Domain Wall Bulk viscous cosmology in $f(R,T)$ theory of gravitation

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Abstract:

We construct Bianchi type-III cosmological models with bulk viscous domain wall in $f(R,T)$ theory of gravity. We solve the field equations by using the variation law for generalized Hubble's parameter proposed by Berman (Nuovo Cimento B, 74, 182, 983). We obtain two types of values for the average scale factor, one is of power law type and the other is of exponential form. We find the values of different physical parameters of the models and compared them with the recent observational data. Further we discuss the physical and geometrical properties of the models.

Keywords: Bianchi type-I, Cosmological constant, Lyra geometry

1. Introduction:

General Relativity was introduced by Einstein's in 1916, which is a geometric theory of gravitation. Till today, it is the most successful theory of gravitation in terms of consistent and completeness. Cosmology is the branch of science, which deals with structure of the universe at large scale. In order to understand the cosmological models, General Relativity is used as a basic building blocks, which provides the gravitational interaction successfully. The important results concluded from SNIa and CMBR[1,2] is that our universe is accelerating in nature. The inflationary scenario is generally restricted to the early universe, which was transformed to the present day universe with a prominent consequence like universe age and other physical parameters of cosmology [3,4]. Results like the value of the deceleration parameter play an important role in determine the accelerated phase and decelerated phase of the universe. The most accepted result on q (deceleration parameter) [5] is $q \approx 0.66$ for perfect fluid universe having equation of state $p = \alpha\rho$. In such scenario scale factor evolve with cosmic time as $a = t^{3(1+\alpha)}$, which leads to $q = \frac{1+3\alpha}{2}$. Thus $q \approx 0.66$ $\Rightarrow \alpha \approx -0.77$.. The negative value of $\alpha \approx -0.77 \Rightarrow p \approx -0.77\rho$, which indicates that the present universe is dominated by a fluid with negative pressure and violate the SEC (strong energy condition) [6]. In order to accommodate such results, several modified theories of gravitation came in to existence and became popular among the astrophysicist and cosmologist. Among all the modified theories found in literature, the $f(R)$ theory is the most discussed one[7]. The $f(R)$ theory is also able to explain the transition phase of the universe. In terms of the solar system test of GR, the functional form $f(R) = \frac{1}{R}$ is incompatible but one should note that any theory of gravitation most pass the solar system test [8]. After thatHarko et al.[9] have introduced the generalize version of $f(R)$, known as $f(R,T)$ gravity. In this case the matter Lagrangian is given by arbitrary function of the R and T representing Ricci scalar and trace of the energy momentum tensor respectively.

In this work, we have explored the bulk viscous cosmological models, which involves "domain wall" in view of $f(R,T)$ gravity. In early universe domain walls are considered as topological defects, which appear during phase transition of the universe. Some of the other topological defects includes monopoles, cosmic strings and textures. Domain walls are form when a discrete symmetry is broken spontaneously [10]. It is

usually characterized through the π_0 homotopy group [11]. The isotropic equation of state $p = -\frac{2}{3}\rho$ may represents a network of domain wall [6]. Vilekin [12] has investigated the problem of vacuum domain wall in the view of GR. He has obtained the analytical solution and shown that the metric has no time geometric singularities and flat everywhere locally except on the wall itself and also has event horizon. Thick domain wall is investigated by Wang [13], where he has derived the analytical solution representing the gravitational collapse of a thick domain wall. Katore et al. have discussed the thick domain wall in modified $f(R,T)$ gravity for both FRW and axially symmetric space-time[14].

On the other hand the concept of bulk viscosity is supposed to be crucial in the early stage of the universe. The inclusion of viscosity in the fluid examine different dynamics of the universe of homogeneous type. The coefficient of bulk viscosity determines magnitude of the viscous stress relative to the expansion. In Saez–Ballester theory, Rao et al. [15] have discussed one dimensional cosmic string in presence of bulk viscosity. Further, Mishra and Dua [16] obtained the solution of the cosmic string cosmological model through the deceleration parameter as a bilinear function of time. Recently, Khadekar et al. [17] have analysed different form of coefficient of bulk viscosity in presence of chaplygin gas. In view of the above literature and importance of domain wall and bulk viscosity in the early universe, motivates us to study cosmological bulk viscous fluid model with domain wall in the theory of $f(R,T)$ gravity.

