HAM FOR INFLUENCE OF MAGNETIC FIELD ON WATER BASED HYBRID NANOFLUID FLOW

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Abstract : In this analysis, effect of magnetic field on heat transfer features of three-dimensional water based hybrid nanofluid flow between two horizontal parallel plates in a rotating system is examined. Flow characteristics of conventional fluid (water), nanofluids CuO-water and Al2O3 and hybrid nanofluid under influence of various parameters are compared. Effects of rotation, nanoparticle volume fraction, nanoparticle size on thermal conductivity are considered. Resulting system of differential equations is solved using HAM. Physics of the problem is understood by discussing effects of Reynolds number, Magnetic parameter, nanoparticle volume fraction, Eckert number, Permeability parameter, Rotation parameter and Prandtl number on Heat transfer.

Key words: **Hybrid Nanofluid; Heat transfer; HAM; MHDIntroduction**

Nomenclature

B	External uniform magnetic field
C_p	Specific heat at constant pressure
g	Acceleration due to gravity
k	Thermal conductivity
k_1	Permeability of the fluid
L	Distance between the plates
M	Magnetic parameter (Ratio of Lorentz force to viscous force)
Pr	Prandtl number (ratio of momentum diffusivity to thermal diffusivity)
T	Temperature
u, v, w	Velocity components along x, y, z axes, respectively

Greek symbols

η	Dimensionless variable
θ	Dimensionless temperature $(\theta = \frac{T-T_L}{T_w-T_L})$
κ	Permeability (dimensionless)
μ	Dynamic viscosity
ν	Kinematic viscosity
Ω	Constant Rotation velocity
ρ	Density
σ	Electrical conductivity (S/m)
φ	Porosity
$\emptyset_1$	Al_2O_3-Nanoparticle volume fraction
$\emptyset_2$	CuO-Nanoparticle volume fraction

Subscripts

f	Fluid phase
nf	Nano-fluid
s	Solid phase

I. INTRODUCTION

The restriction of the conventional fluids to expedite cooling/heating rates give rise to exploration of nanofluids. Generally, water based single phase nanofluids containing nanoparticles such as CuO or Al2O3 are discussed. Enhancement of heat transfer is beneficial in engineering and actual world problems. To achieve this, experiments considering hybrid nanoparticles in place of single nanoparticle based nanofluids are performed. Consequently, investigators are fascinated towards heat transfer properties of hybrid nanofluids. Enhancement of heat transfer is vital in Industrial and engineering processes. High thermal conductivity of nano fluids is a boon in this direction, thus many researchers are working intensively considering heat transfer properties of nanofluids.

Kataria and Mittal [1] analyzed velocity, mass and temperature of nanofluid flow in a porous medium.

In real world problems, thermal radiation is evident which is reflected in study of various researchers. Kataria and Mittal [2] discussed optically thick nanofluid flow past an oscillating vertical plate in presence of radiation.

Kataria and Patel [3-7] studied MHD flow considering different types of fluids. Sheikholeslami et al. [8] analyzed thermal radiation effects on Fe3O4- H2O nanofluid flow.

Effect of magnetic field on heat and mass transfer in presence of thermal radiation between horizontal parallel is studied in the present investigation. System considered is a rotating system.

Novelty of the present work is the inclusion of often neglected effects such as Brownian motion, thermal interfacial resistance, nanoparticle size, nanoparticle volume fraction and micro mixing in suspensions. Homotopy analysis method is employed to solve the resulting system of ordinary differential equations. The effects of relevant parameters on nanofluid flow are discussed in detail.

II. PROBLEM STATEMENT

It is assumed that CuO-Water nanofluid flows between two horizontal parallel plates placed L units apart through a porous medium. A coordinate system (x, y, z) is such that origin is at the lower plate. Here we consider hybrid nanofluid. Where fluid is water based and CuO, Al2O3 are nanoparticles. The lower plate is stretched by two equal forces in opposite directions. The plates along with the fluid rotate about y axis with angular velocity Ω. A uniform magnetic flux with density B is applied along y-axis. Under these assumptions, governing equations are:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \tag{1}$$

$$\rho_{nf}\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+2\Omega w\right)=\mu_{nf}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)-\sigma_{nf}B^2u-\frac{\mu_{nf}\varphi}{k_1}u \tag{2}$$

