An introduction to physical properties of liquid helium

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Abstracts

Helium is an element of many superlatives. It is, on the one hand, the smallest and most inert of all atoms and, on the other, the second most abundant element (after hydrogen) in the universe. The bosonic nature, small mass and weak intermolecular forces explain why it is the only substance that remains liquid down to zero Kelvin and the only naturally occurring superfluid. It has been a subject of extensive theoretical and experimental studies for the last six decades for several reasons including its superfluid behavior and it appears that it will continue to fascinate the researchers for many more decades to come. However, In view of our recent work we have studied the system form a different perspective which has resulted a better microscopic understanding of the system.

Keywords: Helium, Boson, fermion, superfluid, superconductivity

Introduction

There are two types of particles in the universe: Bosons and Fermions. As a general rule, matter is made of fermions, e.g., protons, neutrons and electrons are fermions. Bosons are particles (quanta) associated with interactions, e.g., photons, are Bosons. The key difference between these two types of particles concerns the behavior of identical particles-particles of the same type. Fermions obey the Pauli Exclusion Principle, i.e., no more than one Fermion can occupy a single quantum state. However, there is no such restriction on Bosons and any number of Bosons can occupy the same quantum size. Examples of Bosons include ^{4}He in the ground state, pion, Higgs Boson; ρ meson, photon; ^{16}O in the ground state, graviton etc; and those of Fermions are ^{3}He in ground state, proton, neutron, quark, electron, neutrino; ^{5}He in ground state etc. Liquid Helium-4 is a Boson while liquid Helium-3 is a Fermion. These simple Boson and Fermion, respectively, has been baffling concepts for Physics since their discovery. Liquid Helium has been studied intensively for more than three quarters of a century: indeed one may advance the claim that it is the most studied pure substance in the history of science [1-4].

Background

Helium is found to have two common isotopes ^{4}He and ^{3}He whose nuclei, respectively, contain (two protons + two neutrons) and (two protron +one neutron). It was first liquefied by Kamerlingh Onnes in 1908. Since then its various properties have been investigated extensively. The wealth of experimental and theoretical results is reviewed in various books and review articles [5 (a), 5 (b), 5 (c), 6]. Liquid Helium undergoes a liquid to liquid phase transition when cooled and its high temperature phase behaves like a normal liquid (non zero viscosity) while the low temperature phase assumes several unique properties including superfluidity (flow without resistance).

Current Status

Owing to its unusual behavior, many theories of Liquid Helium have been profound and developed to explain its different properties [6]. ^{3}He was discovered and liquefied much after its much abundant isotope ^{4}He, till then much work had been done on ^{4}He, so many of the properties for Fermi liquid Helium was also studied using same methods and principals with basic modifications. The “two fluid theory" of Tisza was a phenomenological theory developed to explain some of the important experimental results. According to the two fluid model of Tisza Helium can be considered as a mixture of two fluids "superfluid" and "normal". Liquid Helium is purely superfluid i.e. the atoms are in the lowest energy state at zero temperature. With the increase of temperatures excited molecules are formed and these constitute the normal component. Tisza considered the motion of Helium as some kind of turbulent motion because the velocity pressure dependence is more similar to turbulent than to laminar motion. The most important success of Tisza theory was the prediction of second sound in Helium. Based on London's idea Landau also developed another two fluid model. Landau's two fluid theories also assume existence of two independent motions in Helium. These two motions occur without momentum transfer from one to another and treated the superfluid flow by expressing the macroscopic hydrodynamical variables like density and velocity, as quantum mechanical operators. He further showed that the equations of motion for these operators implied the continuity equation and Euler's equation for an ideal fluid. In this two fluid model there are two kinds of excited molecules-phonons or quanta of longitudinal compressible waves and rotons. These molecules needed a minimum energy Δ to excite them. Although Landau's two fluid model can explain a very large fraction of the exotic behavior of Helium without invoking the notion of Bose Einstein Condensate (BEC), it has several shortcomings, inconsistencies and quantitative disagreements with certain experimental results which were pointed out by Putterman [7]. Next significant contribution to the development of microscopic theory of Helium came in 1947 from Bogoliubov [8]. Analysing a system of weakly interacting Bosons he tried to overcome the basic objection to the London's idea of associating BEC to the unique behavior of Helium-II. He considered liquid ^{4}He as a system of degenerate non-perfect Bose gas and used the method of second quantization to show that in the presence of small interactions between the atoms, the low excited state of the gas can be described as a perfect Bose gas of certain "quasi-particles" (elementary excitation). In this theory the fraction of atoms which condense into the p=0 state does not assume value 1 even at absolute zero. Furthermore these atoms can move without any friction with respect to the elementary excitation. This theory can reproduce Landau's "phonon-roton" spectrum of low Q (low wave number). Bogoliubov's theory [8] was based on the assumption that liquid ^{4}He in a system of weakly interacting Bose gas but the inter-atomic interactions between Helium atom to a good approximation are strong. In 1953 Feynman using the quantum path integral method showed that liquid Helium should exhibit a transition analogous to the transition of ideal Bose gas regardless of the strong inter-atomic interaction. Different treatments were developed by different author's viz. De Boer, Chester, Miller et. al., Lee and Mohling etc. In Chester theory of liquid ^{4}He, free energy was expanded in powers of a coupling constant g. The first term of the series gave the free energy of the London theory and lead to all the usual properties. According to this theory the transition at the lambda temperature is of third order but the second term in the expansion of free energy raises the transition to one of second order. Different pictures of roton are given by these authors. While de Boer considers roton as short wavelength longitudinal elastic mode, Chester proposed roton as quasi-particles with modified mass associated with the motion of ^{4}He atom. Lee and Mohling after examining the experimental data of the total cross section for the inelastic scattering of cold neutrons in Helium II concluded that the projection of the angular momentum on the direction of the momentum, p of the roton is zero. This implies that the roton excitation has zero angular momentum and it loses rotational character. The idea of vortex line and quantization of superfluid circulation which was first introduced by Onsager was further developed by Feynman.

