Dynamics of LRS Bianchi Universe-I anisotropic dark energy cosmological model in the presence of general relativity

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Abstract:
Dark energy cosmological models have become very important in modern era which is smallest interrelating has dynamical energy and anisotropic equation of state parameter density to describe the accelerated growth of the cosmos, so in this paper we explore, a dark energy model in the existence of general relativity for the context of locally rotationally symmetric Bianchi form-I cosmological models. For the sake of discovering a deterministic model of the cosmos and to get the particular solutions of the field equations of Einstein, we suppose Hubble parameter which gives the fixed value of deceleration parameter that acquires the required results of the field equations of Einstein. This parameter in the model is observed to be time subordinate which shows the development from initial decelerating period to the present accelerating period of expansion and supplies the biggest value and the high speed at which the universe is expanding, with the interpretations of modern cosmology, this point is in contrast. Model offers dynamically anisotropic expansion to the universe which allows for symmetrical Bianchi metrics, thus the cosmic microwave background anisotropy. We have claimed about the conduct of the anisotropy of the dark energy and the geometrical characteristics of the models. For this objective, we also apply an association between metric potential. It is noticed that these anisotropic and isotropic dark energy cosmological models constantly signifies an accelerated universe and are fixed with the current interpretations. Certain significant structures of the models, hence attained, have been conversed.

Keywords: Cosmological model, Field Equations, Anisotropic dark energy, Constant deceleration parameter

I. INTRODUCTION

The Invention of cosmic acceleration was the most important finding in modern cosmology. The basis of cosmic acceleration is still unknown. As per General Relativity, the slowing rate of expansion should be led by gravity, if the Universe consists of ordinary matter or radiation, then this expansion of powerful explosion is known as “Big Bang”. But, if we observe the chronology of the Universe, after the beginning of cosmic inflation, the Universe still expands in an accelerating rate. After that, Researchers originated three explanations, which were “cosmological constant”, “Dark Energy” and “Dark Matter.” A long-discarded version of Einstein's theory of gravity was “Cosmological constant.” “Dark Energy” was tied to Einstein’s cosmological constant, explaining the unrealistic expansion of the Universe, nevertheless the correct explanation was not known to theorists even then they gave the solution a name, the dark energy. Gravity, being the weakest fundamental force of the Universe, couldn’t have alone played the role. Therefore, a hypothetical form of matter, known as “Dark Matter” was introduced. Hence, the origin of dark forces from the present epoch of the Big Bang was known.

In recent years, an evidence suggests that there is something in our Universe which is invisible, as there is some new form of matter or energy. By the current measurements of NASA’s WMAP (Wilkinson’s Microwave Anisotropy Probe), the dark energy provides 68.3% (approx. 70%) of the total energy, where the mass-energy of dark matter gives 26.8% (approx. 25% ) and ordinary (baryonic) matter) [1-4], contribute 4.9% (approx. 5%) , in today’s universe, which were determined while making the accurate measurements of the cosmic microwave background fluctuations.
The universe is spatially flat.

\[
(\Omega_{\text{total}} = \Omega_{\text{mass}} + \Omega_{\text{relativistic}} + \Omega_{\Lambda} = 0.315 \pm 0.018 + 9.24 \times 10^{-5} + 0.6817 \pm 0.0018 = 1.00 \pm 0.02)
\]

accelerating, composed of photons and neutrinos, dark energy, normal mass (baryonic matter) in the current accepted model of modern cosmology. Therefore the role and description of dark energy and dark matter is concerned with the fate and face of the Universe. They must show a substantial role in the chronology of the Universe in leading it to its present state. The expansion of the universe is accelerated by the small value of cosmological constant. The formation of anisotropic compact stars comes into existence by the help of competent candidates of cosmological constant. The logical solution of Kriri and Barua metric has considered for this purpose. The Tolman–Oppenheimer–Volkoff (TOV) equations (which is Static, spherically symmetric perfect fluid models in general relativity), consists the stability and the surface redshift of the compact stars. It also has the radial dependence on cosmological constant. In order to describe presence of dark matter and acceleration in the universe, the significance accomplishment of General Relativity, yet not finished, different theories being proposed.