Let us consider the Bianchi type III metric in the following form

$$ds^2 = dt^2 - A^2dx^2 - B^2e^{-2px}dy^2 - C^2dz^2 \tag{1}$$

In the expression A, B and C depends on cosmic time t and p is a constant. The energy momentum tensors for bulk viscous domain wall is expressed as

$$T_{ij} = (\rho + \overline{P})u_iu_j - \overline{P}g_{ij} \tag{2}$$

Where $u^i = (0,0,0,1)$ and

$$\overline{P} = P - 3\xi H \tag{3}$$

is known as bulk viscous pressure, $\xi(t)$ is the coefficient of bulk viscosity. We have considered a model:

$$f(R,T) = N_1(R) + N_2(T) \text{ with } N_1 = \mu R \text{ and } N_2 = \mu T$$

2. Field Equations:

In view of the model, the related field equations are expressed as

$$R_{ij} - \frac{1}{2}Rg_{ij} = (\alpha - \beta)T_{ij} + \beta(\rho + \overline{P})g_{ij} \tag{4}$$

Incorporation of equation (2) in equation (4) for the metric (1) takes the form

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} = \alpha\overline{P} - \beta\rho \tag{5}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} = \alpha\overline{P} - \beta\rho \tag{6}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{p^2}{A^2} = \alpha\overline{P} - \beta\rho \tag{7}$$

$$\frac{\dot{A}}{A} - \frac{\dot{B}}{B} = 0 \tag{8}$$

$$\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{A}\dot{C}}{AC}+\frac{\dot{B}\dot{C}}{BC}-\frac{p^2}{A^2}=-\alpha\rho+\beta\overline{P} \tag{9}$$

Using (8), the system of equations (5) to (9) reduces to

$$\frac{\ddot{A}}{A}+\frac{\ddot{C}}{C}+\frac{\dot{A}\dot{C}}{AC}=\alpha\overline{P}-\beta\rho \tag{10}$$

$$\frac{2\ddot{A}}{A}+\left(\frac{\ddot{A}}{A}\right)^2-\frac{p^2}{A^2}=\alpha\overline{P}-\beta\rho \tag{11}$$

$$\left(\frac{\dot{A}}{A}\right)^2+2\frac{\dot{A}\dot{C}}{AC}-\frac{p^2}{A^2}=-\alpha\rho+\beta\overline{P} \tag{12}$$

In terms of the metric potentials, a and V are defined as

$$\text{average scale factor}=a=(ABC)^{1/3} \tag{13}$$

$$\text{Volume}=V=a^3=ABC \tag{14}$$

Scalar expansion θ is proportional to shear scalar (Collins and Hawking), which yields

$$C=A^n \tag{15}$$

The relationship between H and a is given as:

$$H=Da^{-m} \tag{16}$$

where H denotes the generalized mean Hubble parameter and it is defined as

$$H=\frac{1}{3}\left(H_x+H_y+H_z\right) \tag{17}$$

where $D>0$, $m\geq 0$ and $H_x=\frac{\dot{A}}{A}$, $H_y=\frac{\dot{B}}{B}$, $H_z=\frac{\dot{C}}{C}$ are the directional Hubble's parameters in the direction of x, y and z axes respectively. In view of equation (14), H can be recast as

$$H=\frac{\dot{V}}{3V}=\frac{\dot{a}}{a} \tag{18}$$

From (16) and (18) one can have

$$\dot{a}=Da^{-m+1} \tag{19}$$

Integrating equation (19), we obtain

$$a(t)=\begin{cases} a_0e^{Dt} & ,\text{if } m=0 \\ \left[m(Dt+D_0)\right]^{1/m} & ,\text{if } m\neq 0 \end{cases}$$

where a_0 and D_0 are constant of integration.