$$\rho_{nf}\left(v\frac{\partial v}{\partial y}\right)=\mu_{nf}\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right) \tag{3}$$

$$\rho_{nf}\left(u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}-2\Omega w\right)=\mu_{nf}\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}\right)-\sigma_{nf}B^2w-\frac{\mu_{nf}\varphi}{k_1}w \tag{4}$$

$$(\rho c_p)_{nf}\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}+w\frac{\partial T}{\partial z}\right)=k_{nf}\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}\right)+\mu_{nf}\left(2\left[\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial v}{\partial y}\right)^2+\left(\frac{\partial w}{\partial z}\right)^2\right]+\left(\frac{\partial v}{\partial x}\right)^2+\left(\frac{\partial v}{\partial z}\right)^2+\left(\frac{\partial w}{\partial x}+\frac{\partial u}{\partial z}\right)^2\right) \tag{5}$$

where

$$\rho_{hnf}=\{(1-\emptyset_2)[(1-\emptyset_1)\rho_{bf}+\emptyset_1\rho_{s1}]\}+\emptyset_2\rho_{s2} \tag{6}$$

$$\frac{\sigma_{hnf}}{\sigma_{bf}}=\frac{\sigma_{S_2}+2\sigma_{bf}-2\emptyset_2(\sigma_{bf}-\sigma_{S_2})}{\sigma_{S_2}+2\sigma_{bf}+\emptyset_2(\sigma_{bf}-\sigma_{S_2})}, \tag{7}$$

$$\frac{\sigma_{bf}}{\sigma_f}=\frac{\sigma_{S_1}+2\sigma_f-2\emptyset_1(\sigma_f-\sigma_{S_1})}{\sigma_{S_1}+2\sigma_f+\emptyset_1(\sigma_f-\sigma_{S_1})} \tag{8}$$

$$(\rho c_p)_{hnf}=\left\{(1-\emptyset_2)\left[(1-\emptyset_1)(\rho c_p)_f+\emptyset_1(\rho c_p)_{s1}\right]\right\}+\emptyset_2(\rho c_p)_{s2} \tag{9}$$

$$\frac{k_{hnf}}{k_{bf}}=\frac{k_{S_2}+(n-1)k_{bf}-(n-1)\emptyset_2(k_{bf}-k_{S_2})}{k_{S_2}+(n-1)k_{bf}+\emptyset_2(k_{bf}-k_{S_2})}, \tag{10}$$

$$\frac{k_{bf}}{k_f}=\frac{k_{S_1}+(n-1)k_f-(n-1)\emptyset_1(k_f-k_{S_1})}{k_{S_1}+(n-1)k_f+\emptyset_1(k_f-k_{S_1})}, \tag{11}$$

$$\mu_{hnf}=\frac{\mu_{bf}}{(1-\emptyset_1)^{2.5}(1-\emptyset_2)^{2.5}} \tag{12}$$

Therefore, the governing momentum and energy equations for this problem are given in dimensionless form by:

$$a_1f^{iv}-Re(f'f''-ff''')-2Krg'-\left(a_3M^2+\frac{a_1}{\kappa}\right)f''=0, \tag{13}$$

$$a_1g''-Re(f'g-fg')+2Krf'-\left(a_3M^2+\frac{a_1}{\kappa}\right)g=0 \tag{14}$$

$$\theta''+Pr\left(Rea_2f\theta'+Eca_4(4f'^2+g^2)\right)=0 \tag{15}$$

$$Pr=\frac{\mu_{bf}(c_p)_{bf}}{k_{bf}},\quad M^2=\frac{\sigma_{bf}{B_0}^2L^2}{\rho_{bf}\nu_{bf}},\quad \frac{1}{\kappa}=\frac{\nu\varphi^2}{k_1\nu_{bf}},\quad Kr=\frac{\Omega L^2}{\nu_{bf}},\quad Re=\frac{aL^2}{\nu_{bf}},\quad Ec=\frac{(aL)^2}{(c_p)_{bf}(\theta_0-\theta_L)}, \tag{16}$$

$$b_0=(1-\emptyset_1)(1-\emptyset_2) \tag{17}$$

$$b_1=(b_0+(1-\emptyset_2)\emptyset_1\frac{\rho_{s1}}{\rho_f}+\emptyset_2\frac{\rho_{s2}}{\rho_f}), \tag{18}$$

