WAVE PROPAGATION THROUGH ROTATING VISCOELASTIC MEDIUM

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ABSTRACT

In the present paper phase velocities and attenuation coefficients of two dispersive waves have been explored. The viscoelastic media considered to be homogenous, isotropic rotating Kelvin-Voigt type. The secular equation being complex quadratic equation provides us two roots that can be connected with two dispersive waves, which are found to attenuating in existing media. The numerical illustrations and graphical presentation have been carried out for copper material with help of MATLAB code. The present outcomes are advantageous in applications of geophysics.

1. INTRODUCTION

Casula and Carcione [1] studied wave propagation in the generalized mechanical model analogies of linear viscoelastic behaviour. The theory of thermo-viscoelasticity and the solution of boundary value problems were investigated by Biot [2, 3]. Auriault [4] and Sharma et. al [5] studied effect of rotation on the body wave in terms of ratio of wave frequency to angular frequency in elastic and viscothermoelastic media respectively. An analysis of waves in viscoelastic media structure such as anisotropic, half-space and for Rayleigh and Lamb waves, have been reported in literature [6-11].

In present paper, the analytical expressions of various characteristics of waves have been obtained for homogenous isotropic rotating Kelvin-Voigt viscoelastic media. Hence, system of equations with complex coefficients has been solved to obtain the solutions of phase velocity as well as attenuation coefficients.

2. PROBLEM FORMULATION

A homogenous isotropic rotating infinite Kelvin-Voigt type viscoelastic medium with uniform angular velocity $\vec{\Omega}$, which is rotating about $z-$axis, has been considered. The basic governing equations [11] after including the centripetal forces as well as Coriolis forces, in absence of body forces as well as heat sources are given below

$$(\lambda^* + \mu^*)\nabla\left(\nabla.\vec{u}\right) + \mu^*\nabla^2\vec{u} = \rho\left(\ddot{\vec{u}} + \vec{\Omega}\times(\vec{\Omega}\times\vec{u}) + 2\left(\vec{\Omega}\times\dot{\vec{u}}\right)\right) \tag{1}$$

where $\lambda^* = \lambda\left(1+\alpha_0\frac{\partial}{\partial t}\right)$ and $\mu^* = \mu\left(1+\alpha_1\frac{\partial}{\partial t}\right)$ (2)

where derivative with respect to time has been used as superposed dot and

$\vec{u}(x,y,z,t)=(u_x,u_y,u_z)$	displacement vector
λ,μ	Lame's parameter
α_0 and α_1	Viscoelastic relaxation time
ρ	Density
C_e	specific heat at fixed value of strain
α_T	Linear thermal expansion coefficient

Non-dimensional quantities are defined as below

$$x'=\frac{x}{l},y'=\frac{y}{l},z'=\frac{z}{l}\ t'=\frac{c_1}{l}t,\ u'_x=\frac{u_x}{l},u'_y=\frac{u_y}{l},u'_z=\frac{u_z}{l}\ \alpha'_0=\frac{c_1}{l}\alpha_0,\ \alpha'_1=\frac{c_1}{l}\alpha_1,$$

$$\Omega'=\frac{\Omega}{\omega^*},\ \delta^2=\frac{c_2{}^2}{c_1{}^2},\ c'=\frac{c}{c_1} \qquad (3)$$

where $c_1^2=\dfrac{\lambda+2\mu}{\rho},\ c_2^2=\dfrac{\mu}{\rho}$

The characteristic length is represented by l, the wave velocities are c_1 and c_2 respectively. Using quantities (3) in equation (1), we obtain (primes being suppressed for convenience)

$$\left[1-\delta^2+(\delta_0-\alpha_1\delta^2)\frac{\partial}{\partial t}\right]\nabla\left(\nabla.\vec{u}\right)+\delta^2\left(1+\alpha_1\frac{\partial}{\partial t}\right)\nabla^2\vec{u}=\ddot{\vec{u}}+\vec{\Omega}\times\left(\vec{\Omega}\times\vec{u}\right)+2\left(\vec{\Omega}\times\vec{u}\right) \qquad (4)$$

where $\delta_0=\alpha_0+2\delta^2(\alpha_1-\alpha_0)$

3. DISPERSIVE EQUATIONS AND THEIR SOLUTION

Considering the direction of rotation along $z-$axis i.e $\vec{\Omega}=\Omega\hat{k}$. Therefore, $\vec{u}$ is collinear to $\vec{\Omega}$ Hence, analysis of displacements carried out in $xy-$plane, which will remain fixed along $z-$axis. Considering $\phi=\nabla.\vec{u}$ and $\vec{\psi}=\nabla\times\vec{u}=\psi_3\hat{k}$ and applying consequently on equation (4) which yields the following differential equations which are coupled in terms of ϕ and ψ_3 as

$$\left[\left(1+\delta_0\frac{\partial}{\partial t}\right)\nabla^2+\Omega^2-\frac{\partial^2}{\partial t^2}\right]\phi+2\Omega\dot{\psi}_3=0 \qquad (5)$$

$$\left[\delta^2\left(1+\alpha_1\frac{\partial}{\partial t}\right)\nabla^2+\Omega^2-\frac{\partial^2}{\partial t^2}\right]\psi_3-2\omega\dot{\phi}=0 \tag{6}$$