2.1 Case-I, $m=0$. In this case, average scale factor is expressed as

$$a=a_0e^{Dt} \tag{20}$$

From (14) and (20), we get the volume as

$$V=a^3=ABC=\left(a_0e^{Dt}\right)^3 \tag{21}$$

Using (8) and (15) in (21), we get

$$A = B = a_0^{\frac{3}{n+2}} e^{\frac{3}{n+2}Dt} \quad , C = a_0^{\frac{3n}{n+2}} e^{\frac{3n}{n+2}Dt} \tag{22}$$

Using (22), solving equations (10) to (12), we obtain

$$\overline{P} = \frac{1}{\alpha^2 - \beta^2}\left[\left(\frac{3D}{n+2}\right)^2 (3\alpha - \beta(1+2n)) - (\alpha - \beta)\left(\frac{p}{a_0^{\frac{3}{n+2}} e^{\frac{3}{n+2}Dt}}\right)^2\right] \tag{23}$$

and

$$\rho = \frac{1}{\beta}\left[\alpha\overline{P} - \left(\frac{3D}{n+2}\right)^2 (1+n+n^2)\right] \tag{24}$$

For this case $m = 0$, we get the values of constants as follows:

$$p = 0, n = 1, -2, \beta = 3(1+\alpha) \tag{25}$$

where α is any arbitrary constant. Substituting (23) into (3), we get

$$P = \frac{1}{\alpha^2 - \beta^2}\left[\left(\frac{3D}{n+2}\right)^2 (3\alpha - \beta(1+2n)) - (\alpha - \beta)\left(\frac{p}{a_0^{\frac{3}{n+2}} e^{\frac{3}{n+2}Dt}}\right)^2\right] + 3\xi D \tag{26}$$

The effective dark energy equation of state (EoS) becomes $\omega = \frac{p}{\rho}$

$$\omega = \frac{\beta}{\alpha^2 - \beta^2}\left[\frac{\left(\frac{3D}{n+2}\right)^2 (3\alpha - \beta(1+2n)) - (\alpha - \beta)\left(\frac{p}{a_0^{\frac{3}{n+2}} e^{\frac{3}{n+2}Dt}}\right)^2}{\alpha\overline{P} - \left(\frac{3D}{n+2}\right)^2 (1+n+n^2)}\right] + 3\xi\beta D \tag{27}$$

Now, with the help of metric potentials, (1) can be reframed as

$$ds^2 = dt^2 - a\left({}_0^{\frac{3}{n+2}} e^{\frac{3}{n+2}Dt}\right)^2 (dx^2 + e^{-2px} dy^2) - \left(a_0^{\frac{3n}{n+2}} e^{\frac{3n}{n+2}Dt}\right)^2 dz^2 \tag{28}$$

The Ricci Scalar R for the metric is expressed as

$$R = 2\left[\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{B}}{AB} + \frac{\dot{A}\dot{C}}{AC} + \frac{\dot{B}\dot{C}}{BC}\right] \tag{29}$$

$$R = 2\left(\frac{3D}{n+2}\right)^2\left[3 + 2n + n^2 - \left(\frac{p}{a_0^{\frac{3}{n+2}} e^{\frac{3}{n+2}Dt}}\right)^2 \left(\frac{n+2}{3D}\right)^2\right] \tag{30}$$

2.2 Case-II, $m \neq 0$. This case average scale factor yield

$$a = \left(m(Dt + D_0)\right)^{\frac{1}{m}} \tag{31}$$

From (14) and (31), we get the volume as

$$V = a^3 = ABC = \left(m(Dt + D_0)\right)^{\frac{1}{m}} \tag{32}$$

Using (8), (15) in (32), we get

$$A = B = \left(m(Dt + D_0)\right)^{\frac{3}{m(n+2)}} \tag{33}$$

$$C = \left(m(Dt + D_0)\right)^{\frac{3n}{m(n+2)}} \tag{34}$$

Using (33), (34), solving equations (10) to (12), we obtain

$$\begin{aligned}\overline{P} = {} & \frac{1}{\alpha^2 - \beta^2}\left[\left(m(Dt + D_0)\right)^{-2}\left(\frac{3D}{n+2}\right)^2\left(\alpha(3 - \frac{2}{3}m(n+2) - \beta(1+2n)\right)\right] \\ & - \left[(\alpha - \beta)\left(\frac{p}{\left(m(Dt + D_0)\right)^{\frac{3}{m(n+2)}}}\right)^2\right]\end{aligned} \tag{35}$$

$$\rho = \frac{1}{\beta}\left[\alpha\overline{P} - \left(m(Dt + D_0)\right)^{-2}\left(\frac{3D}{n+2}\right)^2\left(1 + n + n^2 - \frac{m}{3}(2 + 3n + n^2)\right)\right] \tag{36}$$

For this case when $m \neq 0$, we get the values of constant as follows:

$D = 1; m = 9; \alpha = 0$.

