On Equitable Power Domination Number of Some Graphs

A. Parthiban[1], G. Samdanielthompson[2], K. Sathish Kumar[3]

[1]Department of Mathematics, School of Chemical Engineering and Physical Sciences, Lovely Professional University, Phagwara-144 411, Punjab, India

[2]Department of Mathematics, Hindustan College of Arts and Science, Padur, Chennai-603 103, Tamil Nadu, India

[3]Department of Mathematics, Madras Christian College (Autonomous), Chenna-600 059, Tamil Nadu, India

Corresponding author: parthiban.23447@lpu.co.in

Abstract

Let G (V, E) be graph. A set $S \subseteq V$ is said to be a power dominating set (PDS) if every vertex $u \in V - S$ is observed by certain vertices in S by the following rules: (i) if a vertex v in G is in PDS, then it dominates itself and all the adjacent vertices of v and (ii) if an observed vertex v in G has $k > 1$ adjacent vertices and if $k - 1$ of these vertices are already observed, then the remaining one non-observed vertex is also observed by v in G. A power dominating set $S \subseteq V$ in $G\,(V, E)$ is said to be an equitable power dominating set (EPDS), if for every vertex $v \in V - S$ there exists an adjacent vertex $u \in S$ such that the difference between the degree of u and degree of v is less than or equal to 1, i.e., $|d(u) - d(v)| \leq 1$. The minimum cardinality of an equitable power dominating set of G is called the equitable power domination number of G, denoted by $\gamma_{epd}(G)$. "An edge is said to be subdivided if the edge xy is replaced by the path: xwy, where w is the new vertex. A graph obtained by subdividing each edge of a graph G is called subdivision of G, and is denoted by $S(G)$". In this paper we establish the equitable power domination number of subdivision of graphs. We also obtain the equitable power domination number of the generalized Petersen graphs and balanced binary tree.

Keywords

Power dominating set, Power domination number, Equitable power dominating set, Equitable power domination number, Generalized Petersen graphs, Balanced binary tree, and Subdivision graph.

1. Introduction

Only simple, finite, undirected, and connected graphs are considered in this paper. A dominating set of a graph $G = (V, E)$ is a set S of vertices such that every vertex v in $V - S$ has at least one neighbor in S. The minimum cardinality of a dominating set of G is called the domination number of G, denoted by $\gamma_d(G)$ [8]. For a few other variants of dominating set refer to [9, 10].

A power dominating set $S \subseteq V$ in $G\,(V, E)$ is said to be an equitable power dominating set, if for every vertex $v \in V - S$ there exists an adjacent vertex $u \in S$ such that the difference between the degree of u

and degree of v is less than or equal to 1, that is $|d(u) - d(v)| \leq 1$. The "minimum cardinality" of an equitable power dominating set of G is called the equitable power domination number of G, denoted by $\gamma_{\text{epd}}(G)$ [2]. For more results one can refer to [3, 4]. In this paper, we obtain the equitable power domination number of the generalized Petersen graphs and balanced binary tree.

2. Main Results

For the sake of convenience, by EPDS and EPDN we mean an equitable power dominating set and the equitable power domination number, respectively.

2.1 EPDN of the Generalized Petersen Graphs and Balanced Binary Tree

First we recall the definition of the generalized Petersen graph for the sake of completeness.

Definition 1 [1]

"The generalized Petersen graph $GP(n,k)$ is defined to be a graph with $V\left(GP(n,k)\right) = \{a_i, b_i: 0 \leq i \leq n-1\}$ and $E(GP(n,k)) = \{a_i a_{i+1}, a_i b_i, b_i b_{i+k}: 0 \leq i \leq n-1\}$, where the subscripts are expressed as integers modulo n $(n \geq 5)$ and k $(k \geq 1)$."

Note:

1. $GP(n,k)$ is isomorphic to $GP(n, n-k)$.
2. Without restriction of generality, one may consider the generalized Petersen graph $GP(n,k)$ with $k \leq \lceil (n-1)/2 \rceil$.

Theorem 2

Let $GP(n,k)$ be the generalized Petersen graph.

Then $\gamma_{\text{epd}}(GP(n,k)) = \begin{cases} 2, & \text{for } k = 1, 2 \text{ and } m \geq 4 \\ 3, & \text{for m} \geq 10 \text{ and k} \geq 3. \end{cases}$

Proof.

Let $GP(n,k)$ be the given GPG with $V = \{a_1, a_2, \ldots, a_n, b_1, b_2, \ldots, b_n\}$ and edge set $E(GP(n,k)) = \{a_i a_{i+1}, a_i b_i, b_i b_{i+k}: 0 \leq i \leq n-1\}$. To obtain the equitable power domination number of $GP(n,k)$, we consider the following two cases:

Case 1: For $k = 1, 2$ and $m \geq 4$.