Considering the waves propagating in x – direction as normal mode form given below

$$(\phi,\psi_3)=(B_1,B_2,B_3)e^{\{\iota(kx-\omega t)\}},\quad \iota^2=-1 \tag{7}$$

Using Normal mode solution (7) in equations (5) - (6), yields

$$-\left[\iota\omega\delta_0^*+\nu^2\left(1+\Gamma^{-2}\right)\right]B_1+2\iota\nu^2\Gamma^{-1}B_2=0 \tag{8}$$

$$2\iota\nu^2\Gamma^{-1}B_1+\left[\iota\omega\alpha_1^*\delta^2+\nu^2\left(1+\Gamma^{-2}\right)\right]B_2=0 \tag{9}$$

The non-trivial solution system of equations (8)-(9) for B_1 and B_2 in view of existing condition yields the following dispersion equation

$$\prod_{i=1}^{2}(1-\nu^2\xi_i^2)=0 \tag{10}$$

Here ξ_i^2 , $i=1,2$ are the zeros of the quadratic equation

$$\xi^4-\hat{S}\xi^2+\hat{P}=0 \tag{11}$$

where $\hat{S}=\dfrac{\iota\omega^{-1}\left(1+\Gamma^{-2}\right)\left(\alpha_1^*\delta^2+\delta_0^*\right)}{\delta_0^*\alpha_1^*\delta^2}$.

$$\hat{P}=-\frac{\left(1-\Gamma^{-2}\right)^2}{\omega^2\delta_0^*\alpha_1^*\delta^2}. \tag{12}$$

where $\Gamma=\omega/\Omega$ is known as Kibel number and $v=\omega/k$ is the phase velocity.

For $\Gamma=1$, $\hat{P}=0$ and equation (11) provides us

$$\zeta^2=\frac{2\iota\omega^{-1}\left(\alpha_1^*\delta^2+\delta_0^*\right)}{\delta_0^*\alpha_1^*\delta^2},\quad \zeta^2=0 \tag{13}$$

On solving above equations, we obtained the following roots

$$\xi_{1,2}=\left[\frac{\hat{S}\pm\sqrt{\hat{S}^2-4\hat{P}}}{2}\right]^{\frac{1}{2}}$$

and $$\zeta_1\to 0,\ \zeta_2=\left[\frac{2\iota\omega^{-1}\left(\alpha_1^*\delta^2+\delta_0^*\right)}{\delta_0^*\alpha_1^*\delta^2}\right]^{\frac{1}{2}} \tag{14}$$

Hence the following roots are associated with the complex roots ξ_i^2 , $i=1,2$ expressed in the form as

$$v_j=\begin{cases}\pm\dfrac{1}{\xi_j},\ \Gamma\neq 1\\[2ex] \pm\dfrac{1}{\zeta_j},\ \Gamma=1\end{cases}\qquad (j=1,2) \tag{15}$$

The two different sets (15), the solution of complex phase velocities yields the two distinct types of waves in viscoelastic materials.

We write

$$v_j^{-1}=V_j^{-1}+\iota\omega^{-1}Q_j,\ j=1,2 \tag{16}$$

Where V_j represents the phase speed and Q_j defines the attenuation coefficient. With help of relation (16), the equation (15) provides

$$V_j = \frac{1}{\mathrm{Re}(\xi_j)}, \quad Q_j = \omega \,\mathrm{Im}(\xi_j), \quad (j = 1, 2) \text{ for } \Gamma \neq 1 \text{ and}$$

$$V_1 \to \infty,\ Q_1 \to 0,\ V_2 = \frac{1}{\mathrm{Re}(\zeta_2)}, \qquad Q_2 = \omega \,\mathrm{Im}(\zeta_2) \tag{17}$$

in case of $\Gamma = 1$

For non-zero value of angular frequency, the QL-wave and QT-wave are coupled waves. The amplitude ratios are expressed below

$$\left(\frac{B_1}{B_2}\right)_j = \frac{2\iota\Gamma^{-1}v_j^2}{\iota\omega\delta_0^* + v_j^2\left(1+\Gamma^{-2}\right)}, \quad j = 1, 2$$