$$b_2 = \frac{1}{{b_0}^{2.5}} \tag{19}$$

$$b_3 = \left(b_0 + (1-\emptyset_2)\emptyset_1 \frac{(\rho)_{s1}(c_p)_{s1}}{(\rho)_{bf}(c_p)_{bf}} + \emptyset_2 \frac{(\rho)_{s2}(c_p)_{s2}}{(\rho)_{bf}(c_p)_{bf}} \right), \tag{20}$$

$$b_4 = \frac{k_{hnf}}{k_{bf}}, \tag{21}$$

$$b_5 = \frac{\sigma_{hnf}}{\sigma_{bf}} \tag{22}$$

$$a_1 = \frac{1}{{b_0}^{2.5} b_1}, \tag{23}$$

$$a_2 = \frac{b_3}{b_4}, \tag{24}$$

$$a_3 = \frac{b_5}{b_1}, \tag{25}$$

$$a_4 = \frac{b_2}{b_4}, \tag{26}$$

III. SOLUTION BY HOMOTOPY ANALYSIS METHOD

HAM is a fundamental concept of topology. Equations (13) – (15) are coupled non-linear ordinary differential equations and exact solutions are not possible. To solve these equations together with the boundary conditions, the modified homotopy analysis method (HAM) suggested by Liao [09] is employed.

Subject to

$$f = 0, f' = 1, g = 0, \theta = 1 \; at \; \eta = 0, \qquad f = 0, f' = 0, g = 0, \theta = 0 \; at \; \eta = 1 \tag{27}$$

Initial guess is given by:

$$f_0(\eta) = \frac{-2}{e^2-4e+3} + \frac{e-1}{e-3}\eta + \frac{2-e}{e^2-4e+3} e^{\eta} + \frac{e}{e^2-4e+3} e^{-\eta}; \; g_0(\eta) = 0; \; \theta_0(\eta) = 1-\eta; \tag{28}$$

with auxiliary linear operators:

$$L_f = \frac{\partial^4 f}{\partial \eta^4} - \frac{\partial^2 f}{\partial \eta^2}, \; L_g = \frac{\partial^2 g}{\partial \eta^2} - \frac{\partial g}{\partial \eta}, \; L_\theta = \frac{\partial^2 \theta}{\partial \eta^2} \tag{29}$$

Satisfying

$$L_f(C_1 + C_2\eta + C_3 e^{\eta} + C_4 e^{-\eta}) = 0, \; L_g(C_5 + C_6 e^{\eta}) = 0, \; L_\theta(C_7 + C_8\eta) = 0. \tag{30}$$

where $c_1, \; c_2, \ldots, c_8$ are the arbitrary constants.

Convergence of the HAM solutions depend on the values of the auxiliary parameters $\hbar_f, \hbar_g$ and $\hbar_\theta$.

IV. RESULTS AND DISCUSSION:

This section is dedicated to the physics of the problem through graphical representation of velocity and temperature profiles. Solutions are obtained using Mathematica. Effects of different parameters: Magnetic parameter M, Rotation parameter Kr, Permeability parameter κ, Prandtl number Pr and Schmidt Number Sc on fluid flow is represented through Figs 1 - 8.

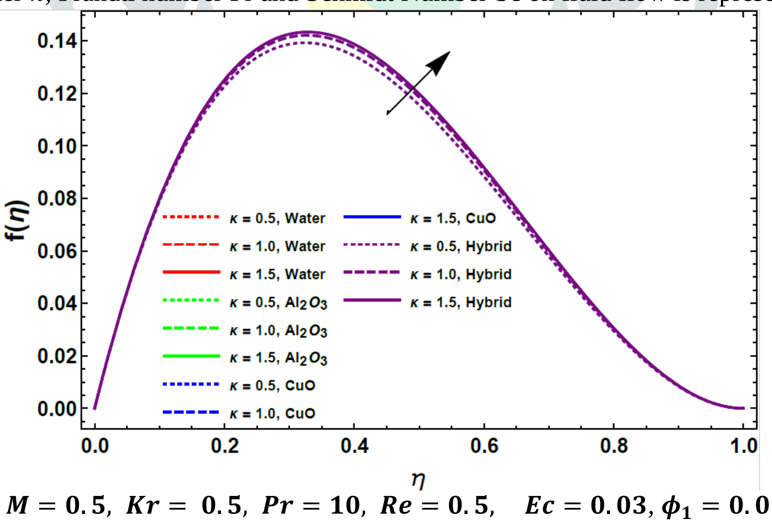

Fig 1: f for $\boldsymbol{\eta}$ **and** $\boldsymbol{\kappa}$ **at** $\boldsymbol{M = 0.5}$, $\boldsymbol{Kr = 0.5}$, $\boldsymbol{Pr = 10}$, $\boldsymbol{Re = 0.5}$, $\boldsymbol{Ec = 0.03}$, $\boldsymbol{\phi_1 = 0.03}$, $\boldsymbol{\phi_2 = 0.03}$.

