ML ESTIMATION OF RELIABILITY INDICES FOR IDENTICAL TWO-COMPONENT SYSTEM IN PRESENCE OF CCFS FOLLOWS WEIBULL-LAW

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Abstract :

This study aimed to assess the maximum likelihood estimation method for the Reliability indices of Identical two component repairable system . The system is assumed to be under the influence of common-cause failures (CCFs). The CCFs and individual failures follows weibull law with occurrence of chance .Numerical evidences are provided to justify the use of M L estimation procedure in the cause of system Reliability and Frequency identical Failure functions .

Keyword, Reliability, Identical series, parallel system, common cause failure, MLE.

1. INTRODUCTION

System Reliability plays a vital role in nuclear power plants, electrical, electronics and industrial sectors . the common cause failures have identified in recent time (1971) which have a significant contribution to highly risk in complexity systems. like nuclear power plants. The multi-component failures due to external causes like radiation, humidity etc this type causes is called CCF . CCFs are greatly reducing the reliability indices under its influence. Billiton &Allan [1983] discussed the role of CCFS. Atwood & Stevenson [1982] and Meacham and Atwood [1983] used BFR model for CCFS in the area of nuclear power plants. The Quantification and estimation of CCFs rates were discussed by A.A.Chari[1991]. The number of failures in relative time of exposure of each component are used in Various papers of James-Stein (1992). U S Manyam & A.A.Chari [2003] have studied the concept of CCFs to arrive at exponential law . The expression of Reliability indices like Reliability R(t), meantime between failures E(t) and Frequency Failure F(t) functions using markovian approach. This paper attempts the estimation of Reliability R(t), meantime between failure E(t) and Frequency Failure F(t) functions for parallel and series system in the context of Common Cause Failures to arrive at weibul-law.

2. ASSUMPTIONS

1. The system has two components, which are stochastically independent.
2. The system is affected by individual as well as common cause failures.
3. The components in the system will fail singly at the constant rate β_a and Failure probability is P_1
4. The components may fail due to common causes at the constant rate β_c and with failure probability is P_2 such that $P_1 + P_2 = 1$.
5. Time occurrences of CCS failures and individual failures follow Wei-bull law.
6. The individual failures and CCS failures occurring independent of each other.
7. The failed components are serviced singly and service time follows exponential distribution with rate of service .

2. NOTATIONS

β_I : Individual failure rate.

β_c : Common cause failure rate.

μ:Service rate of individual components

R(t):System Reliability function.

$R_s(t)$:Reliability function of series system.

$\hat{R}_S(t)$:ML estimate of Reliability function of series system.

$R_p(t)$:Reliability function of two component parallel system.

$\hat{R}_p(t)$: Maximum Likelihood Estimate of time dependent Reliability function for parallel system.

$E_S(t)$: Expected time of failure for series system (MTTF / MTBF)

$\hat{E}_S(t)$: ML Estimate of Expected mean time of failure for series system.

$E_p(t)$: Expected time of failure for parallel system (MTTF / MTBF)

$\hat{E}_p(t)$: M L Estimate of Expected mean time of failure for parallel

$F_s(t)$: Steady state Frequency of Failure function of series system.

$\hat{F}_s(t)$: ML Estimate of steady state Frequency Failure function of series system.

F_p (t): Steady state Frequency Failure function for parallel system.

$\hat{F}_p(t)$: ML Estimate of steady state frequency of Failure function for parallel system.

$\hat{\beta}_I$:sample estimation of individual failure rate

$\hat{\beta}_c$:sample estimation of common cause failure rate

$\hat{\mu}$: Sample estimate of service time of the components

n = Sample size.

N = Number of simulated samples.

3. MODEL

The assumptions of Markova model can be to drive and be formulated the Reliability function R(t) under the influence of individual as well as CCF.

