Design and Analysis of Fringing Field Effect on Capacitance of Capacitor

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Abstract

The consideration of fringing field is very crucial for the design of parallel capacitors when the gap of the parallel plates is comparable to the geometrical dimensions. This work presents the finite element modelling of the effect of fringing field on parallel plate capacitor. The accurate prediction of the capacitance can be done only when the domain used to model fringing field is large enough and suitable boundary conditions are applied. The electric field distribution of parallel plate capacitor for different boundary conditions has observed. Also, the variation in the size of air domain has done to observe the effect on the capacitance. In the modelling the size of the air domain, the effect on the capacitance and the choice of boundary are addressed as well.

Keywords: Fringing filed; Capacitance; Electric flux; Parallel plate capacitor; Finite element modelling;

1. Introduction

The effect of the fringing field on the capacitor's capacitance has been the interest of many researchers due to the applications like microstrip circuit and antenna elements. In 1877, Kirchhoff obtained the estimated solution for the circular capacitor [1]. He used conformal mapping technique to encounter the effect of fringing field. Later on, the research on capacitance estimation has increased substantially [2-4]. In 1970, some researchers used integral equations to obtain the capacitance [5]. Though a lot of research was done to encounter the fringing field effect on capacitance of parallel plate capacitor, but conformal mapping method was mostly used [5]. The estimation of the electrostatic field at the edges of the plate is necessary to evaluate the capacitance of parallel plate capacitor. Previously, the analysis fringing field of parallel plate capacitor has been done by assuming the plate to be semi-infinite and then conformal mapping was applied to the geometric configuration [6]. Afterward, plates were supposed to be of finite width to accurately account the fringing field capacitance [7].

The bending of the electric flux lines near the edges of the parallel plates is known as fringing or also known as edge effect. The electric flux lines inside the parallel plates are uniform but at the edges, the flux lines are bend slightly upward. The capacitance of the parallel plate capacitor is estimated by equation 1.

$$C = \frac{\varepsilon A}{g} \qquad\qquad (1)$$

where A is the area of the parallel plates, g is the gap between the plates, and ε is the permittivity in the gap. The capacitance of the parallel plate capacitor calculated by the above formula is less than the actual capacitance of capacitor due to fringing effect [8]. The capacitance of capacitor including the fringing field effect can be calculated by the most accurate method i.e. Laplace formula. Several approximations like zero thickness of the plate has been done to estimate the fringing filed capacitance [9][10]. By taking the finite

thickness of the electrodes, some other formulae have also been derived [11-16]. All these derived mathematical equations are complex and provide only the approximate solutions. The FEM of the parallel plate capacitor is an efficient method to model and view the distribution of electric field around the structure [17-18].

In this paper, finite element modelling of a parallel plate capacitor has been done. The effect of variation in the air domain on the capacitance has observed. The electric field distribution for insulator and conductor boundary conditions is determined. All the simulations are performed in FEM based tool COMSOL Multiphysics. The electrostatics physics has used to apply the potential difference between the plates of the capacitor.

2. Fringing field Effect

A typical capacitor structure consists of two conductive plates separated by a dielectric material. When a voltage is applied between the two plates, an electric field is produced between them. In fringing field, the electric field occurs not only in between the conducting plates but spreads some distance away. The figure 1 illustrates gold disc capacitor positioned in spherical air domain. The size of the air domain truncates the modelling space.
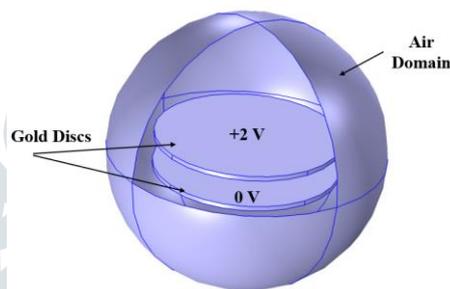

Figure1: Schematic diagram of parallel plate capacitor

The gold disc with the radius 10 cm and the height of 0.5 cm are placed in spherical air domain. The radius of the air domain is 15 cm. The spherical domain in which the capacitor is placed, can be assumed as the two different physical conditions: it can be treated as perfect insulator or perfect conductor. In perfect insulator, the charge does not redistribute itself. The modelling of the perfect insulator is done by using Zero Charge boundary condition. In this, the electric field lines are always tangential to the boundary.