Figures 1 shows effects of Permeability parameter κ on velocity profiles. It is seen that velocity increases in x direction. This is true due to the effects of drag force.

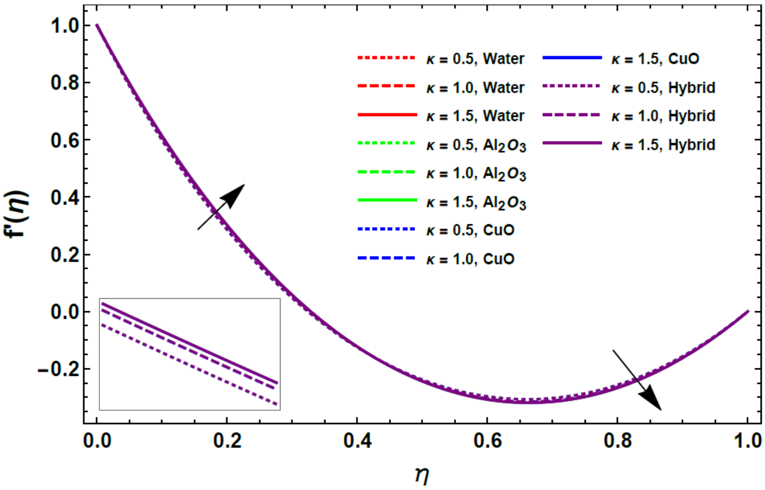

Fig 2: f' *for* $\boldsymbol{\eta}$ *and* $\boldsymbol{\kappa}$ *at* $M = 0.5,\ Kr = 0.5,\ Pr = 10,\ Re = 0.5,\ Ec = 0.03,\ \phi_1 = 0.03,\ \phi_2 = 0.03.$

Figure 2 shows effects of permeability of porous medium κ on velocity profiles in y-direction. It is evident that, motion of the fluid is increasing in some interval of η after that motion of the fluid decreasing.

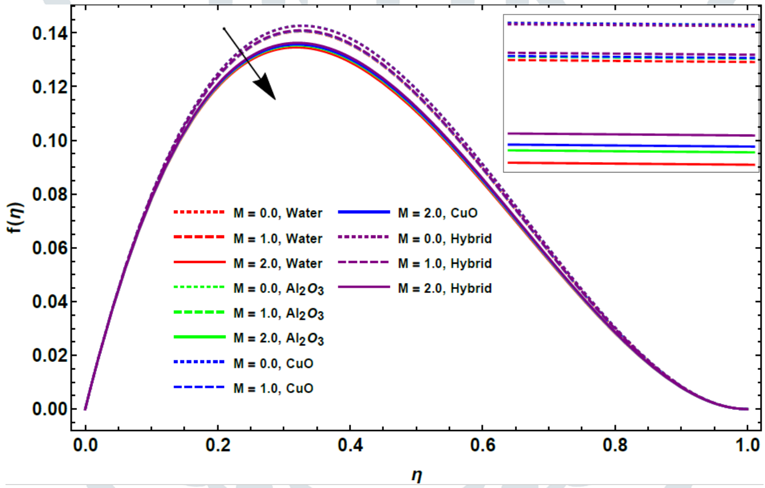

Fig 3: f *for* $\boldsymbol{\eta}$ *and* $\boldsymbol{M}$ *at* $Pr = 10,\ \kappa = 0.5,\ Re = 0.5,\ Kr = 0.5,\ Ec = 0.03,\ \phi_1 = 0.03,\ \phi_2 = 0.03.$