The quantities β_0, $\beta_{1,}$, β_2 are as follows

$\beta_0 = \beta_I P_1$, β_1=2$\beta_I P_1$ β_2= $\beta_c P_2$, μ_1= μ, μ_2= 2 μ,

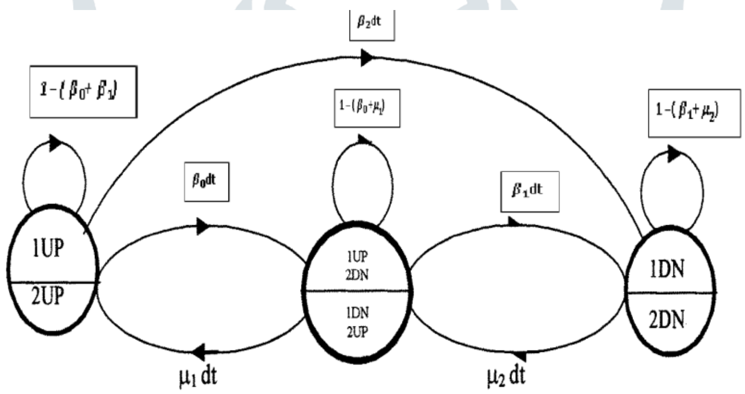


Fig. 3.1: Markov Graph For Two Component System With Individual And Common Cause Failures.

From the Markov graph the equations were formed and the probabilities of the Various state of the systems i.e. $P_o(t)$, P_1 (t), P_2(t) are derived (see Chari [1991]).

5 RELIABILITY FUNCTION OF TWO-COMPONENT IDENTICAL SYSTEM

The Reliability function for transient state for series and parallel systems are derived using the probabilities mentioned in the section

(a) Series System

The transient state expression of Reliability function for series system is given by

$$R_s(t) = \left(\frac{(\gamma_1 + \beta_I P_1)\, e^{-(\gamma_1 t)} - (\gamma_2 + \beta_I P_1)\, e^{-(\gamma_2 t)}}{\gamma_2 - \gamma_1} \right) \qquad \text{------(1)}$$

where

$$\gamma_1 = \left(\frac{-(3\beta_I P_1+\beta_c P_2) + \sqrt{(\beta_I P_1+\beta_c P_2)^2+8\beta_c P_2}}{2} \right) \quad \text{------ (2)}$$

$$\gamma_2 = \left(\frac{-(3\beta_I P_1+\beta_c P_2) - \sqrt{(\beta_I P_1+\beta_c P_2)^2+8\beta_c P_2}}{2} \right) \quad \text{------ (3)}$$

β_I& $\beta_c$are the individual failure and Common cause failure rates respectively.

(b) Parallel System

Thus The transient state expression of Reliability function for parallel system is given by

$$R_p(t) = \left(\frac{(\gamma_1+\mu+3\,\beta_I P_1)\, e^{-(\gamma_1 t)} - (\gamma_2+\mu+3\beta_I P_1)\, e^{-(\gamma_2 t)}}{\gamma_2-\gamma_1} \right) \quad \text{------ (4)}$$

where

$$\gamma_1 = \left(\frac{-(3\beta_I P_1+\beta_c P_2+\mu) + \sqrt{(\beta_I P_1+\beta_c P_2-\mu)^2+8\beta_c\, \mu P_2}}{2} \right) \quad \text{------ (5)}$$

$$\gamma_2 = \left(\frac{-(3\beta_I P_1+\beta_c P_2+\mu) - \sqrt{(\beta_I P_1+\beta_c P_2-\mu)^2+8\beta_c \mu P_2}}{2} \right) \quad \text{------ (6)}$$

where β_I, β_c& μ are the individual ,Common cause failure and service rates respectively.

6. MEAN TIME BETWEEN FAILURE (MTBF/ MTTF)-TWO COMPONENT IDENTICAL SYSTEMS

The Mean Time Between failure function for series and parallel systems are derived using the probabilities mentioned in the section

(a) Series System

The expression of Mean time between failure function for series system is give by

$$E_s(t) = \frac{1}{(2\beta_I P_1+\beta_c P_2)} \quad \text{------- (7)}$$

(b) Parallel System:

The expression of Mean time between failure function for parallel system given by

$$E_p(t) = \frac{(3\beta_I P_1+\mu)}{(2\beta_I^2 P_1^2 + \beta_I P_1 \beta_c P_2+\mu\beta_c P_2)} \quad \text{------- (8)}$$

Where

The β_I, β_c & μ are individual, Common cause failure rates and repair rate respectively and P_1& P_2 are the occurrence of probability of individual and CFS failure events.

