MULTIFACILITY LOCATION PROBLEM: A Review

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Abstract: We recognize the multi-facility location problem in this paper. The aim here is to put several facilities in addition to a collection of existing facilities to reduce the total travel costs between the new facilities and the collection of existing facilities.

Keyword: *Facility location, multifacility, cost minimization*

1. Introduction

The ideal arrangement can frequently be practiced in an iterative mode by explaining various single office area issues, contingent upon the attributes of a specific rectangular situating issue. Besides, a two dimensional direct programming issue (p1) will make the rectangular MFLP progressively effective (Francis et al. 1992). Francis (1964) fathomed a specific MFLP case, with a straight length where the loads were equivalent. The position issue was separate by Cabot et al. in 1970 into two separate subproblems, each like a direct issue of programming that basically speaks to a double issue with negligible cost arrange stream. The x and y facilitates for the new offices give the double factors in every one of the perfect tableaux for both stream issues. The strategy for this issue was presented in Pritsker and Ghare (1970). The major commitment was to determine the conditions for an ideal arrangement and a calculation for ideal answers for deteriorated issues. Rao (1973) thought about the strategy for direct pursuit of the RMFLP top to bottom, yet demonstrated that it was basically an essential simplex-based straight programming arrangement and that ideal conditions were not sufficient within the sight of decadence. In unique cases, an adequate condition for optimality was talked about and the key issues identified with direct research were examined. She indicated that the issue of direct impediment could be unraveled by straight programming from Wesolowsky and Love (1971a, b) and Morris (1975). Juel and Love (1976) built up a point by point set of proper and sufficient ideal conditions. For the limited multi-office area issues with blended principles, Alternatively to straight programming, (Francis et al. 1992) proposed a direct methodology, with regularly perfect areas. The issue is transformed into a solitary issue in recognizing the office, to which we can apply middle measures, by disposing of the term that demonstrates the connection between the new offices in target activity. The main variable we pick is the first, and the subsequent variable is the second, and so on. This is proceeded until, by coordination plummet, we have acquired a similar vector that we had recently gotten by organize plunge. Allen (1995) built up a double based low-bound connection with the topic of the p-separation position of the multi-offices. The problem of a multi-facility position with Euclidean distances can be solved by expanding the well known Weiszfeld

algorithm. Like the Weiszfeld method, though, the iterations generated by this technique converge only under any specific hypothesis. Several authors have used the generalized Weiszfeld algorithm to solve the problem with the position of multi-facility with distances where ideal criteria for problem(P) have been defined for any norm.

2.Multifacility Location Problem

Let, m be the number of existing facility which are situated at known separate points $P_1, P_2, P_3 \ldots, P_m$ in the X-Y plane, and

N be the amount of new facilities to be installed in X1, X2, X3, ..., Xn, X-Y.d(Xj,Pi) be the distance between the new deployment site j and existing deployment i.

$d(X_j\ X_k)$ be the distance between the most recent k and j setup. w_{ji} is the distance from the current facility j and k.v_{jk} be annual unit gap costs for new installations k to j.

if the new facilities are located at $X_1, X_2, X_3, \ldots, X_n$, then the total transportation cost is as given below.

$$f(X_1, X_2, \ldots, X_n) = \sum_{1 \le j < k \le n} v_{jk}\ d(X_j, X_k) + \sum_{j=1}^{n} \sum_{i=1}^{m} w_{ji}\, d(X_j, P_i) \qquad (1)$$

The aim of the multifacility location problem is to pick $X_1^*, X_2^* \ldots X_n^*$ of new facilities in order to reduce the annual total cost.

The goal function is two cost components: (i) total cost due to transportation between the new facilities, (ii) Complete transport costs for new installations and existing installations. If all the terms of v_{jk} are zero, then the first component of the objective function will be the zero and modified objective function is as stated in the following.

$$f(X_1, X_2, \ldots, X_n) = \sum_{j=1}^{n} \sum_{i=1}^{m} w_{ji}\, d(X_j, P_i) \qquad (2)$$

i.e

$$f(X_j) = \sum_{j=1}^{n} f_j(X_j)$$

where

$$f_j(X_j) = \sum_{i=1}^{m} w_{ji}\, d(X_j, P_i)$$

It alludes to the location of a solitary new office that doesn't affect the location of any new office. Consequently, the location of the new offices can be seen as a solitary location issue. Here, information for

existing offices will be joined with each new office to survey the location of each new office. Decide the places of n new offices by rehashing the single location process.

3.Model for multi-facility location problem

Let us assume the rectilinear distance between all the facilities.

$X_j = (x_j, y_i)$ is the new facility site.$P_i = (a_i, b_i)$ is the location of the ith existing facility.

$d(X_j, X_k)$ = distance from new facilities j to k. $d(X_j, P_i)$ = Distance from new facility j to current facility i.