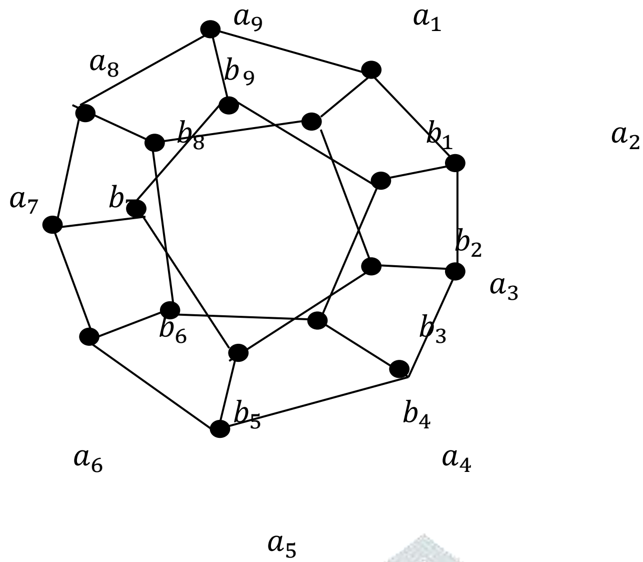

Fig. 1: $GP(9,2)$

Without loss of generality, we choose any one of b_i's, $1 \leq i \leq n$ to be in S, say b_1. Note that b_1 equitably power dominates b_3, a_1, and b_{n-1}. Now the observed vertices b_3, a_1 and b_{n-1} have more than one non-observed vertices and so fail to observe their neighboring vertices which leads to choose another vertex to be in EPDS. Then one can choose either b_2 or b_n to be in S for the sake of minimum cardinality. Now it is easy to see that all the remaining non-observed vertices are observed by their respective neighbors and therefore $|S| = 2$.

Case 2: For $m \geq 10$ and $k \geq 3$
Construction of EPDS is similar to Case 1.

2.2 Equitable Power Domination Number of the Balanced Binary Tree

We recall a few relevant definitions needed for this section for the sake of convenience.

Definition 3 [5]

"A graph without cycles is called an acyclic graph and a connected acyclic graph is called as a tree."

Definition 4 [5]

A binary tree is a tree in which each vertex has at most 2 pendant vertices.

Definition 5 [5]

A balanced binary tree is a binary tree in which the left and right sub trees of every vertex differ in height by no more than one.

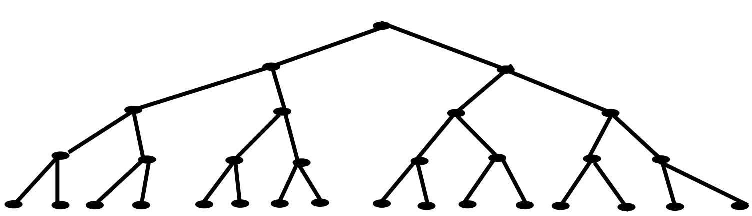

Fig.2: Balanced Binary Tree

Theorem 6

Let $B(1,k)$ be a balanced binary tree. Then $\gamma_{epd}\big(B(1,k)\big) = \sum_{n=0}^{n=k} 2^n - 2^{n-1}$.

Proof.

Let $B(1,k)$ be the given balanced binary tree on k levels with vertex set $V = \{a_0, a_1, a_2, a_1', a_2', a_3', a_4', a_1'', a_2'', a_3'', a_4'', a_5'', a_6'', a_7'', a_8'', \ldots, a_1^n, a_2^n, a_3^n, a_4^n, a_5^n, a_6^n, a_7^n, a_8^n, \ldots, a_n^n\}$

where $a_1^n, a_2^n, a_3^n, a_4^n, a_5^n, a_6^n, a_7^n, a_8^n, \ldots, a_n^n$ are the pendant vertices. To obtain an equitable power dominating set S, without loss of generality, we choose a_0 to be in S. The vertex a_0 equitably power dominates a_1 and a_2. Now the vertices a_1 and a_2 have two non-observed vertices a_1', a_2' and a_3', a_4', respectively. So one has to choose any one between a_1 and $a_{2,}$ say a_1, then a_2 is observed by $a_{0.}$ Again as a_2 has two non-observed vertices a_3' and a_4', so one has to choose any one between a_3' and a_4', say a_3'. Also a_1 in S observes a_1' and a_2'. Proceeding in the same way, finally we need to choose $a_1^n, a_2^n, a_3^n, a_4^n, a_5^n, a_6^n, a_7^n, a_8^n, \ldots, a_n^n$ as they are the pendent vertices and there are no adjacent vertices satisfying the desired equitable property. Thus we obtain the sequence of vertices, namely $a_{0,}$ a_1, a_1', a_3', a_1'', a_3'', a_5'', a_7'', ... and so on. That is.,

$\gamma_{epd}(B(1,1)) = 1$

$\gamma_{epd}(B(1,2)) = 1 + 2$

$\gamma_{epd}(B(1,3)) = 1 + 2 + 2^3$

$\gamma_{epd}(B(1,4)) = 1 + 2 + 2^2 + 2^4$

$\gamma_{epd}(B(1,5)) = 1 + 2 + 2^2 + 2^3 + 2^5$

…

Thus $\gamma_{epd}(B(1,k)) = \sum_{n=0}^{n=k} 2^n - 2^{n-1}$.