7. TWO COMPONENT IDENTICAL SYSTEM FREQUENCY OF FAILURES

The frequency of failures function for series and parallel systems are derive using the probabilities mentioned in the section

(a) Series System

Thus, the expression of frequency of failure function for series system is give by

$$F_s(t) = \frac{\left(2\mu^2(2\beta_I P_1+\beta_c P_2)\right)}{\left(2\mu(\mu+2\beta_I P_1+\beta_c P_2)+\mu\ \beta_c P_2+\beta_I P_1(2\beta_I P_1+\beta_c P_2)\right)} \quad \text{------ (9)}$$

(b) Parallel System

The expression of Frequency of failure function of parallel system is

$$F_p(t) = \frac{\left(2\mu(2\beta_c P_2(\mu+\beta_I P_1)+\beta_I^2 P_1^2)\right)}{\left(2\mu(\mu+2\beta_I P_1+\beta_c P_2)+\mu\ \beta_c P_2+\beta_I P_1(2\beta_I P_1+\beta_c P_2)\right)} \quad \text{------ (10)}$$

Where

β_I, β_c, μ, P_1&P_2 are individual failures rate, Common cause failure rate, repair rate and probability of occurrence of individual as well as CCS failures.

8. ESTIMATION OF RELIABILITY FUNCTION-ML ESTIMATION APPROACH

This section discusses the Maximum likelihood estimation approach for estimating Reliability function of two component parallel and series systems, which is under the influence of Individual as well as common cause failures.

Let X_1, X_2, X_3...X_n,, be a sample of ‘n' number of times between individual failures which will obey weibul law.

Let Y_1 Y_2, Y_3 ...Y_n,, be a sample of ‘n number of times between common cause system failures assume to follow wei-bul law.

Let Z_1 Z_2, Z_3 ...Z_n,, be a sample of ‘n` number of times between service of the component assume to follows exponential law.

$$\hat{\mu} = \frac{1}{\frac{\sum z_i}{n}} \quad \hat{\beta}_I = \frac{\sum_{i=1}^{n} x_i^K}{n} \qquad \hat{\beta}_c = \frac{\sum_{i=1}^{n} y_i^K}{n} \quad \text{------(11)}$$

are sample estimates of rate of the individual failure rate ($\hat{\beta}_I$) ,common cause failure rate ($\hat{\beta}_c$) and service rate $\hat{\mu}$ of the components respectively.

9. ESTIMATION OF RELIABILITY FUNCTION OF TWO SYSTEMS - M L ESTIMATION APPROACH

The Reliability function of two component identical series and parallel systems by using ML approach as follows

(a) Series System

The time dependent expression of Reliability function for series system is given by

$$\hat{R}_s(t) = \left(\frac{(\hat{\gamma}_1 + \hat{\beta}_I P_1)\, e^{-(\hat{\gamma}_1 t)} - (\hat{\gamma}_2 + \hat{\beta}_I P_1)\, e^{-(\hat{\gamma}_2 t)}}{\hat{\gamma}_2 - \hat{\gamma}_1}\right) \quad \text{------ (12)}$$

(b) Parallel System

Thus the expression of Reliability function for parallel system is given by

$$\hat{R}_p(t) = \left(\frac{(\hat{\gamma}_1 + \hat{\mu} + 3\hat{\beta}_I P_1)\, e^{-(\hat{\gamma}_1 t)} - (\hat{\gamma}_2 + \hat{\mu} + \hat{\beta}_I P_1)\, e^{-(\hat{\gamma}_2 t)}}{\hat{\gamma}_2 - \hat{\gamma}_1}\right) \quad \text{------ (13)}$$

where

$$\hat{\gamma}_1 = \left(\frac{-(3\hat{\beta}_I P_1 + \hat{\beta}_c P_2) + \sqrt{(\hat{\beta}_I P_1 + \hat{\beta}_c P_2)^2 + 8\hat{\beta}_c P_2}}{2}\right) \quad \text{------ (14)}$$

$$\hat{\gamma}_2 = \left(\frac{-(3\hat{\beta}_I P_1 + \hat{\beta}_c P_2) - \sqrt{(\hat{\beta}_I P_1 + \hat{\beta}_c P_2)^2 + 8\hat{\beta}_c P_2}}{2}\right) \quad \text{------ (15)}$$