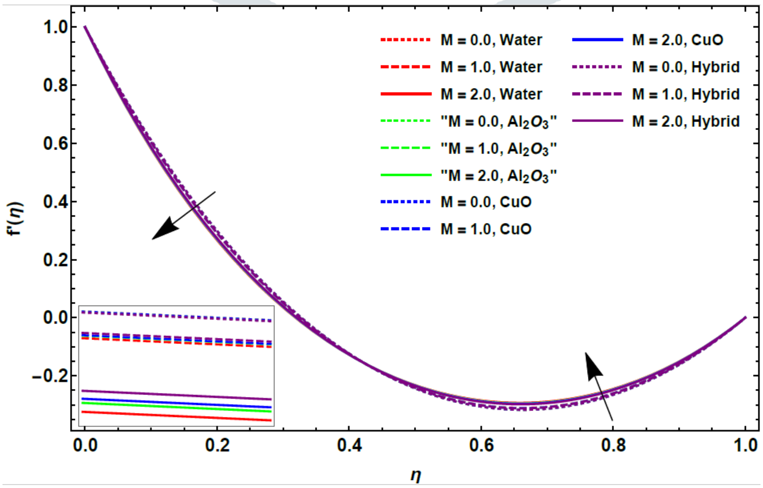

Fig 4: f' *for* $\boldsymbol{\eta}$ *and* $\boldsymbol{M}$ *at* $Pr = 10,\ \kappa = 0.5,\ Re = 0.5,\ Kr = 0.5,\ Ec = 0.03,\ \phi_1 = 0.03,\ \phi_2 = 0.03.$

Figure 3 show effects of magnetic field M on velocity profiles in all directions. It is seen that, velocity in x- direction is decrease with increase in M. From Figure 4, we conclude that velocity in y- direction improve in some interval of η after that reduce with increase in M.

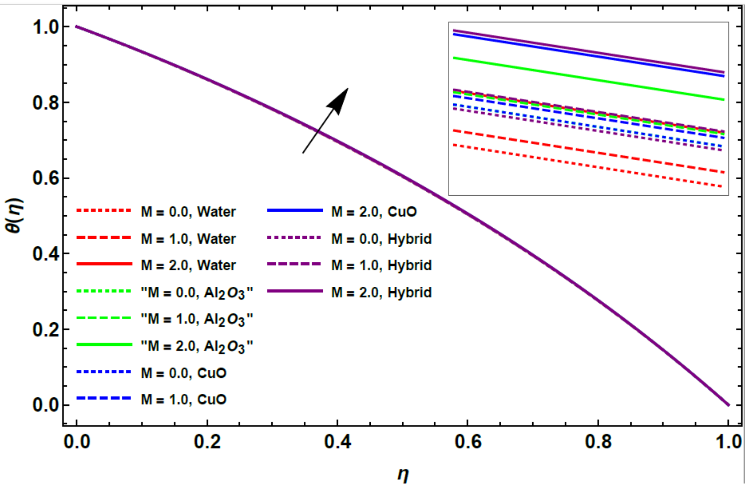

Fig 5: $\boldsymbol{\theta}$ *for* $\boldsymbol{\eta}$ *and M at* $Pr = 10,\ \kappa = 0.5,\ Re = 0.5,\ Kr = 0.5,\ Ec = 0.03,\ \phi_1 = 0.03,\ \phi_2 = 0.03.$

Figures 5 illustrates that magnetic parameter M having negative impact on temperature profile and hence temperature increases.

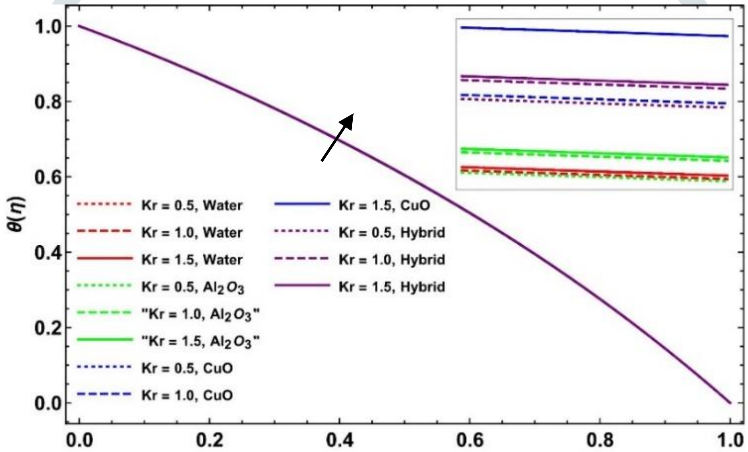

Fig 6: $\boldsymbol{\theta}$ *for* $\boldsymbol{\eta}$ *and Kr at* $M = 0.5,\ Pr = 10,\ \kappa = 0.5,\ Re = 0.5,\ Ec = 0.03, \phi_1 = 0.03,\ \phi_2 = 0.03.$

Figures 6 illustrate that rotation parameter Kr on temperature profiles. It is seen that; rotation parameter tends to improve heat transfer process while reduce mass transfer process.