The floating potential boundary condition is used to model perfect conducting surface. In this, the electric potential value on the surface of the sphere is fixed to a constant value that is computed during the solution. Here, the electric field lines are normal to the boundary.

3. Results and Discussion

The design and modelling of parallel plate capacitor has done in finite element modelling (FEM) based tool COMSOL Multiphysics. In this two-gold disc are placed in spherical air domain whose size is varied to observe the effect on capacitance. Figure 2 (a) shows the electric field distribution when the air domain is considered as perfect insulator. The electric field lines move from the positive plate to the plate having zero potential as shown in figure 2 (b). The electric field lines are much intense at the edges of the parallel plate is due to the curvature.
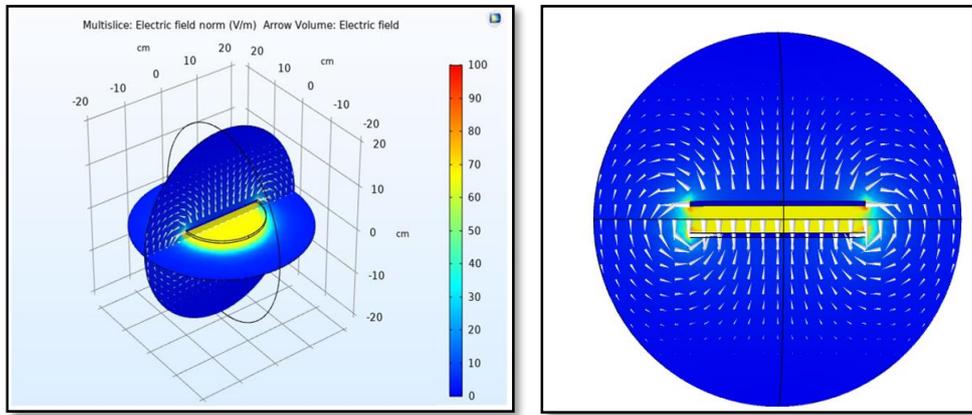
Figure 2: Electric field distribution (a). for zero charge boundary condition (b). Electric field lines direction

Figure 3 shows the electric field distribution when the air domain is considered as perfect conductor.
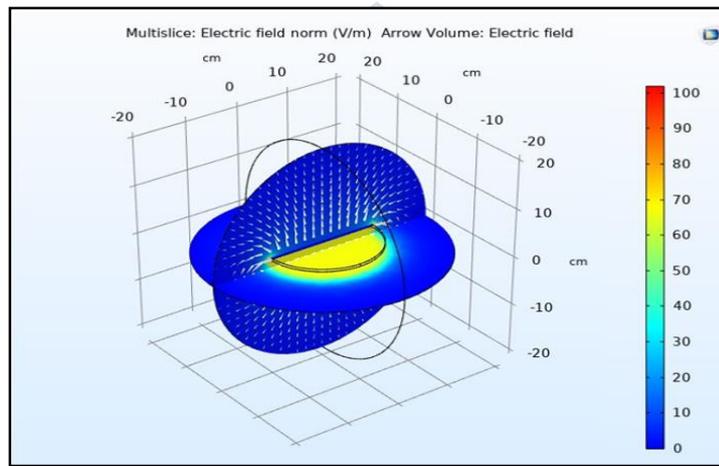

