EOQ model based on Trapezoidal Fuzzy Number

*Vikramjeet Singh (1), Seema Saxena *(2), Rajesh Kumar Gupta(3),*

Pushpinder Singh(4), Nitin Kumar Mishra (5)

[2,3,5] *Department of Mathematics, School of Chemical Engineering and Physical Sciences, Lovely Professional University, Phagwara, Punjab, India.*

[1]*Department of Mathematics, IKGPTU Campus, Batala, Punjab(India) 143506.*

[4]*Department of civil engineering University of Windsor, Windsor, Canada.*

1.Abstract

The basic objective of modeling an inventory system is to find an optimal value to the cost of the system and the optimized ordering quantity. The present work deals with the uncertainty in the model which arises due to the crisp nature of the cost values involved. This unpredicted behavior of the various factors in the inventory problem is defined by the trapezoidal fuzzy number. The optimized value of the total cost along with the optimized value a replenishing quantity is determined for both the crisp and the fuzzy model. The results are compared for the insights.

Keywords: Trapezoidal Fuzzy Number, shortages, inventory, Graded Mean Representation Method.

2.Introduction

The introduction of fuzzy theory was done by Zadeh(1965) [13]. He together with his colleagues Bellman(1970) [6] developed a model on decision science in fuzzy form. After this the introduction of fuzzy in the research work started. Zimmerman(1983) [14] introduced the fuzzy theory in operation research. Later many researchers introduced this fuzzy concept in their models including or not including shortages. Considering triangular and trapezoidal fuzzy form of numbers [2] formulated an inventory model with shortages. With a trapezoidal form of fuzzy numbers and completely back locked shortages inventory model was presented by [3]. Involving credit delays supply chain model was framed by [4]. An optimum value to the ordering quantity and their respective cost is determined in this study. Analyzing the risk in the management fuzzy model was developed by [5]. An optimized value of the ordering quantity with their respective cost is determined by [10] with partial shortages. Taking weibull form of deterioration and minimize value to the parameters was determined by [11]. Along with trade credit [8] use the concept of inflation to find an optimized value to the parameters in inventory system. A comparative study of inventory system with crisp Network and fuzzy network was done by [7]. Other zestful research works done in the field of fuzzy theory along with credit and inflation in the inventory system is done by the [9], [1] and [12].

3.Presumptions

1. Demand is taken as a constant value.
2. There is zero lead time.
3. Completely backlogged shortages are considered.
4. Fuzzification of the cost values is done.
5. Trapezoidal fuzzy number is used for fuzzification.
6. Graded Mean Representation Method is used for defuzzification.

4.Symbols Used

1. $c_1 = cost\ of\ carrying\ inventory$
2. $c_2 = cost\ of\ inventory$ shortage
3. $c_3 = cost\ of\ ordering\ inventory$
4. $Q_x = ordering\ quantity$
5. $S_x = Shortage\ quantity$
6. $t_x = Cycle\ length$
7. $R_x = Net\ Demand$
8. $T_x = Total\ time\ for\ cycle$
9. Z = $Net\ Cost$

5. Mathematical Inventory Model in crisp form

The total cost in crisp environment is given by the sum of the ordering, carrying and the shortage cost.

Net Cost Value $Z(Q_x,S_x) = [c_1 t_1 \frac{(Q_x - S_x)}{2} + c_2 t_2 \frac{S_x}{2} + c_3] \frac{r_x}{Q_x}$

$$= \frac{c_1 (Q_x - S_x)^2 T}{2Q_x} + \frac{c_2 {S_x}^2 T}{2Q_x} + \frac{c_3 r_x}{Q_x}$$

Net value to the parameters in the crisp form of the inventory function is given by

Optimized value to the quantity ordered $Q_x^* = \sqrt{\frac{2(c_1+c_2)c_3 r_x}{c_1 c_2 T}}$

Optimized value to the quantity backlogged $S_x^* = \sqrt{\frac{2c_1 c_3 r_x}{c_2(c_1+c_2)T}}$

After obtaining the optimized values to the ordering and the shortage quantity, the minimized valuation of the net value of cost function is

$$Z(Q_x^*, S_x^*) = \sqrt{\frac{2c_1 c_2 c_3 r_x T}{(c_1+c_2)}}$$

6. Proposed Fuzzy Inventory Model

The cost parameters are fuzzified on the basis of the trapezoidal fuzzy number.

