Mathematical Review Of FDE And Comb Type Pilot

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***Abstract*— Scientific survey of the Orthogonal Frequency Division Multiplexing is shown as far as Inter image impedance, Multi transporter regulated framework and cyclic prefix. Demonstrating of the numerical condition of the Orthogonal Frequency Division Multiplexing, Inverse quick Fourier change and quick Fourier change is clarified with the appropriate model. Bit mistake rate execution of OFDM is likewise given the assistance of measurable calculation. This paper will enable the scientist to all the more likely comprehend the idea of the OFDM in 3G and 4G/LTE frameworks.**

Keywords—Cyclic prefix, Inter symbol interference, Multi carrier modulated systems.

I. Introduction

Symmetrical recurrence division multiplexing (OFDM) is a broadband key innovation that is utilized in "4G remote correspondence framework". It is additionally utilized in "Long haul development" (LTE) 4G cell standard and Worldwide interoperability for microwave get to (Wi max) moreover. These are currently day's dominants benchmarks and both depend on OFDM [1].

LTE – A long haul development propelled which is the most recent correspondence standard in broadband is likewise founded on OFDM. LTE is a remote correspondence standard that can bolster a huge data transfer capacity. As a rule, GSM has a data transmission of about 200KHz yet OFDM can have the transfer speed of about 100MHz. Normally, the information rate in OFDM will be higher which is utilized in 3G and 4G to empower information pace of up to 100MBps or all the more then 500MBps. A few IEEE Wi-Fi remote neighborhood WLAN (802.11, 802.11G, 802.11N, 802.11AC) gauges are utilized for high information rate which depends on the OFDM [2].

II. OFDM using FDE technique

To remove Inter symbol interference following methods are used.

1. Previously we have seen time domain equalization also known as zero forcing equalizer.
2. OFDM
3. Frequency domain equalization

Frequency domain equalization

Let N=4 symbols are to be transmitted $x(0), x(1), x(2), x(3)$ by adding one modification which is adding the cyclic prefix. Remember in OFDM these were the time domain samples. In FDE we directly transmit the symbols. There is no loading of the symbols on the subcarrier [3].

So, adding the cyclic prefix it will be

$x(3), x(0), x(1), x(2), x(3)$

These are the time domain symbols + CP. This will be transmitted across the ISI channel.

Therefore

$$y(k) = h(0)x(k) + h(1)x(k-1) + v(k)$$

2 taps ISI channel

And we will have

$$y = h \otimes x + v$$

Aries due to cyclic prefix.

Now once we take the FFT of the output

$$Y(l) = H(l) \times X(l) + V(l)$$

V(l) is the Lth FFT point of the noise samples.

Time domain symbols are $x(0), x(1), x(2), x(3)$

Once if we take the FFT of these N=4 pt, we have simply FFT coefficient of transmitted symbols.

And hence $X(0), X(1), X(2), X(3)$ are simply frequency domain FFT coefficient of the time domain symbols [4].

$$X(l) = \frac{1}{N}\sum_{k=0}^{N-1} x(k) e^{-j2\pi\frac{kl}{N}}$$

Let N=4

We have

$$X(l) = \frac{1}{4}\sum_{k=0}^{3} x(k) e^{-j\pi\frac{kl}{2}}$$

Which is the Lth FFT point of the time domain transmitted symbol?

And hence as we know

$y(0), y(1), y(2), y(3)$ are the time domain output symbols

$Y(0), Y(1), Y(2), Y(3)$ are frequency domain output samples.

Similarly for the channel taps

$h(0), h(1), 0, 0$

$\downarrow\downarrow\downarrow\downarrow\downarrow\downarrow\downarrow\downarrow\downarrow\downarrow\downarrow\downarrow\downarrow\ N = 4pt, FFT$

$H(0), H(1), H(2), H(3)$

$H(0), H(1), H(2), H(3)$ are frequency domain channel coefficient

$h(0), h(1), 0, 0$ are time domain channel taps with zero padded

Finally for noise

$v(0), v(2), v(3), v(4)$

$\downarrow\downarrow\downarrow\downarrow\downarrow\downarrow\downarrow\downarrow\downarrow\downarrow\downarrow\downarrow\downarrow\ N = 4pt, FFT$

$V(0), V(1), V(2), V(3)$

Where

$V(0), V(1), V(2), V(3)$ are the noise samples across the subcarrier [5].