or

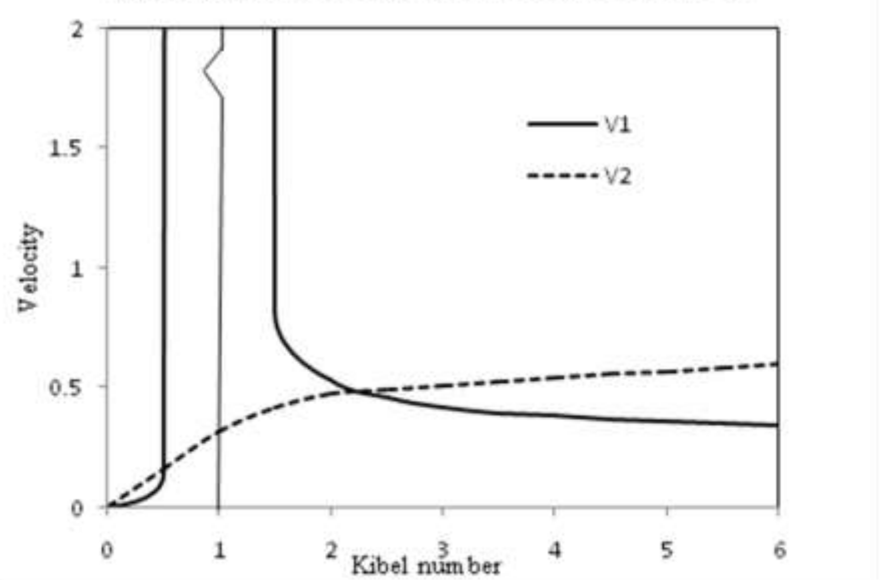
$$\left(\frac{B_2}{B_1}\right)_j = -\frac{2\iota\Gamma^{-1}v_j^2}{\iota\omega\alpha_1^*\delta^2 + v_j^2\left(1+\Gamma^{-2}\right)}, \quad j = 1, 2 \tag{18}$$

4.NUMERICAL ILLUSTRATIONS AND GRAPHICAL PRESENTATION

The numerical illustrations and graphical presentations have been carried out for Copper material [5] whose physical data is given below

$\lambda = 8.2 \times 10^{10}$ N m^{-2}, $\mu = 4.2 \times 10^{10}$ N m^{-2}, $\rho = 8.950 \times 10^{3}$ Kg m^{-2}, $\alpha_0 = \alpha_1 = 6.8831 \times 10^{-13}$ s.

Fig. 1: Velocity in viscoelastic media versus Kibel number



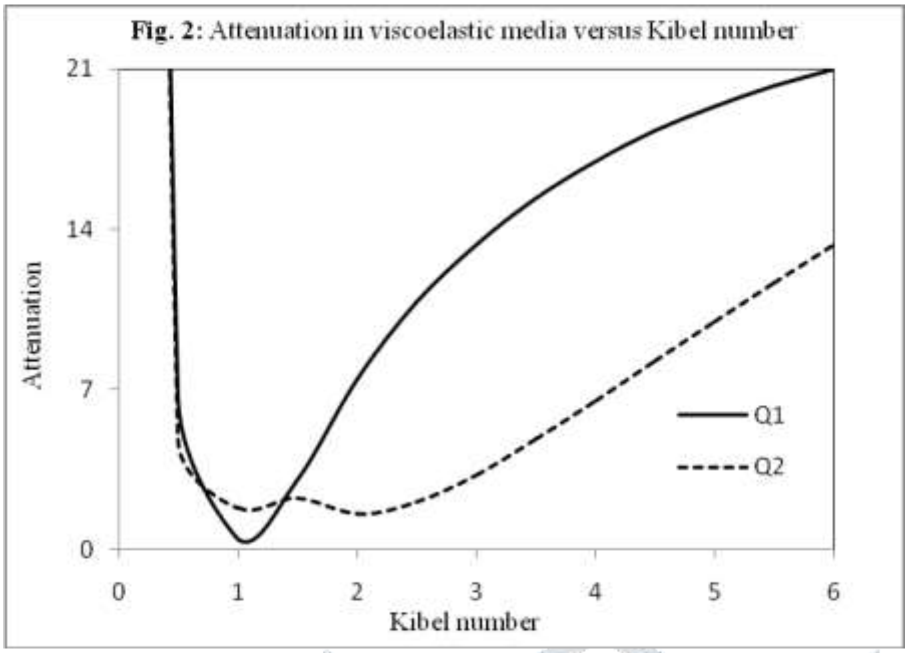


Fig. 1 represents profile of velocity of quasi-longitudinal (QL) wave attains resonance for equal value of wave and angular frequencies. Initially value increases to attain maximum value followed by decrease in value to attain stable value for $\Gamma \geq 3$ and the behaviour of QT-wave, which will increase value of ratio of wave frequency and angular frequency and finally approaches to stable value δ for its large values.

Fig. 2, reveals that due to viscous property in addition to elastic behaviour, the attenuation coefficient shapes of quasi-longitudinal and quasi transverse waves. As per analytical results the relation of phase speed and attenuation coefficients, we obtained the maximum values of attenuation coefficients whereas minimum values of phase velocities and at equal value of wave frequency phase velocity obtained maximum values then attenuation reduces to zero for quasi-longitudnal wave. In case of QT-wave, it has been observed that the attenuation profile reduces to attain minimum value at point the point whenever wave frequency is twice of angular frequency and then rises with increasing value of ratio of wave frequency to angular frequency.

CONCLUSION

The phase velocity in form of dimensionless quantity, suffers resonance for quasi-longitudinal wave for the equal vales of wave frequency and angular frequency i.e. Kibel number approaches to value equal to unity and for quasi-transverse (QT) wave due to energy dissipation in a homogenous isotropic rotating viscoelastic media attains increases in magnitude in comparison of elastic media.

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