Scalar Field In Cosmology

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Abstract

Cosmic real scalar field play a vital role in cosmology in the form of governing dynamics of the universe. Several class of scalar field proposed as in different context in physics. Among the all of class of scalar field the simplest one is quintessence. One can write the Generalized Lagrangian of few particular class of scalar field as $\mathcal{L} = Y\left(\frac{Y}{M^4}\right)^{\alpha-1} - V(\phi)$.

In this short review we discuss the Lagrangian structure of two major class of scalar field namely quintessence and tachyonic scalar field.

Introduction

Dark energy appears in cosmology introduced as in the theoretical explanation of the observed accelerated expansion of the universe. This breakthrough observation supported by the Supernova Ia[1, 2] in the year 1998. Apart from Supernova Ia some other potential cosmological observations [3, 4, 5, 6, 7] also support observation of cosmic acceleration. The nature of dark energy is repulsive gravity due to its negative pressure provide it an exotic characteristic. Although Einstein introduce first time cosmological constant to make physically relevant solution of his gravity equation. The cosmological constant introduced by Einstein to make static universe in his cosmological constant model. The cosmological constant has been consider as the first candidate of dark energy.

Scalar field in cosmology

In the standard cosmology, as ΛCDM model suffer flatness and horizon problem. The cosmic inflation introduced[8] to overcome these two issues of standard cosmology. Cosmic inflation demands constant energy density of the universe. The constant energy density of the universe leads to exponential expansion of the cosmic spacetime termed as cosmic inflation. It has been observed that the rolling homogeneous real scalar field [9] also give cosmic inflation. There are number of class of scalar field proposed by different authors.

One can write the general form of Lagrangian of scalar field as follows

$$\mathcal{L} = Y\left(\frac{Y}{M^4}\right)^{\alpha-1} - V(\phi) \qquad (1)$$

where M is constant and Y = $\frac{\dot{\phi}^2}{2}$ for spatially homogeneous scalar field and α = 1 leads to the Lagrangian (1) for quintessence(simplest form of class of scalar field).

The Action of this generalized Lagrangian can be written as

$$\mathcal{A} = \int d^4x\sqrt{-g}\left(\frac{R}{16\pi G} - Y\left(\frac{Y}{M^4}\right)^{\alpha-1} + V(\phi)\right). \qquad (2)$$

In particular the another class of scalar field appears in string theory [10, 11, 12, 13] is called tachyonic scalar field. The Lagrangian of this tachyonic scalar field can be written as

$$\mathcal{L} = -V(\phi)\sqrt{1 - \partial^i\phi\partial_i\phi}. \qquad (3)$$

The energy momentum tensor for the generalized Lagrangian can be written as

$$T^{ik} = \frac{\partial\left(Y\left(\frac{Y}{M^4}\right)^{\alpha-1} - V(\phi)\right)}{\partial(\partial_i\phi)}\partial^k\phi - g^{ik}Y\left(\frac{Y}{M^4}\right)^{\alpha-1} - V(\phi). \qquad (4)$$

This energy momentum tensor gives pressure and energy density of scalar field as follows

$$\rho_\phi = \left(\frac{\partial\mathcal{L}}{\partial Y}\right)(2Y) - \mathcal{L} \qquad (5)$$

$$P_\phi = \mathcal{L}. \qquad (6)$$

By using Lagrangian (3) in (5) and (6) we have following expression for pressure and energy density of spatially homogeneous tachyonic scalar field

$$P = -V(\phi_{tach})\sqrt{1-\dot{\phi}_{tach}^2} \qquad (7)$$

$$\rho = \frac{V(\phi_{tach})}{\sqrt{1-\dot{\phi}_{tach}^2}}. \qquad (8)$$

Dynamical role of scalar field

Scalar field, specially quintessence introduced in cosmology in the context as the solution of cosmological constant problem i.e., the solution of this problem is, why the observed value of cosmological constant at present epoch is very small? The dynamics of any scalar field can be understood via their equation of motion. The stationary action principle in the case of non-minimally coupled scalar field (quintessence) in the flat FRLW model provide us the equation of motion (the Klein-Gordon equation) of scalar field for spatially homogeneous approximation as follows

$$\ddot{\phi}_{quint} + 3H\dot{\phi}_{quint} + V'(\phi) = 0. \qquad (9)$$

The same condition of the stationary action principle also give the following equation of motion of tachyonic scalar field for spatially homogeneous approximation as follows

$$\frac{\ddot{\phi}_{tach}}{\dot{\phi}_{tach}} + \frac{(1-\dot{\phi}_{tach}^2)V'(\phi_{tach})}{\dot{\phi}_{tach}V(\phi_{tach})} + 3H(1-\dot{\phi}_{tach}^2) = 0. \qquad (10)$$

Further the conservation principle of energy momentum tensor leads to the conservation of energy equation in the following form

$$\dot{\rho_y} + 3H(1+w_y) = o, \qquad (11)$$

where "*wy*" is equation of state i.e the ratio of pressure and energy density of corresponding scalar field(either quintessence or tachyonic). This conservation of energy equation (11) provide us the scaling solution of energy of scalar field i.e the functional form energy density with scale factor of the universe. The scaling solution of scalar field depict the dynamical behaviour of scalar field in the universe under non-minimal coupling approximation.

Conclusion

In this short review we discuss the generalized form of Lagrangian of real cosmic scalar field. This scalar field minimally coupled to gravity in the action. The variation of action under stationary action approximation gives the equation of motion of scalar field. The energy momentum tensor provide us the expression for the energy density and pressure of scalar field. For simplicity we assume the scalar field is spatially homogeneous. The dynamical behaviour of non-minimally coupled scalar field under spatially homogeneous approximation can be given by their equation of motion and conservation of energy equation.

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