The field equations of Einstein are a system of simultaneous nonlinear differential equations. In the field equations, physical solutions have been identified in the context of cosmology and astrophysics. Einstein introduced to a cosmological term \( \Lambda \) in physics to explain the general theory of relativity. There has been significant and important evidence in recent years for the recognition of Einstein's cosmological constant (\( \Lambda \)). \( \Lambda \) is the function of temperature which is associated with the impulsive symmetry breaking procedure. There are two fundamental concepts; firstly, vacuum energy density that is directly related to the cosmological term which is also one of the fundamental problems of physics. Secondly, enormous scale conduct of the universe could be understood from the cosmological term that tells us that the noticeable universe is a major one and almost level. In addition the estimated worth of the cosmological concept and the expected value of the Quantum Field Theory (QFT) of the standard model is one of the outstanding problems in Physics. Many authors [5-14] have gone through the problem of cosmological term and after many attempts they have suggested that it takes on dynamical attributes and some of the followed the ideal of decreasing vacuum energy density with cosmic expansion, in order to understand the incredible of little estimation of the term-\( \Lambda \), from its Quantum field theory calculated value which is recently observed, time dependent can be helpful to understand development of \( \Lambda \). With regard to such a vast entropy per baryon, physical phenomenon, as perceived, the noticeable degree of radiation of cosmic background in cosmology, indicates dissipative effect. As it is observed today, for a long time, the process of dissipation continued, and in the early eras of cosmic development, many causes the high degree of isotropy. There are two main elements, the dissipative effect and the coefficient of bulk and shear viscosity, that are supposed to be particularly significant in the creation and foundation of the universe. On the account of this assumption, Weinberg[15] estimates and applies bulk and shear viscosity formulations that determine the role of cosmological entropy production because of the basis of this assumption. Padmanabhan and Chitre [16] noticed the existence of bulk viscosity with reference to general relativity, which causes solutions to be formed such as inflation etc. Riess[17] observed a basic description of the bulk viscosity which behaves as a real representative in the growing universe, that is called the theory of the negative entropy field. Referencing the viscosity dissipation method, Bilinski and Khalatnikov [18] considered the existence of the cosmological solutions for the homogeneous Bianchi type-I system. The Bianchi type-I solution with shear viscosity being the power function of entropy density, in the case of stiff matter, explored by Banerjee et al [19]. Referencing the cosmological development, Bali[20-22] and Baghel [23-26], Peebles [27] and Singh et. al. [28-31] revealed the outcome of bulk viscosity.

Cosmological concepts and scientific application of the outcome of FRW models encourages the concept of uniformity and isotropy of the universe. According to the viewpoint of above symmetries, the universe is neither uniform nor isotropic. The presence of anisotropic phase supported by many theoretical opinions Lima [32], Lornez [33] and current experimental data regarding cosmic background radiation anisotropies which approaches an isotropic discussed by Lukash[34]. The FRW model plays a very important role physically as well as geometrically in the dissipation of the initial universe because of the cosmological models that are spatially uniform and anisotropic with the abundant formation.
Flat FRW models were determined as anisotropic and exciting for further investigation just like Bianchi Type I models. These models supported by the facts from study of low density universe. Mark and Harko [35] has revealed the subtleties of a normal bulk viscous fluid cosmological model with continuously slowing cosmological models of Bianchi form I. Saha [36-37] considered with viscous fluid of the cosmological models of Bianchi form I.

By using Law of variation of Hubble’s parameter, Einstein’s field equations can be solved which gives deceleration parameter is constant suggested by Berman [38-39]. Singh and Desikan [40-41]. Maharaj and Naidoo [42], Pradhan and et al [43-44] has considered the previous literature, a fixed deceleration parameter in cosmological models. There is chequered history for red shift magnitude test. It was used to draw very unconditional decisions for the duration of the 1960 and 1970. It was declared that deceleration parameter stated to lie between 0 and 1 and the universe is decelerating. Pradhan and et al [45-46] observed kindIa supernovae, and concluded about the extension of the universe is increasing and the cosmological models with variable deceleration parameter has been also examined by them. Vishwakarma and Narlikar[47] and Virely et al [48] also considered the cosmological models with variable deceleration parameter.