A generalized mathematical description of BEC was developed by Penrose and Onsager. Based on the first principle, Penrose and Onsager indicated that liquid Helium-II in equilibrium shows BEC. The hard core repulsive part of the He-He interaction and the macroscopic occupation of a particular state pose serious difficulties in developing a viable microscopic theory of Helium-II. Efforts were made by Beliaev, Hugenholtz and Pines to overcome these difficulties. It was followed by the classic work of Bogoliubov on the dilute weakly interacting gas. However, since He atoms experiences strong repulsive which forbids any two Helium atoms from occupying same point in real space, Bogoliubov theory is found to be inconsistent with the known experimental results. Brueckner and Sawada tried to overcome these difficulties by considering the system to be dilute one. The method was used by Goble and Trainer to show that the condensate fraction varies from 0.37 to 0.79 as the hard core radius altered from 3.0 to 1.0^{o}A. Similar calculations were carried out by Bycking but he allowed for all partial waves of Lennard-Jonnes potentials with a hard core radius of 2.6^{o}A. The numerical value obtained for excitation curve agrees qualitatively with those of the experimental results. Liu et. al. showed that if two-body pseudo-potential is used then one can get a qualitative correct excitation spectrum. Another alternative treatment to study strongly interacting Bosons such as LHe-4 is the Correlated Basis Function (CBF) introduced by Feenberg and collaborators. Various Monte Carlo methods such as Variational Monte Carlo used by Masserini et. al., diffusion Monte Carlo used by Caperly et. al. and Moroni et. al., Path Integral Monte Carlo used by Caperley and Pollock and Green-function Monte-Carlo used by Whitlock and Panoff to calculate the properties of liquid Helium [1, 3, 4, 6-8]. Some exclusive studies were done on Helium-3 [2, 5 (a), 5(b), 5(c), 6, 7]. Fairbank and co-workers in 1954 performed an experiment which showed unusual magnetic properties in ^{3}He at 0.1°K. At this temperature and low pressure, susceptibility of ^{3}He was found to be nearly independent of temperature, a result consistent with the behavior of ^{3}He as a Fermi liquid.

According to the Landau's theory of Fermi liquid ^{3}He was thought of a gas of weakly interacting quasi particles moving in a self consistent field due to other particles. The form of distribution function of quasi particles and the Fermi function of a gas of free particles are the same, but the energy is a function of the distribution function. The theory is not expected to be valid above some temperature because of excessive collision rates but the theory is unable to predict this temperature region. Electrons in metals, which have a Fermi temperature of ~ 10^4–10^5 K, may sometimes, at temperatures ≤ 20 K, enter the so-called superconducting state, in which they can flow without apparent resistance; this is just the analog, for a charged system, of the superfluidity of liquid ^{4}He. Since for liquid ^{3}He the Fermi temperature is only a few Kelvin, it would have been reasonable to speculate that the atoms might undergo a similar transition at temperatures of the order of mK, since the atoms are electronically neutral, the result would be not superconductivity but rather superfluidity, as in ^{4}He. However, in the absence of a microscopic theory of superconductivity no quantitative approach to this question suggested itself. This phenomenon was well studied by Bardeen, Cooper and Schrieffer (1957) and also by Bogoliubov (1958) who included the concept of Cooper pairs Cooper pairing in superconductors and superfluid ^{3}He are very different entities, in the former case pairs are formed by point like, structureless electrons and are spherically symmetric, while in the other case cooper pairs are made up of actual atoms and have internal structure themselves.

The three superfluid phases of ^{3}He, phase A, B and A1 though have different properties, but the cooper pairs in all three phases are in a state with parallel spin (s = 1) and relative orbital angular momentum l= 1 the pairing being known as "spin triplet p- wave pairing". In 1978, studies showed that ^{3}He under a certain temperature $T \leq T_A$ (T_A ~ 2.6 mK being the second order phase transition temperature), the material exhibits spontaneous magnetization M in the absence of an external magnetic field [9]. So, what we can conclude though numerous theoretic studies have been done so far is that there is still not one concrete microscopic theoretical approach to explain a number of experimental finding at a quantitative level. Also the present theories are mainly based on different presumptions like existence of p = 0 state at absolute zero which itself has serious doubt [10].

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