronts: An exact solution of a nonlinear diffusion equation. Physics Letters A, 97(8), 311-314.