We have claimed about the conduct of the anisotropy of the dark energy and the geometrical characteristics of the models. To get this objective, we also apply an association between metric potential. It is noticed that these anisotropic and isotropic dark energy cosmological models constantly signifies an accelerated universe and are fixed with the current interpretations.

The Prime purpose of proposed work is to explored in this paper we explore, a dark energy model in the existence of general relativity for the context of locally rotationally symmetric Bianchi form-I cosmological models.

The manuscript leads by the series of sections as mentioned: In section 2, the basic definitions of anisotropic models are mentioned. In the division 3, field equation’s results are obtained in the existence of general relativity for the context of LRS Bianchi form-I cosmological models, in section 4, the conclusion drawn from the results.

II. METRIC AND FIELD EQUATIONS

Line element
\[ ds^2 = dt^2 - R_1^2(t) dx^2 - R_2^2(t) [dy^2 + dz^2] \] .... (1)

defines the spatially uniform and anisotropic locally rotationally symmetric Bianchi type-I metric space time where \( c = 1, 8\pi G = 1 \).

where the \( R_1, R_2 \) are metric potentials, that denotes cosmic time functions t.

The cosmic matter represented by a perfect fluid of the energy momentum tensor
\[ T_{ij} = (\rho + p) v_i v_j - p g_{ij} \] .... (2)

"p" specifies its pressure, The four velocity vector is denoted by \( v_i \), therefore \( v_i v^i = 1 \), energy density of the cosmic matter is denoted by \( \rho \).

The equation of state is defined as \( p = \omega \rho, 0 \leq \omega \leq 1 \) .... (3)

The field equations of Einstein is identified as given below with time dependent G and lambda,
\[ R_{ij} - \frac{1}{2} R g_{ij} = T_{ij} (8\pi G(t)) + \Lambda(t) g_{ij} \] .... (4)
In co-moving co-ordinate system, by using equation of (1) and equation (2), the field equation of (4) gives as

\[ 2 \frac{R_2^{**}}{R_2} + \left[ \frac{R_2}{R_2} \right]^2 = -8\pi G \rho + \Lambda \] 

\[ \text{.....(5)} \]

\[ \frac{R_1^*}{R_1} + \frac{R_2^{**}}{R_2} + \frac{R_1^* R_2^*}{R_1 R_2} = -8\pi G \rho + \Lambda \] 

\[ \text{.....(6)} \]

\[ \frac{2R_1^* R_2^*}{R_1 R_2} + \left[ \frac{R_2^{**}}{R_2} \right]^2 = 8\pi G \rho + \Lambda \] 

\[ \text{.....(7)} \]

“ * ” signifies the ordinary differentiation, with respect to the cosmic time \( t \).

Einstein tensor’s equation, with respect to disappearing of divergence is given as

\[ 8\pi G^* \rho + 8\pi G \left( \rho^* + (\rho + p) \left[ \frac{R_1^*}{R_1} + \frac{2R_2^*}{R_2} \right] \right) = -\Lambda^* \] 

\[ \text{.....(8)} \]

The equation of energy conservation gives (\( T_{ij}^j = 0 \))

\[ \rho^* + (\rho + p) \left[ \frac{R_1^*}{R_1} + \frac{2R_2^*}{R_2} \right] = -\Lambda^* \] 

\[ \text{.....(9)} \]

By substituting the value of equation (9) in the equation (8), we acquire \( G \) and \( \Lambda \) coupled field specified by

\[ \Lambda^* = -8\pi G^* \rho \] 

\[ \text{.....(10)} \]

The equation (10) representing as lambda (\( \Lambda \)) is a constant when \( G \) is a constant.

By substituting the value of equation (3) in (9) equation, also by integrating,

\[ \rho = \frac{k}{R^{3(\omega+1)}} \] 

\[ \text{.....(11)} \]

denoting \( k \) signifies as the constant of integration that is \( k > 0 \).

Volume of this model is specified by

\[ V(t) = R^3 = \left[ R_1 R_2^2 \right] \] 

\[ \text{.....(12)} \]

We define Average scale factor \( R \) is \( \left[ R_1 R_2^2 \right]^{1/3} \) of locally rotationally symmetric Bianchi type- \( I \) universe.