Now the frequency domain model across the Lth FFT pt

$$Y(l) = X(l)H(l) + V(l)$$

Now we can estimate the Lth FFT coefficient of $X(l)$

Which will be

$$\overleftrightarrow{X}(l)=\frac{Y(l)}{H(l)}$$

$$\overleftrightarrow{X}(0)=\frac{Y(0)}{H(0)} \qquad \overleftrightarrow{X}(1)=\frac{Y(1)}{H(1)}$$

$$\overleftrightarrow{X}(2)=\frac{Y(2)}{H(2)} \qquad \overleftrightarrow{X}(3)=\frac{Y(3)}{H(3)}$$

These are the FFT coefficients of the symbols sequence $x(0),x(1),x(2),x(3)$

Now if you perform the IFFT of received FFT coefficient of the symbols

$\overleftrightarrow{X}(0),\overleftrightarrow{X}(1),\overleftrightarrow{X}(2),\overleftrightarrow{X}(3)$

$\downarrow\downarrow\downarrow\downarrow\downarrow\downarrow\downarrow\downarrow\downarrow\downarrow\downarrow\downarrow\downarrow\ N=4, pt, IFFT$

$\overleftrightarrow{x}(0),\overleftrightarrow{x}(1),\overleftrightarrow{x}(2),\overleftrightarrow{x}(3)$

We will able to generate the estimate of the symbols in the time domain $\overleftrightarrow{x}(0),\overleftrightarrow{x}(1),\overleftrightarrow{x}(2),\overleftrightarrow{x}(3)$

Similarly

$$\overleftrightarrow{x}(k)=\frac{1}{N}\sum_{l=0}^{N-1}\overleftrightarrow{X}(l)e^{j2\pi\frac{kl}{N}}$$

$$\overleftrightarrow{x}(k)=\frac{1}{N}\sum_{l=0}^{N-1}\frac{Y(l)}{H(l)}e^{j2\pi\frac{kl}{N}}$$

Which is the final symbol estimated in time domain.

For N=4 channel taps, Estimate of the Kth symbol in the time domain [6].

$$\overleftrightarrow{x}(k)=\frac{1}{4}\sum_{l=0}^{3}\frac{Y(l)}{H(l)}e^{j\pi\frac{kl}{2}}$$

III. OFDM USING COMB TYPE PILOT

OFDM channel estimation using comb type pilot

N=4 subcarrier

N=4 symbols loaded into subcarrier

L=2 Tap ISI channel

If L=2 pilot subcarrier then N-L=4-2=2 are information subcarrier.

$$y(k)=h(0)x(k)+h(1)x(k-1)+v(k)$$

$h(0),h(1)=L=2$ Taps

Channel coefficient $H(l)$ in frequency domain are given by zero padded FFT of channel taps $h(k)$ in time domain [7].

N-L=4-2=2 Zeros

$$H(l)=\frac{1}{N}\sum_{k=0}^{N-1}h(k)e^{-j2\pi\frac{kl}{N}}$$

$$H(l)=\frac{1}{N}\sum_{k=0}^{3}h(k)e^{-j\frac{\pi}{2}kl}$$

After solving

$$H(0)=h(0)+h(1)$$

$$H(1)=h(0)-j*h(1)$$

$$H(2)=h(0)-h(1)$$

$$H(3)=h(0)+j*h(1)$$

Only L=2 unknown, therefore L=2 pilot symbols are needed.

Let,

$X(0),X(2)$ are not pilot symbols, $l=2$ therefore 0,2 are regular information bearing symbols.

$X(l)$ is loaded on to the l[th] subcarrier.

At the transmitter

$$x(k)=\frac{1}{N}\sum_{l=0}^{N-1}X(l)e^{j2\pi\frac{kl}{N}} \qquad \text{IDFT}$$

For N =4

$$x(k)=\frac{1}{4}\sum_{l=0}^{3}X(l)e^{j\frac{\pi}{2}kl} \qquad \text{for } k^{th} \text{ sample}$$

$x(3)$ is the prefix and known as cyclic prefix CP.