The cost values in fuzzy form are represented as

$\widetilde{c_1} = (c_{11}, c_{12}, c_{13}, c_{14})$, $\widetilde{c_2} = (c_{21}, c_{22}, c_{23}, c_{24})$, $\widetilde{c_3} = (c_{31}, c_{32}, c_{33}, c_{34})$

Net Cost Value $Z(Q_x,S_x) = \frac{\widetilde{c_1}(Q_x - S_x)^2 T}{2Q_x} + \frac{\widetilde{c_2} {S_x}^2 T}{2Q_x} + \frac{\widetilde{c_3}\, r_x}{Q_x}$

Optimized value to the quantity ordered $Q_x^* = \sqrt{\frac{2(\widetilde{c_1}+\widetilde{c_2})\widetilde{c_3} r_x}{\widetilde{c_1}\widetilde{c_2} T}}$

Optimized value to the quantity backlogged $S_x^* = \sqrt{\frac{2\widetilde{c_1}\widetilde{c_3} r_x}{\widetilde{c_2}(\widetilde{c_1}+\widetilde{c_2})T}}$

The defuzzification of the fuzzified value to the total cost is given by

$$Z_d(Q_x, S_x) = \frac{a + 2b + 2c + d}{6}$$

7. Numerical Example

Taking the numerical values as taken in [7] and making a comparison with the fuzzified model with the graded mean representation method.

$$T_x = 6, c_1 = 4, c_2 = 10, c_3 = 20, r_x = 1000$$

Table 1: Optimized values to parameters in Crisp Environment

Net Demand	Optimum order quantity Q_x^*	Optimum shortage quantity S_x^*	Optimum Cost value Z
1000	48.3046	13.8013	828.0787

The numerical values in the fuzzy inventory model with Trapezoidal Fuzzy Number are

$$T_x = 6, c_1 = (1,3,5,6), c_2 = (8,9,11,12), c_3 = (15,18,22,25), r_x = 1000$$

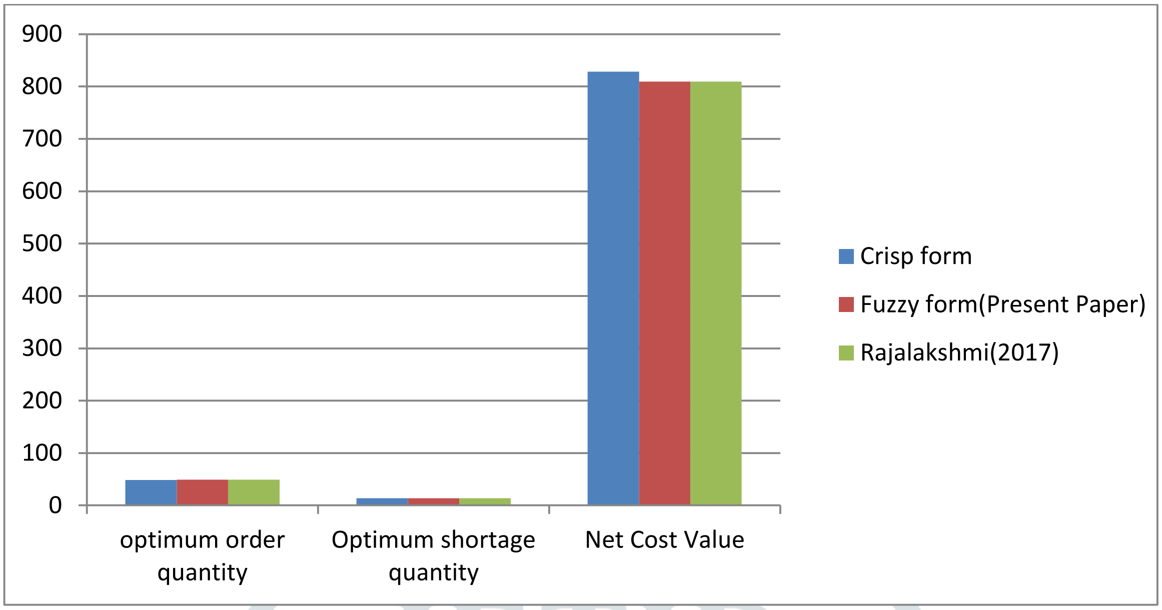
Using the equations for optimized order quantity, shortage quantity and the minimized cost value, we get