From the equations (5), (6) and (7)

\[ \frac{R_1^*}{R_1} - \frac{R_2^*}{R_2} = k_1 \frac{k_1}{R^3} \] 

\[ \text{.....(13)} \]

The Constant of integration is denoted by \( k_1 \). The Hubble-parameter is denoted by ‘H’, and deceleration parameter is denoted by \( q \), \( \sigma \) is shear and \( \theta \) is the volume expansion.

\[ \theta = 3H = \frac{3R^*}{R}, \]
\[ H = \frac{\dot{R}}{R} = \frac{1}{3} \frac{\dot{V}}{V} = \frac{1}{3} \left[ \frac{R_1^*}{R_1} + \frac{2 R_2^*}{R_2} \right] \]

\[
\sigma = \frac{K}{\sqrt{3} R^3} ,
\]

\[
q = -1 - \frac{H^*}{H^2} = -\frac{RR^*}{[R]^2}
\]

In terms of \( H, \sigma \) and \( q \), the equations (5) to (7) and also equation (9) can be specified by

\[
(2q - 1)H^2 - \sigma^2 = 8\pi pG - \Lambda , \quad \ldots (14)
\]

\[
3H^2 - \sigma^2 = 8\pi \rho G + \Lambda , \quad \ldots (15)
\]

\[
\rho^* + 3(\rho + p) \frac{R^*}{R} = 0 \quad \ldots (16)
\]

\[
\rho_c = \frac{3H^2}{8\pi G} \quad \text{consider as critical density} \quad \ldots (17)
\]

\[
\rho_v = \frac{\Lambda}{8\pi G} \quad \text{specifies as vacuum density} \quad \ldots (18)
\]

\[
\Omega = \frac{\rho}{\rho_c} = \frac{8\pi G \rho}{3H^2} \quad \text{consider as density parameter} \quad \ldots (19)
\]

By substituting these values in the equation (15), we acquire

\[
\frac{3 \sigma^2}{\theta^2} = 1 - \frac{(24\pi G\rho)}{H^2} - \frac{3\Lambda}{\theta^2}
\]

Signifies that for \( \Lambda \geq 0 \)

\[
0 \leq \frac{\sigma^2}{\theta^2} \leq \frac{1}{3} ,
\]

\[
0 \leq \frac{8\pi G \rho}{\theta^2} \leq \frac{1}{3}
\]

Therefore the existence of negative lambda gives more for anisotropy whereas positive lambda lowers the upper limit of anisotropy.

From (14), and (15), we have

\[
\frac{d\theta}{dt} = -12\pi pG - \frac{3}{2} \sigma^2 + \frac{3}{2} \Lambda - \frac{\theta^2}{2}
\]

\[
= -12\pi G(p + p) - 3\sigma^2
\]

That indicates, during the evolution of time, the rate of volume extension reduces and therefore the existence of positive lambda implies that the universe is slowing down its rate of decay whereas a negative lambda would support it.

\[
\sigma^* = -\frac{3\sigma R^*}{R}
\]

It indicates that in the growing universe, the value of \( \sigma \) reduces and for infinitely value of \( R \).
the value of $\sigma$ is insignificant.

### III. SOLUTION OF THE FIELD EQUATIONS

Five equations in six unknowns ($R_1, R_2, \rho, p, G$ and $\Lambda$) are provided by the system of equations (3), (5) – (7) and (10). For completely solving the system, an extra equation is required.

\[ \frac{\Lambda}{R^m} = \frac{a}{R^m}, \quad \text{signifies a decaying vacuum energy density.} \quad \text{\ldots (20)} \]

By substituting the equations (11) and (20) in the equation (10), then

\[ G = \left[ R^{3\omega+3-m} \right] \frac{a m}{8\pi k} \quad \text{\ldots (21)} \]

By using the equations (14), (15), (20) and (21), we acquire

\[ \frac{R^{*2}}{R} + 2 \left( \frac{R^{*2}}{R} \right)^2 - \frac{am(1-w)}{2(3\omega+3-m)R^m} - \frac{a}{R^m} = 0 \quad \text{\ldots (22)} \]

Take the value $\omega = \frac{1}{3}$

To get cosmological models for dense matter, this relates to the equation $\rho = 3p$, in general relativity, this equation of state has been extensively useful.