Therefore,

$$d(X_j, X_k) = |x_j - x_k| + |y_j - y_k| \quad (3)$$

$$d(X_j, P_i) = |x_j - a_i| + |y_j - b_i| \quad (4)$$

By substituting Eqs. (3) and (4) in Eq. (1), we get the following.

$$f(X_1, X_2, \ldots X_n) = f_1(x_1, x_2, \ldots x_n) + f_2(y_1, y_2, \ldots y_n) \quad (5)$$

where,

$$f_1(x_1, x_2, \ldots, x_n) = \sum_{1 \le j < k \le n} v_{jk} \left|x_j - x_k\right| + \sum_{j=1}^{n} \sum_{i=1}^{m} w_{ji} \left|x_j - a_i\right| \quad (6)$$

$$f_2(y_1, y_2, \ldots, y_n) = \sum_{1 \le j < k \le n} v_{jk} \left|y_j - y_k\right| + \sum_{j=1}^{n} \sum_{i=1}^{m} w_{ji} \left|y_j - a_i\right| \quad (7)$$

The functions f_1 and f_2= represent the total cost of transportation in *X* direction and *Y* direction, respectively.

The objective is to minimize $f(X_1, X_2, \ldots, X_n)$ therefore,

$$\min f(X_1, X_2, \ldots, X_n) = \min \{ f_1(x_1, x_2, \ldots, x_n) + f_2(y_1, y_2, \ldots, y_n) \}$$
$$\min f_1(x_1, x_2, \ldots, x_n) + \min f_2(y_1, y_2, \ldots, y_n)$$

So, if we can establish a technique to find optimal X-coordinate of the new facilities, the same technique can be use to find optimal Y-coordinates of the new facilities.

Now, let us develop a linear programming model to determine the optimal X-coordinates of the new facilities.

Consider the function, minimize $f_1(x_1, x_2, \ldots, x_n)$

$$Minimize\ f_1(x_1, x_2, \ldots, x_n) = min\{\sum_{1\le j<k\le n} v_{jk}\,|x_j - x_k| + \sum_{j=1}^{n}\sum_{i=1}^{m} w_{ji}\,|x_j - a_i|\}$$

as given Eq. (6).

In the above function, both the components are in absolute form which are known to be non-linear. So, these are to be transfer ed into linear form.

4. Method of Transformation

Assume that a, b, p and q are given. If

(i) a - b – p + q = 0.

(ii) p ≥ 0,

(iii) q ≥ 0,

(iv) pq = 0

Then

$$|a - b| = p+q$$

one can verify the justification of the above transformation with suitable data (Example: a = 8, b =14, p = 0 and q = 6).

5.Model to Determine X-coordinates of New facilities

Based on the above transformation technique. (6) is transformed as follow: (Eq. (6) is reproduced here for quick reference).

$$f_1(x_1, x_2, \ldots, x_n) = \sum_{1\le j<k\le n} v_{jk}\,|x_j - x_k| + \sum_{j=1}^{n}\sum_{i=1}^{m} w_{ji}\,|x_j - a_i|$$

Use x_j, x_k, p_{jk} and q_{jk} as data set for transforming the first components of the Eq. (6) Similarly, use x_j, a_i, r_{ji} and s_{ji} as data set for transforming the second component of Eq.(6).

$$Minimize \sum_{1 \le j < k \le n} v_{ij}\left(p_{jk} - q_{jk}\right) + \sum_{j=1}^{n}\sum_{i=1}^{m} w_{ji}\ (r_{ji} + s_{ji})$$

Subject to

$x_j - x_k - p_{jk} + q_{jk} = 0,\ 1 \le j \le k \le n$

$x_j - a_k - r_{ji} + s_{ji} = 0,\ I = 1, 2, .., m$

$j = 1, 2, .., n$

$p_{jk}, q_{jk} \ge 0,\ 1 \le j \le k \le n$

$r_{ji}, s_{ji} \ge 0,\ i = 1, 2, \ldots, m$

$j = 1, 2, .., n$

x_j is unrestricted in sign, $j = 1, 2, .., n$

$p_{jk}, q_{jk} = 0,\ 1 \le j \le k \le n$

$r_{ji}, s_{ji} \ge 0,\ i = 1, 2, \ldots, m$

$j = 1, 2, .., n$

The unrestricted nature can be avoided by the following assumption.

If we have any coordinate of existing facilities in the second or third or fourth quardents. then we have suitably altered the coordinates such that the points $P_1, P_2, P_3, \ldots, P_m$ lie in the first quadrant. Then we have the constant $x_j \ge 0$ for all j instead of unrestricted nature in sign for x_js.

One can notice that except the last two sets of multiplicative constraints. The model is in linear form.

For any possible simple solution in the above model. If pji is the simple solution that can be achieved. Qjk would not be in the basic solution and vice versa, too. If rji in the simple solutions feasible is not in the simple solution conceivable and vice versa. The pjk variable coefficients are -1 times the qjk variable coefficients. Thus the two variables depend on each other. Similarly, rji and sji rely on linerly. But there are linear (variable) vectors on the basis. Thus, both pjk and qjk are not available at the same time. Similarly, rji and sji won't be avialable at the same time.