Are sample estimates of rate of the individual failure ($\hat{\beta}_I$)common cause failure($\hat{\beta}_c$) and service rate $\hat{\mu}$ of the components respectively.

10. ESTIMATION OF MEAN TIME BETWEEN FAILURES TWO COMPONENT SYSTEM-M L ESTIMATION APPROACH

The Mean Time Between failure function of two component identical series and parallel systems by using ML approach as follows

(a) Series System

The expression of Mean time between failure function for series system is give by

$$\hat{E}_s(t) = \frac{1}{(2\hat{\beta}_I P_1 + \hat{\beta}_c P_2)} \quad \text{------ (16)}$$

(b) Parallel System:

The expression of Mean time between failure function for parallel system given by

$$\hat{E}_p(t) = \frac{(3\hat{\beta}_I P_1 + \hat{\mu})}{(2\hat{\beta}_I^2 P_1^2 + \hat{\beta}_I P_1 \hat{\beta}_c P_2 + \hat{\mu}\hat{\beta}_c P_2)} \quad \text{------ (17)}$$

Where

Where $\hat{\beta}_I$, $\hat{\beta}_c$, $\hat{\mu}$ P_1&P_2 are individual failures rate, Common cause failure rate, repair rate and probability of occurrence of individual as well as CCS failures

11. ESTIMATION OF TWO COMPONENT SYSTEM FREQUENCY FAILURES M L ESTIMATION APPROACH

The frequency failure functions for two component identical series and parallel systems by using ML approach as follows.

(a) Series System

Thus, the expression of frequency of failure function for series system is give by

$$\hat{F}_s(t) = \frac{\left(2\hat{\mu}^2(2\hat{\beta}_I P_1+\hat{\beta}_c P_2)\right)}{\left(2\hat{\mu}(\hat{\mu}+2\hat{\beta}_I P_1+\hat{\beta}_c P_2)+\hat{\mu}\,\hat{\beta}_c P_2+\hat{\beta}_I P_1(2\hat{\beta}_I P_1+\hat{\beta}_c P_2)\right)} \quad \text{------ (18)}$$

(b) Parallel System:

Thus, the expression of frequency of failure function for parallel system is give by

$$\hat{F}_p(t) = \frac{\left(2\hat{\mu}(2\hat{\beta}_c P_2(\hat{\mu}+\hat{\beta}_I P_1)+\hat{\beta}_I^2 P_1^2)\right)}{\left(2\hat{\mu}(\hat{\mu}+2\hat{\beta}_I P_1+\hat{\beta}_c P_2)+\hat{\mu}\hat{\beta}_c P_2+\hat{\beta}_I P_1(2\hat{\beta}_I P_1+\hat{\beta}_c P_2)\right)} \quad \text{------ (19)}$$

Where

$\hat{\beta}_I$, $\hat{\beta}_c$, $\hat{\mu}$ P_1&P_2 are individual failures rate, Common cause failure rate, repair rate and probability of occurrence of individual as well as CCS failure

Now Maximum likelihood estimates of Reliability, Frequency and Mean time between failure functions are $R_S(t)$, $R_P(t)$, $E_s(t)$, $E_p(t)$,$F_s(t)$ and ,$F_p(t)$ & which are given in equations (1), (4), (7), (8), (9) & (10) are given by the functions $\hat{R}_s(t)$, $\hat{R}_p(t)$, $\hat{E}_s(t)$, $\hat{E}_p(t)$ $\hat{F}_s(t)$ and $\hat{F}_p(t)$ which are shown in equations (11) ,(12) , (15) (16), (17)& (18).respectively. Obviously, these estimates are functions of $\hat{\beta}_I$, $\hat{\beta}_c$, $\hat{\mu}$ which are differentiable. Now from multivariate central limit theorem .