The value of the equation (22) turn into

\[ \frac{R^{*2}}{R} + 2 \left( \frac{R^{*2}}{R} \right)^2 - \frac{a}{R^m} \left[ \frac{2(m-6)}{3(4-m)} \right] = 0 \quad \text{\ldots (23)} \]

To acquire the time progression of Hubble parameter, integrate the equation (23)

\[ \frac{R}{R} = H = \sqrt{\frac{4a}{3(4-m)}} \frac{m}{2} \sqrt{\left( \frac{4a}{3(4-m)} \right) t + t_0}^{-1} \quad \text{\ldots (24)} \]

here $t_0$ is a constant.

To acquire the value of scale factor, using the equation (24)

\[ R = \left( \frac{m}{2} \sqrt{\left( \frac{4a}{3(4-m)} \right) t + t_0} \right)^{2/m} \quad \text{\ldots (25)} \]

By substituting the equation (25) into the equation (13), the equation of metric (1) exist as

\[ ds^2 = dt^2 + \left( \frac{m}{2} \sqrt{\left( \frac{4a}{3(4-m)} \right) t + t_0} \right)^{m/2} \times \]

\[ \left\{ -m_1^2 \exp 2 \left\{ \frac{8k_1}{3} \left[ \sqrt{\frac{3(4-m)}{a}} \right] \frac{1}{m-6} \left( \frac{m}{2} \sqrt{\left( \frac{4a}{3(4-m)} \right) t + t_0} \right)^{m-6} \right\} dx^2 - \right. \]

\[ m_2^2 \exp 2 \left\{ \frac{4k_1}{3} \left[ \sqrt{\frac{3(4-m)}{a}} \right] \frac{1}{m-6} \left( \frac{m}{2} \sqrt{\left( \frac{4a}{3(4-m)} \right) t + t_0} \right)^{m-6} \right\} (dy^2 + dz^2) \] \quad \text{\ldots (26)}
Here \( m_1 \) and \( m_2 \) signifies as constants.

From the above model (26), \( \rho \) is denoted as matter density, \( p \) is denoted as pressure and \( G \) signifies as gravitational constant, \( \Lambda \) is denoted as cosmological constant and the spatial volume is denoted as \( V \) are specified as

\[
V = \left( \frac{m}{2} \sqrt{\left( \frac{4a}{3(4-m)} \right) t + t_0} \right)^{6/m}, \quad \ldots (27)
\]

\[
\rho = \frac{k}{\left[ \frac{m}{2} \sqrt{\left( \frac{4a}{3(4-m)} \right) t + t_0} \right]^{\frac{m}{6}}} \quad \ldots (28)
\]

\[
p = \frac{k}{\left[ \frac{m}{2} \sqrt{\left( \frac{4a}{3(4-m)} \right) t + t_0} \right]^{\frac{m}{6}}} \quad \ldots (29)
\]

\[
G = \frac{am}{8\pi k (4-m)} \left[ \frac{m}{2} \sqrt{\left( \frac{4a}{3(4-m)} \right) t + t_0} \right]^{-\frac{2}{m}} \left( 4-m \right)^{2/m} \quad \ldots (30)
\]

\[
\Lambda = a \left[ \frac{m}{2} \sqrt{\left( \frac{4a}{3(4-m)} \right) t + t_0} \right]^{-2} \quad \ldots (31)
\]

\( \theta \) is denoted as expansion scalar, \( \sigma \) signifies by shear

\[
\theta = 3 \left[ \frac{4a}{3(4-m)} \right] \left[ \frac{m}{2} \sqrt{\left( \frac{4a}{3(4-m)} \right) t + t_0} \right]^{-1} \quad \ldots (32)
\]

\[
\sigma = \frac{k}{\sqrt{3}} \left[ \frac{m}{2} \sqrt{\left( \frac{4a}{3(4-m)} \right) t + t_0} \right]^{-6/m} \quad \ldots (33)
\]

\( \Omega = \frac{\rho}{\rho_c} = \frac{m}{4} \) is the density parameter \ldots (34)

For acquired model, value of deceleration parameter \( q \) specifies as

\[
q = \frac{m}{2} - 1 \quad \ldots (35)
\]