Table 2: Optimized values to parameters in Fuzzy Environment

Net Demand	Optimum fuzzy order quantity Q_x^*	Optimum fuzzy shortage quantity S_x^*	Optimum fuzzy Cost value Z
1000	49.4413	13.4839	809.0398

Having a comparison with the values obtained by [7] in the research work done using signed distance method for defuzzification, it is seen that very slight decrease in seen in the values obtained in the present model. This implies that graded mean representation gives more accuracy towards the values of the parameters.

8. Comparative graph



9.Conclusion

The paper presents a study in an inventory system with a crisp form and a fuzzy sense of developing an inventory problem. The method for defuzzification is through trapezoidal form with graded mean representation method used for defuzzification. The results show that the fuzzy model not only removes the uncertainty but also provides low values to the parameters. Among a comparison with the model framed by [7], it is clearly seen that the defuzzy methods signed distance method and graded mean representation method provides similar results.

10. References

[1] N.Kazemi, E.Ehnsani, M. (2010). An inventory with backorders with fuzzy parameters and decision variables. *International Journal of Approximate Reasoning*, 51(8):964-972.

[2] Nabendu Sen, S. M. (2015). A fuzzy inventory model with shortage using different fuzzy numbers. *American journal of Mathematics and Statistics*, 5(5):238-248.

[3] Pushpinder Singh, Nitin Kumar Mishra, Vikramjeet Singh, and Seema Saxena. An eoq

model of time quadratic and inventory dependent demand for deteriorated items with partially backlogged shortages under trade credit. In AIP Conference Proceedings, volume 1860, page 020037. AIP Publishing, 2017.

[4] Pushpinder Singh, Nitin Kumar Mishra, Manoj Kumar, Seema Saxena, and Vikramjeet Singh. Risk analysis of flood disaster based on similarity measures in picture fuzzy environment. Afrika Matematika, 29(7-8):1019-1038, 2018.

[5] R. E. Bellman, L. A. Z. (1970). Decision-making in a fuzzy environment. *Management Science*, 17(4):B141-B164.

[6] Rajalaxmi, R. (2017). A fuzzy inventory model with allowable shortage using different fuzzy numbers. *International Journal of Computational and Applied Mathematics*, 12(1).

[7] Vikramjeet Singh, Seema Saxena, Pushpinder Singh, and Nitin Kumar Mishra. Replenishment policy for an inventory model under ination. In AIP Conference Proceedings, volume 1860, page 020035. AIP Publishing, 2017.

[8] Vikramjeet Singh, Seema Saxena, Rajesh Kumar Gupta, Nitin Kumar Mishra, and Pushpinder Singh. A supply chain model with deteriorating items under inflation. In 2018 4th International Conference on Computing Sciences (ICCS), pages 119-125. IEEE, 2018.

[9] Zadeh, L. (1965). Fuzzy set. *Information and Control*, 8:338-353.

[10] Zimmerman, H. (1983). Using fuzzy sets in operational research. *European Journal of Operational Research 13*, 13:201-206.