2.2 Equitable Power Domination Number of Subdivision of Certain Classes of Graphs

The concept of subdivision in graphs was introduced by Trudeau, Richard J in 1993 [11]. We recall the definition of subdivision of a graph.

Definition 7 [11]

"An edge is said to be subdivided if the edge uv is replaced by the path:uwv, where w is the new vertex. A graph obtained by subdividing each edge of a graph G is called subdivision of the graph G, and is denoted by $S(G)$."

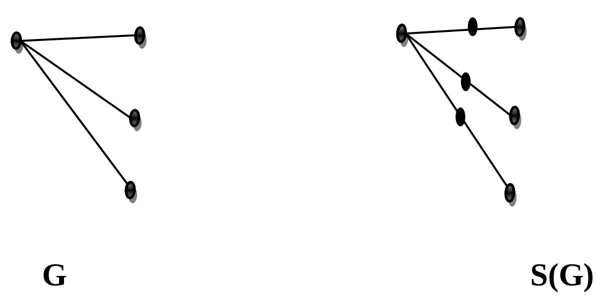

Fig.3: A graph G and subdivision of G, S(G)

Theorem 8

Let G be graph on n vertices. Then $\gamma_{epd}\big(S(G)\big) \geq \gamma_{epd}(G)$.

Proof.

Let G be the given graph with V = $\{ v_1, v_2, \ldots, v_n\}$ and edge set E = $\{ e_1, e_2, \ldots, e_n\}$. Obtain the subdivision of G, denoted $S(G)$, as follows: $V(S(G)) = V(G) \cup E(G)$ and $E\big(S(G)\big) = \{(v_i e_i), (e_i v_j)$: for $1 \leq i \leq n$ and $i + 1 \leq j \leq m - 1\}$. We consider the following two cases in obtaining an EPDS of S(G).

Case 1: For a vertex v_i incident with e_i for which $|d(v_i) - d(e_i)| \geq 1$ for at least one 'i'. Then one has to choose e_i to be in S. Thus $\gamma_{epd}\big(S(G)\big) \geq \gamma_{epd}(G)$.

Case 2: For a vertex v_i incident with e_i for which $|d(v_i) - d(e_i)| < 1$ for $1 \leq i \leq n$. Then S remains the same. Thus $\gamma_{epd}\big(S(G)\big) = \gamma_{epd}(G)$.

Theorem 9 [2]

Let C_n , $n \geq 3$ be a cycle. Then $\gamma_{epd}(C_n) = 1$.

Theorem 10

Let C_n , $n \geq 3$ be a cycle. Then $\gamma_{epd}\big(S(C_n)\big) = 1$.

Proof.

Let C_n be a cycle with $V(C_n) = \{v_1, v_2, \ldots, v_n\}$. When one performs the subdivision on C_n, the resultant graph is again a cycle on $2n$ vertices. So by Theorem 9, $\gamma_{epd}\big(S(C_n)\big) = 1$.

Theorem 11 [2]

Let P_n , $n \geq 1$ be a path. Then $\gamma_{epd}(P_n) = 1$.

Theorem 12

Let P_n , $n \geq 3$ be a path. Then $\gamma_{epd}\big(S(P_n)\big) = 1$.

Proof.

Let P_n be a path with $V(P_n) = \{v_1, v_2, \ldots, v_n\}$. An easy check shows that when one performs the subdivision on P_n, the resultant graph is again a path on $2n$ -1 vertices. So by Theorem 11, we deduce that $\gamma_{epd}\big(S(P_n)\big) = 1$.

Definition 13 [5]

"Any two distinct vertices of a graph G are adjacent then G is said to be complete graph and it is denoted by K_n."

Theorem 14 [2]

For a complete graph K_n, $\gamma_{epd}(K_n) = 1$.

Theorem 15

Let $S(K_n)$ be the subdivision of a complete graph K_n. Then $\gamma_{epd}\big(S(K_n)\big) = m + n$, for $n \geq 5$.

Proof.

Let K_n be a complete graph with $V(K_n) = \{v_1, v_2, \ldots, v_n\}$ and $E(K_n) = \{e_1, e_2, \ldots, e_m\}$. By the definition of a complete graph, the degree of each vertex v_i, $d(v_i) = n - 1$ for $1 \leq i \leq n$. Obtain the subdivision of a complete graph K_n, denoted by $S(K_n)$ as follows: $V(S(K_n)) = V_1 \cup V_2$, where $V_1 = V(K_n) = \{v_1, v_2, \ldots, v_n\}$ and $V_2 = E(K_n)$. One can notice that the subdivided graph of a complete graph K_n gives rise to the graph such that no two adjacent vertices with $|d(u) - d(v)| \leq 1$ and violate the equitable property. So to obtain an equitable power dominating set, one has to choose the entire vertex set to be in EPDS. Thus $|S| = m + n$.

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