\( \rho_v \) is denoted as the vacuum energy, \( \rho_c \) is denoted as density critical density are

\[
\rho_v = \frac{k(4-m)}{m} \left[ \frac{m}{2} \sqrt{\left( \frac{4a}{3(4-m)} \right) t + t_0} \right]^{-8/m} \quad \ldots (35)
\]

\[
\rho_c = \frac{4k}{m} \left[ \frac{m}{2} \sqrt{\left( \frac{4a}{3(4-m)} \right) t + t_0} \right]^{-8/m} \quad \ldots (36)
\]

From the given model (26), it has been noticed that for \( 0 < m < 4 \), If \( t = t'' \) then value of the spatial volume \( V = 0 \) and

Also the value of \( t' = \frac{-t_0}{\frac{m}{\sqrt{4a}} \frac{4}{3(4-m)}} \) and the value of \( \theta \) which is denoted as expansion scalar is infinite, that shows that with zero volume, our space begins to grow at \( t=t'' \) and an infinite rate of development. As \( t \) increases,
however expansion scalar decreases but the spatial volume increases. Thus, the expansion rate falls as time rises. When \( t \) approaches to infinity, then the spatial volume become extremely huge. All the parameters \( p, \rho, \sigma, \rho_0, \rho_c, \sigma \) reduce with rising time and tend to zero asymptotically. Hence, the model essentially gives an empty universe for large \( t \). The proportion \( \frac{\sigma}{\theta} \) approaches to zero as \( t \) approaches to \( \infty \) that displays that the model approaches isotropy for large values of \( t \). At \( t = t' \), the value of \( G (t) \), the gravitational constant is zero and in the past era, it becomes extremely huge as \( t \) rises. \( G \) rises, that is identically result as that is achieved by Levitt [9], Abdel-Rahan [49], Chow[50], and Milne[51].\( \text{When} \ \rightarrow \infty \ \text{then} \ \frac{\sigma}{\theta} \rightarrow 0, m < 4 \), so the model tends to isotropy for a big value of \( t \). Therefore, with a big bang start approaching the isotropy at late times, the model signifies a shearing, non-rotating and expanding model of the universe.

IV. CONCLUSION

In this manuscript we have achieved to explore, a dark energy model in the existence of general relativity for the context of locally rotationally symmetric Bianchi form-I cosmological models. In order to discover a deterministic model of the cosmos and to acquire the particular solutions of the field equations of Einstein, we suppose the Hubble parameter which gives the fixed value of deceleration parameter that acquires the required results of the field equations of Einstein. This parameter in the model is observed to be time subordinate which shows the development from initial decelerating period to the present accelerating period of expansion and supplies the biggest value and the high speed at which the universe is expanding, with the interpretations of modern cosmology, this point is in contrast. Model offers dynamically anisotropic expansion to the universe which permits for symmetrical Bianchi metrics, therefore the cosmic microwave background anisotropy. The dark energy is slightly interrelating is presumed has dynamical energy density and anisotropic equation of state parameter. For the deviation from isotropic equation of state parameter, a special law has been supposed that is consistent with the supposition on the conservation of the energy-momentum tensor of the Dark energy. By supposing a special law of variation for the mean Hubble parameter, particular solutions of Einstein’s field equations have been achieved that gives a constant value of the deceleration parameter and is consistent with interpretations. Cosmological models that shows the homogeneous and isotropic Bianchi form-I Local Rotational Symmetry with variables \( G \) and lambda. The corporal properties of corresponding cosmological models and the behavior of the anisotropic model has been conferred. The universe starts to accelerate and increases exponentially with infinite time \( t \). Model represents a tends to isotropy for the hefty value of \( t \). Thus it signifies shearing, non-rotating and growing model for the cosmos with a big bang start approaching to isotropy at late times. Hence the space starts through a decelerating extension deviates and the growth deviates through decelerating phase to accelerating one. The value of the gravitational constant \( G (t) \) is zero at the initial singularity and it becomes extremely huge as \( t \) rises. In the conclusion, in Bianchi type-I space-time Local Rotational Symmetry with variables \( G \) and lambda, the results obtained in this manuscript are new and beneficial for a better consideration of the evolution of the universe.

REFERENCES