As we know that $y(k)=h(0)x(k)+h(1)x(k-1)+v(k)$ therefore

This is circular shift of the channel taps or channel filter over sample $x(0),x(1),x(2),x(3)$

Output $y=h\otimes x+v$

$$Y(l)=H(l)\times X(l)+V(l)$$

In this example in CTP since pilots are transmitted only on $l=1,3$ subcarrier which are known as pilot subcarrier [8].

$$Y(1)=H(1)\times X(1)+V(1)$$

$$Y(3)=H(3)\times X(3)+V(3)$$

$$\begin{bmatrix}Y(1)\\Y(2)\end{bmatrix}=\begin{bmatrix}X(1),0\\0,X(3)\end{bmatrix}\begin{bmatrix}H(1)\\H(3)\end{bmatrix}+\begin{bmatrix}V(1)\\V(3)\end{bmatrix}$$

2X1 2X2 2X1 2X1

X=diagonal matrix of pilot symbols X(1) and X(3)

$$Y=XH+V$$

$$\min\|Y-XH\|$$

$$H=\left(X^H X\right)^{-1}X^H Y$$

$$H=X^{-1}\left(X^H\right)^{-1}X^H Y$$

$$H=X^{-1}Y$$

$$\begin{bmatrix}Hhet(1)\\Hhet(3)\end{bmatrix}=\begin{bmatrix}X(1),0\\0,X(3)\end{bmatrix}^{-1}\begin{bmatrix}Y(1)\\Y(3)\end{bmatrix}$$

$$\begin{bmatrix}Hhet(1)\\Hhet(3)\end{bmatrix}=\begin{bmatrix}1/X(1),0\\0,1/X(3)\end{bmatrix}\begin{bmatrix}Y(1)\\Y(3)\end{bmatrix}$$

$$Hhet(1)=\frac{Y(1)}{X(1)}$$

$$Hhet(3)=\frac{Y(3)}{X(3)}$$

Hhet(1) and Hhet(3) are known as estimate of channel coefficient H(1) and H(3) corresponds to the pilot subcarrier.

$$H(1)=h(0)-j*h(1)$$

$$H(3)=h(0)+j*h(1)$$

$[h(0), h(1), 0, 0]$ channel tap in time domain

$\Downarrow$ N=4 pt FFT/DFT

$[H(0), H(1), H(2), H(3)]$ Channel coefficient in frequency domain

$$\begin{bmatrix} Hhet(1) \\ Hhet(3) \end{bmatrix} = \begin{bmatrix} 1, -j \\ 1, +j \end{bmatrix} \begin{bmatrix} hhet(0) \\ hhet(1) \end{bmatrix}$$

$$\begin{bmatrix} hhet(0) \\ hhet(1) \end{bmatrix} = \begin{bmatrix} 1, -j \\ 1, +j \end{bmatrix}^{-1} \begin{bmatrix} Hhet(1) \\ Hhet(3) \end{bmatrix}$$

$$\begin{bmatrix} hhet(0) \\ hhet(1) \end{bmatrix} = \frac{1}{2j} \begin{bmatrix} +j, +j \\ -1, +j \end{bmatrix}^{-1} \begin{bmatrix} Hhet(1) \\ Hhet(3) \end{bmatrix}$$

hhet (0) and hhet(1) are estimate of time domain channel taps.

$$hhet(0) = \frac{1}{2j}\{ j * Hhet(1) + j * Hhet(3) \}$$

$$hhet(1) = \frac{1}{2j}\{ -Hhet(1) + Hhet(3) \}$$

And

$$Hhet(0) = hhet(0) + hhet(1)$$
$$Hhet(2) = hhet(0) - hhet(1)$$

are estimates of channel coefficients for information subcarrier.

Advantage: -

In comb type pilot symbols are loaded onto fewer subcarriers. Rest of the subcarrier are loaded with the information symbols. Therefore, information rate is high, this improve the bandwidth efficiency of the system [9].

CONCLUSION

OFDM is very efficient technique because it employs IFFT/FFT, fast efficient algorithm and no matrix inversion calculation is there. Orthogonal frequency division multiplexing is used in the following technologies, Modern wireless technology, 4 G Standards and LTE, Wi max, Wi Fi and 802.11 etc.

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