Based on the above discussion, the last two multiplicative constraint sets can be deleted from the model.

So, the final model to determine the X-coordinates of the new facilities is given below:

$$Minimize \sum_{1 \le j < k \le n} v_{ij}\left(p_{jk} - q_{jk}\right) + \sum_{j=1}^{n}\sum_{i=1}^{m} w_{ji}\ (r_{ji} + s_{ji})$$

$x_j - x_k - p_{jk} + q_{jk} = 0,\ 1 \le j \le k \le n$

$x_j - a_i - r_{ji} + s_{ji} = 0,\ i = 1, 2, .., m$

$j = 1, 2, .., n$

$p_{jk}, q_{jk} \ge 0,\ 1 \le j \le k \le n$

$r_{ji}, s_{ji} \ge 0,\ i = 1, 2, \ldots, m$

$j = 1, 2, .., n$

$x_j \ge 0^*,\ j = 1, 2, \ldots, n.$

*(if all P_1, P_2, P_3,…., P_m are in the first quadrant; even if they are not in the first quadrant, one can suitably modify them such that all are in the first quadrant.)

6. Model to determine Y-coordinate

Consider the function which is give in eq. (7)

$$f_2(y_1, y_2, \ldots, y_n) = \sum_{1 \le j < k \le n} v_{jk} \left| y_j - y_k \right| + \sum_{j=1}^{n} \sum_{i=1}^{m} w_{ji} \left| y_j - a_i \right| \qquad (7)$$

Use y_j, y_k, p_{jk} and q_{jk} as data set for transforming the first term of the function 7 into linear form.

Similarly, assume y_j, b_i, r_{ji} and s_{ji} as data set for transforming the second term of the Eq. (7) into linear form. Based on these guidelines, a model to determine the Y-coordinates of the new facilities is given below.

$$Minimize \sum_{1 \le j < k \le n} v_{ij} \left(p_{jk} - q_{jk} \right) + \sum_{j=1}^{n} \sum_{i=1}^{m} w_{ji} \left(r_{ji} + s_{ji} \right)$$

Subject to

$y_j - y_k - p_{jk} + q_{jk} = 0,\ 1 \le j \le k \le n$

$y_j - b_k - r_{ji} + s_{ji} = 0,\ I = 1, 2, .., m$

$j = 1, 2, .., n$

$p_{jk}, q_{jk} \ge 0,\ 1 \le j \le k \le n$

$r_{ji}, s_{ji} \ge 0,\ i = 1, 2, \ldots, m$

$j = 1, 2, .., n$

y_j is unrestricted in sign, $j = 1, 2, \ldots, n$

$p_{jk}, q_{jk} = 0,\ 1 \le j \le k \le n$

$r_{ji}, s_{ji} \ge 0,\ i = 1, 2, \ldots, m$

$j = 1, 2, .., n$

The unrestricted nature can be avoided by the following assumption.

If we have any coordinate of existing facilities in the second or third or fourth quadrants, then we have suitably alter the coordinates such that the points $P_1, P_2, P_3, \ldots, P_m$ lie in the first quadrant. Then we have the constant $y_j \geq 0$ for all j instead of unrestricted nature in sign for y_js.

One can notice that except the last two sets of multiplicative constraints. The model is in linear form.

As explained in the earlier model, the last two sets of multiplicative constraints can be ignored. So, the final model to determine the Y-coordinates of the new facilities is given below.

$$Minimize \sum_{1 \leq j < k \leq n} v_{ij} \left(p_{jk} - q_{jk}\right) + \sum_{j=1}^{n} \sum_{i=1}^{m} w_{ji} \left(r_{ji} + s_{ji}\right)$$

Subject to

$$y_j - y_k - p_{jk} + q_{jk} = 0, \; 1 \leq j \leq k \leq n$$

$$y_j - b_k - r_{ji} + s_{ji} = 0, \; I = 1, 2, .., m$$

$$j = 1, 2, .., n$$

$$p_{jk}, q_{jk} \geq 0, \; 1 \leq j \leq k \leq n$$

$$r_{ji}, s_{ji} \geq 0, \; i = 1, 2, \ldots, m$$

$$j = 1, 2, .., n$$

$$y_j \geq 0, \; j = 1, 2, \ldots, n.$$

Conclusion

A significant number of the techniques intended for tackling the multi-facility location problem have been ineffectively organized, imperfect or aimless. In this paper, we audit a technique for tackling the multi-facility location problem. This strategy can without much of a stretch be reached out to oblige problems including thing developments that are other than Computational experience shows that this technique beats methods as of now being used. Moreover, the proposed strategy targets position problem structure. Apart from the multi-facility position issue discussed in our article, there are many combinatorial optimization problems that can be listed for median and cubical polyhedrons. One of them is discovering the shortest rectilinear route in a specified axis-parallel polyhedron. This problem is analyzed extensively in rectangular polygons. In multifaceted spaces, with similar outcomes, shortest path problems become significantly tougher. We expect that cubic and median polyhedrons are the only polyhedron type to solve this question effectively.

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