12. INTERVAL ESTIMATION- RELIABILITY, MTBF AND FREQUENCY OF FAILURES

$\sqrt{n}\,[(\hat{\beta}_I, \hat{\beta}_c, \hat{\mu}\ )-(\beta_I, \beta_c, \mu)] \sim N_3(0, \Sigma)$ for $n \to \infty$

Where

$\Sigma = (\sigma_{ij})_{3\times 3}$ co-variance matrix

$\Sigma = \text{dig}\,(\beta_{I,}^2\ \beta_c^2,\ \mu^2)$

Also from Rao (1974) we have

$\sqrt{n}\,[\,F(t) - \hat{F}(t)] \sim N(0,\sigma_\theta^2)$ as $n \to \infty$ and θ is the vector

By the properties of M L method of estimation $\hat{F}$ (t)is CAN estimate of F(t) respectively also $\sigma^2_{(\theta)}$ be the estimator of $\sigma^2_{(\theta)}$

Where $(\hat{\theta}) = (\hat{\beta}_I, \hat{\beta}_c, \hat{\mu})$

Nature of estimates are not established so far.

9. SIMULATION AND VALIDITY :

Reliability indices are simulated For a range of specified values of the rates of common cause failures(β_c), individual (β_I) and service rates(μ) and for the samples of sizes n=5 (5) 30 by using computer package developed and the sample estimates are computed for N = 10000 (20000) 100000 and mean square error (MSE) of the estimates for $F_s(t)$&$F_p(t)$ were obtained and given in table Tab.1The tables and graphs are seen in the Appendix–I. For large samples Maximum Likelihood estimators are undisputedly better since they are CAN estimators. However the M L estimate is still seen to be reasonably good giving near accurate estimate for a sample size five(n=5) also. This shows that ML method of estimator is quite useful in this context.

Table-1:Result sample of the simulations for steady availability function for series system with $\beta_I = 0.02$, β_c=0.03, μ=1,p_1 =0.5 and k=1

Sample size n=5				
N	$F_S(T)$	$\hat{F}_p(t)$	M.S.E	95%CONFIDENCE INTERVAL
10000	0.015000	0.034094	0.000191	(0.00000, 0.146481)
30000	0.015000	0.014987	0.000000	(0.00000, 0.146481)
50000	0.015000	0.017569	0.000011	(0.00000, 0.146481)
70000	0.015000	0.018636	0.000014	(0.00000, 0.146481)
90000	0.015000	0.013161	0.000006	(0.00000 , 0.146481)

Sample size n=10				
N	$F_S(T)$	$\hat{F}_p(t)$	M.S.E	95%CONFIDENCE INTERVAL
10000	0.015000	0.013170	0.000018	(0.000000, 0.107971)
30000	0.015000	0.016209	0.000007	(0.000000, 0.107971)
50000	0.015000	0.012145	0.000013	(0.000000, 0.107971)
70000	0.015000	0.022172	0.000027	(0.000000, 0.107971)
90000	0.015000	0.016774	0.000006	(0.000000, 0.107971)

Sample size n=15				
N	F_S(T)	$\hat{F}_p(t)$	M.S.E	95%CONFIDENCE INTERVAL
10000	0.015000	0.022648	0.000076	(0.000000, 0.090910)
30000	0.015000	0.018216	0.000019	(0.000000, 0.090910)
50000	0.015000	0.017279	0.000010	(0.000000, 0.090910)
70000	0.015000	0.024509	0.000036	(0.000000, 0.090910)
90000	0.015000	0.011517	0.000012	(0.000000, 0.090910)