Where n and β are any arbitrary constants. Substituting (35) into (3), we get

$$\begin{aligned}\overline{P} = {} & \frac{1}{\alpha^2 - \beta^2}\left[\left(m(Dt + D_0)\right)^{-2}\left(\frac{3D}{n+2}\right)^2\left(\alpha(3 - \frac{2}{3}m(n+2) - \beta(1+2n)\right)\right] \\ & - \left[(\alpha - \beta)\left(\frac{p}{\left(m(Dt + D_0)\right)^{\frac{3}{m(n+2)}}}\right)^2\right] + 3\xi D\left(m(Dt + D_0)\right)^{-1}\end{aligned} \tag{37}$$

The effective dark energy equation of state (EoS) ω becomes

$$\begin{aligned}\omega = {} & \frac{\beta}{\alpha^2 - \beta^2}\left[\frac{\left(m(Dt + D_0)\right)^{-2}\left(\frac{3D}{n+2}\right)^2\omega_0 - \alpha - \beta)\left(\frac{p}{\left(m(Dt + D_0)\right)^{\frac{3}{m(n+2)}}}\right)^2}{\alpha\overline{P} - \left(m(Dt + D_0)\right)^{-2}\left(\frac{3D}{n+2}\right)^2\left(1 + n + n^2 - \frac{m}{3}(2 + 3n + n^2)\right)}\right] \\ & + 3\xi\beta D\left(m(Dt + D_0)\right)^{-1}\end{aligned} \tag{38}$$

and here $\omega_0 = \alpha(3 - \frac{2}{3}m(n+2) - \beta(1+2n)$.

Now, with the help of metric potentials, (1) can be reframed as

$$ds^2 = dt^2 - a\left(\left(m(Dt+D_0)\right)^{\frac{3}{m(n+2)}}\right)^2 (dx^2 + e^{-2px}dy^2) - \left(\left(m(Dt+D_0)\right)^{\frac{3n}{m(n+2)}}\right)^2 dz^2 \qquad (39)$$

Following equation (29) the Ricci scalar R for the metric is expressed as

$$R = 2\left(m(Dt+D_0)\right)^{-2}\left(\frac{3D}{n+2}\right)^2\left[R_0 - \left(\frac{p}{\left(m(Dt+D_0)\right)^{\frac{3}{m(n+2)}}}\right)^2\left(\frac{n+2}{3D}\right)^2\right] \qquad (40)$$

And $R_0 = 3 + 2n + n^2 - \frac{m(n+2)^2}{3}$

Name of Physical Parameter(s)	Case-I	Case-II
Scalar expansion (θ)	$3D$	$3D\left(m(Dt+D_0)\right)^{-1}$
Shear scalar (σ^2)	$\frac{1}{3}\left(\frac{3D}{n+2}\right)^2(n-1)^2$	$\frac{1}{3}\left(\frac{3D}{n+2}\right)^2(n-1)^2\left(m(Dt+D_0)\right)^{-2}$
mean Hubble parameter (H)	D	$D\left(m(Dt+D_0)\right)^{-1}$
mean anisotropy parameter (A_m)	$\left(\frac{n-1}{n+2}\right)^2$	$2\left(\frac{n-1}{n+2}\right)^2$

Table-I: Physical parameters for both the cases.

3. Conclusion

In this manuscript, we have presented the effects of bulk viscosity and domain wall in the context of $f(R,T) = N_1(R) + N_2(T)$ gravity. In view of the evolution of scale factor, the solutions are derived for $m = 0$ and $m \neq 0$. The cases $m = 0$ and $m \neq 0$ corresponds to two laws namely exponential and power law. The physical quantities involve in these models are constant in case one whereas for case two, except mean anisotropy parameter all others are depends on cosmic time. It is noted that, the volume of the universe is increasing exponentially and some power $\frac{3}{m}$ of cosmic time for case-I and II respectively (see Table-I). The mean anisotropic parameter is independent of time in both the cases, which indicates that the constructed models are anisotropic in nature throughout the evolution. Further, in both the cases the effect of bulk viscosity coefficient is noticed on the pressure of the domain wall and the EoS parameter but it does not affect the density of the domain wall and effective pressure.

4. **References**:

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