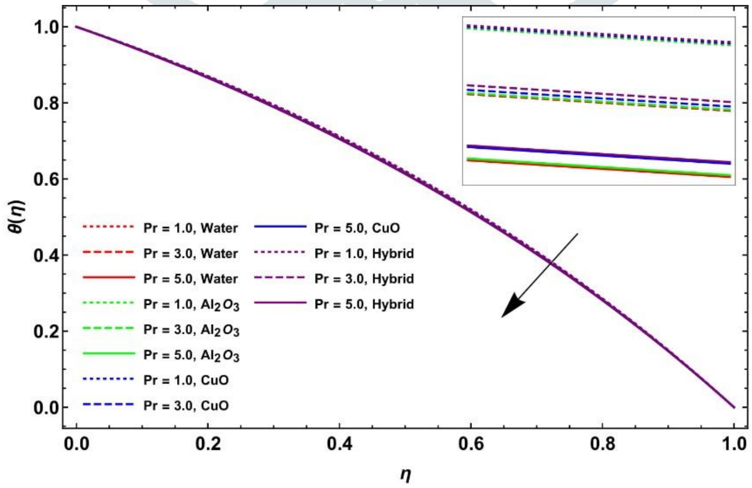

Fig 7: $\boldsymbol{\theta}$ *for* $\boldsymbol{\eta}$ *and Pr at* $M = 0.5,\ Kr = 0.5,\ \kappa = 0.5,\ Re = 0.5,\ Ec = 0.03, \phi_1 = 0.03,\ \phi_2 = 0.03.$

Figure 7 exhibits the temperature for different values of Prandtl number Pr. It is observed that heat transfer process decreases with increase in Pr It is justified due to the fact that thermal conductivity of the fluid decrease with increase in Prandtl number Pr.

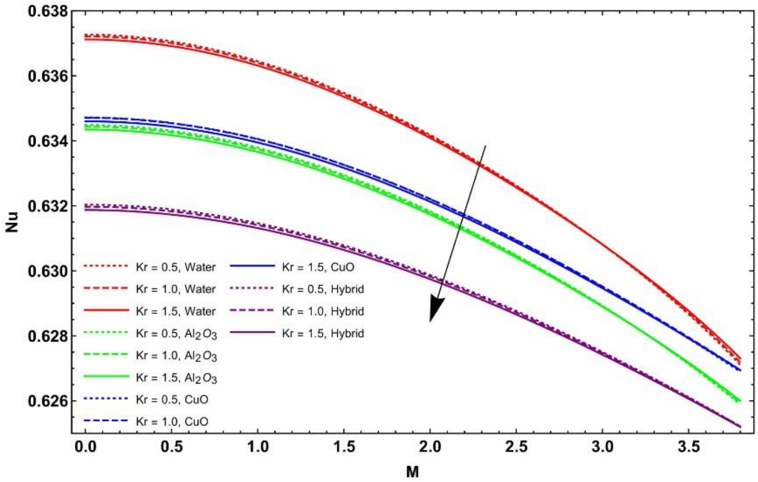

Fig 8: Effect of M and $\boldsymbol{Kr}$ *on Nu at* $\boldsymbol{Pr = 10}$, $\boldsymbol{Re = 0.5}$, $\boldsymbol{\kappa = 0.5}$, $\boldsymbol{Ec = 0.03}$, $\boldsymbol{\phi_1 = 0.03}$, $\boldsymbol{\phi_2 = 0.03}$.

Figure 8 illustrates effect of Kr on Nusselt number. It is evident that Kr decreases the values of Nusselt number.

CONCLUSION:

The most important concluding remarks can be summarized as follows:

- Velocity profile increases by increasing permeability parameter.
- Velocity profile decreases by increasing Magnetic parameter.
- Temperature profile increases by increases Magnetic parameter and rotation parameter.
- Temperature profile decreases with increase in Prandtl number.
- Nusselt number decreases with increase in rotation parameter.

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