Variation in growth rates of various modes (GL, Marangoni) for different value of deformability parameter and Re.

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Abstract

Linear stability analysis of fluid flowing down an inclined rigid and solid surface in presence of interfacial insoluble surfactant. Here, we have three modes like gas-liquid (GL), liquid-liquid and solid-liquid. When the Newtonian and incompressible fluid flows down an inclined rigid surface then we find only single mode that is known as gas-liquid mode. This mode remain stable for low Reynolds number and low wavenumber limit but becomes unstable at higher value of Reynolds number and in finite wave limit. When gas-liquid interface is coated with an insoluble surfactant then GL mode becomes stable even in finite wave range but new mode occurs that is known as surfactant or Marangoni mode. This Marangoni mode remains stable for all the values of Reynolds number for all wave limits. If rigid surface is coated by a deformable solid then gas-liquid mode becomes further more stable in finite wave limit but becomes unstable in short wave limit. Marangoni mode also becomes unstable in presence of soft solid. In methodology, we use Mathematic and MATLAB for producing initial guess values and finally C code is written for generating data in detail.

Key words: Stability, gas-liquid (GL) mode, Marangoni mode, Liquid-solid mode (LS), surfactant.

Introduction

The stability of gravity-driven fluid flow research problem is widely studied by many researcher and scientists because of its relevance with many scientific and industrial problems like coatings [1], heat exchanger [2] and in biological flows [3]. Now, it is also essential to discuss the properties of insoluble surfactant because in this research article, insoluble surfactant plays a very important role to decrease or increase the surface tension of gas-liquid and/or liquid-liquid interface but it doesn’t affect the surface tension of liquid-solid interface. Surfactant is an organic compound. It can be an amphiphilic- means that contains both hydrophilic groups and hydrophilic groups. Hence, a surfactant contains both groups, one water insoluble another water soluble. When surfactant is introduced on gas-liquid or liquid-liquid interface then it breaks the bonds between gas molecules and water molecules or it enters into the troughs and tries to stabilize the interfacial modes. Because it fills the vacant space (in form of troughs) so the fluid particles flows over it and does not go downside. Many researcher and scientists have studied the falling film problem in which they have used a surface active agents like surfactant which plays a very important role in the
applications of flow instabilities [4, 5, 6]. When the surfactant flows on various interfaces they greatly affects the surface instabilities, it works as a stabilizing agents for the gas-liquid interface for low and finite Reynolds numbers and produces the new mode that become stable for low Reynolds number and in finite wave limit[7, 8]. The effects of deformable solid on various modes like gas-liquid etc. have been studied by Shankar and his groups [9, 10, 11, 12, 13, 14]. In this research article, we are showing the effect of surfactant and deformable solid on the growth rate of various unstable or stable modes as well. The liquid-solid mode instabilities are very strong. These instabilities cannot be suppressed by any type of surfactant (soluble or insoluble surfactant) but the presence of soft solid plays a very important role to stabilize the gas-liquid mode instabilities specially in finite wave limit for low Reynolds number that is why we also opted soft solid and coated rigid surface by it.

**Problem Formulation**

The gravity-driven Newtonian and incompressible fluid is considered in this research article which flows down a tilted solid surface, the rigid surface is coated by a deformable solid. The density and viscosity of fluid is \( \rho \) and \( \mu_1 \) respectively. The shear modulus of soft solid is \( E_s' \) and characteristic length/thickness is \( R' \).

The equation of continuity and momentum as given below are considered which govern the fluid flow.

\[
\nabla^* \cdot \mathbf{V}^* = 0 \\
\rho \left[ \partial_t V_1^* + V_1^* \nabla^* \cdot \mathbf{V}^* \right] = \nabla^* \cdot \mathbf{T}^* + \rho \mathbf{g} \tag{2}
\]

The configuration, other parameters and linearised governing equations are written in detail in reference [15].

**Result and Discussion**

In this problem, we used MATLAB and MATHEMATICA codes for generating the initial guess value by solving the linearised governing equation including fourth order Orr-Summerfield equation. Finally we wrote C code and used initial guess to generate data in detail.

Figure 1 show the effect of inertia on the growth rate of Marangoni mode for the parameters solid thickness (\( H = 10 \)), Mrangoni number (\( M = 1 \)) and \( \theta = 45^0 \). In this figure, it is clear that the growth rate of Marangoni mode increases with increasing the Reynolds number from 2.5 to 10. It means, inertial shows destabilizing effect on the Marangoni mode but the value of inertia is decreased then Marangoni mode becomes stable and it becomes completely stable at zero Re or in creeping flow limit. Reynolds number increases when the fluid is inertial force dominating (the viscosity of fluid is less comparatively then fluid flow with higher velocity compare to higher viscous fluid).
Figure 1: Effect of inertial force on the growth rate of Marangoni mode for the data $H = 10$, $G = 0.1$, $Ma = 1$, surface tensions ($\Sigma_1 = 0$, $\Sigma_2 = 0.5$) and $\theta = 45^0$.

Figure 2 shows the effect of inertia on the growth rate of gas-liquid mode for the data soft solid thickness ($H = 10$), Marangoni number ($M = 1$), solid deformability parameter ($G = 0.005$) and inclination angle $\theta = 45^0$. Figure 2 shows clearly that the growth rate of gas-liquid mode also increases or flow becomes unstable when Reynolds number is increases from 2.5 to 10. Finally, it can be predicted that the growth rate of GL mode instabilities increases when the fluid becomes less viscous or when flow is inertial force domination but flow becomes stable when Reynolds number decreases and it becomes completely stable at zero Re.
Figure 2: Effect of inertia on the growth rate of gas-liquid mode for the data: $H = 10$, $G = 0.005$, $Ma = 1$, surface tensions ($\Sigma_1 = 0$, $\Sigma_2 = 0.5$) and $\theta = 45^\circ$.

**Conclusion**

We have studied the problem of gravity-driven of Newtonian liquid layer flowing down a tilted solid surface in presence of insoluble surfactant on gas-liquid interface. Here, we discussed only about two modes (GL and Marangoni mode). GL mode becomes unstable when inertial force increases ($Re$ increases from 2.5 to 10) and vice-versa. Marangoni mode also becomes unstable at higher value of Reynolds number. Finally, it can be predicted that flow becomes unstable at higher value of inertial force or $Re$ and can be stabilize by reducing the value of $Re$.

**References**