Figure 3: Electric field distribution of floating potential boundary condition

The radius of air sphere is changed from 15 cm to 39 cm to observe its impact on the capacitance of the capacitor. As radius of the air domain is changed, the electric field lines intensity seems to be decreasing as shown in figure 4. For small diameter of the spherical air domain, the electric field penetrates at the boundary also, but as the size increases, the effect of the electric field lines decreases at the boundary.
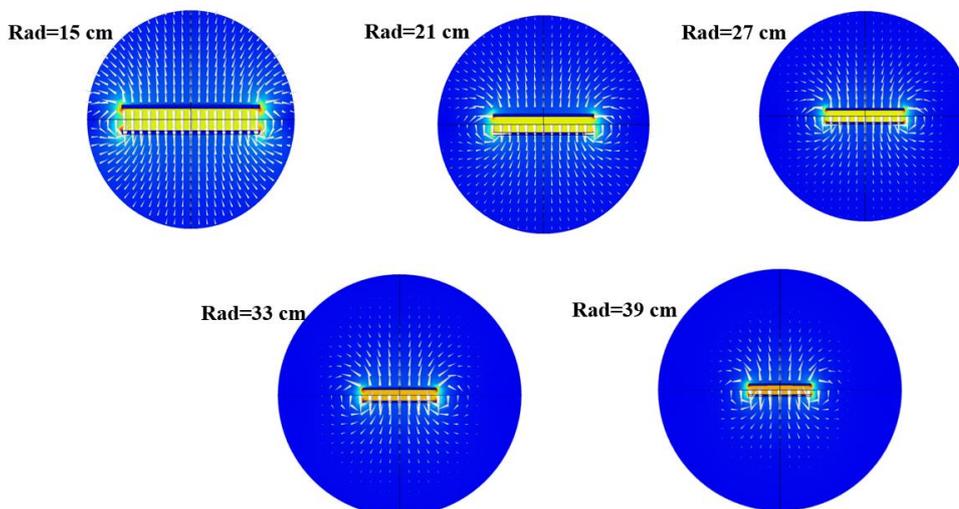

Figure 4: Intensity variation of Electric field lines on changing the air domain radius

Figure 5 show the convergence plot and compared the capacitance values as function of sphere radius for both the boundary conditions. The blue and green lines show the variation in capacitance for insulating and conducting sphere respectively. The average of the two plot is taken and is presented by the red line.

All the three plots converge to the same value as the radius of the air sphere increases. Practically, modelling of small air sphere is enough for both the boundary conditions and average of the two can be taken.
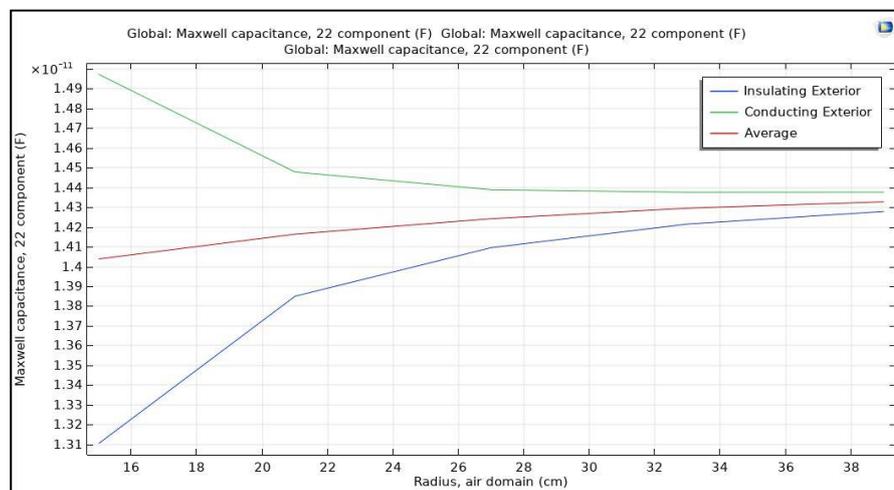

Figure 4: Convergence of capacitance when size of air domain is increased

Conclusion

This work presents the design and analysis of a parallel plate capacitor using FEM based tool COMSOL Multiphysics. The analysis of the parallel plate capacitor for perfect insulator and perfect conductor domain has been done. The electric field lines are denser at the plate edges because of the curvature of the plates. For both the boundary conditions, the capacitance of the capacitor converges as the size of the air domain is increased. An average graph can be plotted for both the boundary conditions since there is small variation in the capacitance is observed.

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