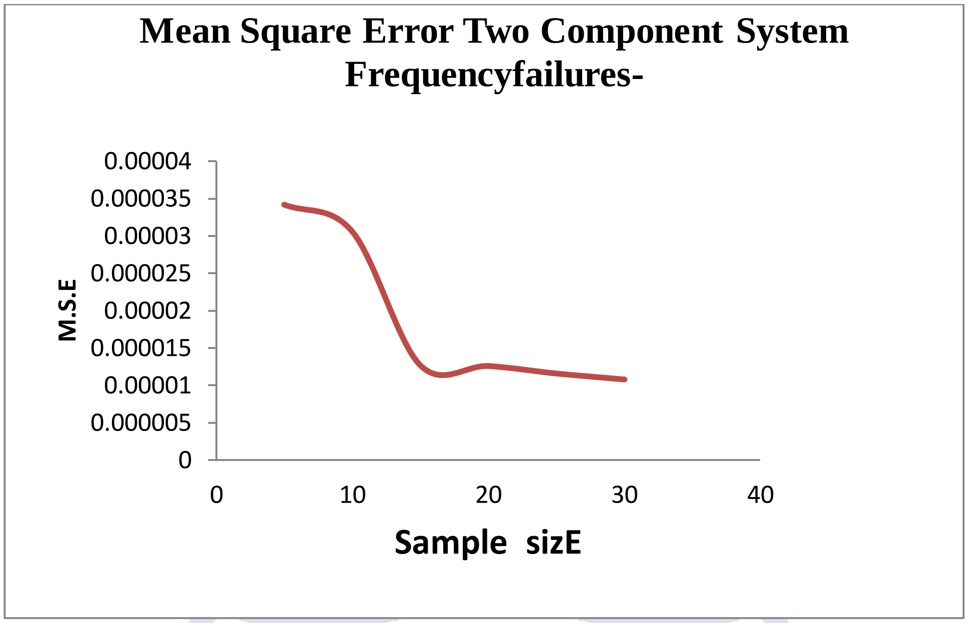
Sample size n=20				
N	F_S(T)	$\hat{F}_p(t)$	M.S.E	95%CONFIDENCE INTERVAL
10000	0.015000	0.015487	0.000005	(0.000000 , 0.080740)
30000	0.015000	0.012203	0.000016	(0.000000, 0.080740)
50000	0.015000	0.022139	0.000032	(0.000000, 0.080740)
70000	0.015000	0.015516	0.000002	(0.000000 , 0.080740)
90000	0.015000	0.013121	0.000006	(0.000000 , 0.080740)

ample size n=25				
N	F_S(T)	$\hat{F}_p(t)$	M.S.E	95%CONFIDENCE INTERVAL
10000	0.015000	0.018342	0.000033	(0.000000. 0.073800)
30000	0.015000	0.015961	0.000006	(0.000000 . 0.073800)
50000	0.015000	0.018059	0.000012	(0.000000 . 0.073800)
70000	0.015000	0.020242	0.000020	(0.000000 . 0.073800)
90000	0.015000	0.017113	0.000007	(0.000000 . 0.073800)

Sample size n=30				
N	F_S(T)	$\hat{F}_p(t)$	M.S.E	95%CONFIDENCE INTERVAL
10000	0.015000	0.017227	0.000022	(0.000000, 0.068677)
20000	0.015000	0.012830	0.000013	(0.00000, 0.068677)
50000	0.015000	0.016338	0.000006	(0.000000 , 0.068677)
70000	0.015000	0.016298	0.000005	(0.000000 , 0.068677)
90000	0.015000	0.017519	0.000008	(0.000000 , 0.068677

The graphic for the simulations of steady availability function for series system

with $\beta_I = 0.02$, β_c=0.03, μ=1,p_1 =0.5 and k=1



CONCLUSIONS:

This paper attempts to evaluate the estimate of the Reliability, Meantime between failures and frequency failure functions in the presence of common cause failures and individual failure the ML method proposed here is giving almost accuracy estimation in case of sample size 10 and above which is verified by the simulation in the absence analytical approach. Also these results suggested the ML estimate is reasonable good and gives accurate estimates even for sample size n=5 therefore this paper identifies the use of thee an ML method of estimator justified through empirical means estimation of the Reliability ,mean time between failure and Frequency failure function of two component system in presence of CCFs as well as individual failures

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