Comparison of Stress in a Curved Beam using Straight and Curved Beam Theory

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Abstract

Beams are members subjected to transverse loading and are found in several applications. Curved beams find their applications in a number of structures, automobiles and machines. Due to curvature, additional stresses are present in curved beams. This paper presents a comparison between curved beam theory and straight beam theory applied to beams of different curvatures.

Keywords

Curved beam, Stress

Introduction

In automotive and structural applications, there are members subjected to transverse loading. Such members are referred as beam. Depending on the application, beams can be straight or curved. Different theories are available for analysis of stresses in beams. For proper design of beam, it is important to use correct theory. Straight beam theory gives bending and shear stresses in beam while curved beam theory also considers radial stresses in the beam.

In the paper, bending stresses in beam with different curvatures is determined using straight beam and curved beam theory. A beam with circular section is taken for analysis. The curvature to depth ratio of beam (R/d) is varied from 2 to 10 and its effect on bending stress is studied.

Due to curvature, two differences are observed in the stresses. First the distribution of stress becomes non-linear and second one more component of stress – radial stress is generated in the beam.

Literature Review

For straight beam, Euler and Bernoulli have given a solution to determine bending stresses. Solution for curved beam has been given by Timoshenko and Goodier based on theory of elasticity. Bernoulli Beam Theory[1], proposed in 18th century, is a basic or elementary theory in the field of beam theories that provides reliable solution for common beam. Timoshenko Beam Theory, proposed in 1921[2], is a first-order shear deformation beam theory that considers shear stresses in beam along with bending stress. Levinson Beam Theory [3] (1981) and Reddy Beam Theory[4] (1984) are the higher-order shear deformation beam theories. The first order and higher order shear deformation beam theories are the refined version of Euler-Bernoulli Beam Theory, which are developed by researchers to overcome the drawbacks posed by the elementary beam theory.

Problem Formulation

A cantilever beam with force acting on free end is taken for analysis. The beam is subjected to point load initially and solved using straight beam theory to determine bending stresses. Then, curved beam theory is used to determine radial and circumferential (bending) stresses in the beam. The procedure is repeated by taking different values of radius of curvature.

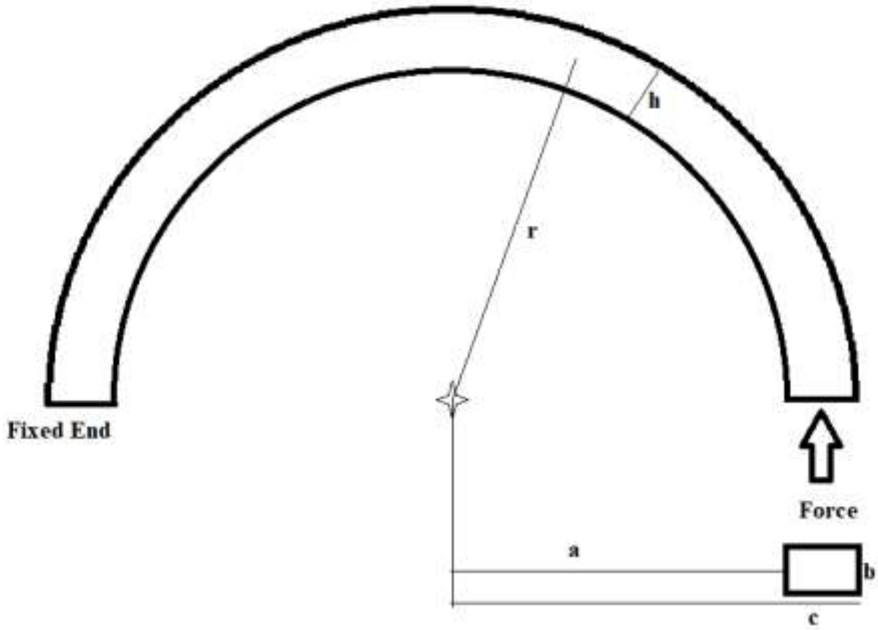

Figure 1 Beam Dimensions

Following dimensions and values of forces are used for calculations:

Table 1: Dimensions

R	radius	0.5 m – 2 m
h	height/thickness	100 mm
b	width	60 mm
A	Area of cross-section	6000 mm^2
A_m		60*ln[(R+50)/(R-50)]
P	Force	1000 N
N		1000 N
M	Moment	2000*R
I	Moment of inertia	$bh^3/12$

Calculations:

For straight beam, following flexure formula is used to determine bending stress:

$$\sigma = \frac{My}{I}$$

σ = bending stress

M = bending moment

y = distance of point where stress is to be calculated from neutral axis

I = moment of inertia

For curved beam, following expressions[1] are used to determine stresses:

$$\sigma_{\theta\theta} = \frac{N}{A} + \frac{M(A - rA_m)}{Ar(RA_m - A)}$$

$$A_m = \int \frac{dA}{r} = b \ln\left(\frac{c}{a}\right)$$

Results and Discussion

The values of stresses for different dimensions of beams are presented in Table 3. Bending stress in inner and outer fibres is calculated using curved beam theory. For straight beam theory, same value of bending stress exists for inner and outer fibre except for the difference in direction. Stress on outer fibres is tensile (+ve) while stress on inner fibres is compressive (-ve). The ratio of stress on outer fibre calculated from curved beam theory and straight beam theory is compared for different values of R/h ratios.

Table 2: Geometry Details

R/h	b	R	h		I	A	A_m	M	N	r_{inner}	r_{outer}
Units	m	m	m		m^4	m^2	m	N-m		m	m
2.5	0.06	0.25	0.1	2.5	0.000005	0.006	0.024327906	500	1000	0.2	0.3
5	0.06	0.5	0.1	5	0.000005	0.006	0.012040242	1000	1000	0.45	0.55
7.5	0.06	0.75	0.1	7.5	0.000005	0.006	0.008011884	1500	1000	0.7	0.8
10	0.06	1	0.1	10	0.000005	0.006	0.006005008	2000	1000	0.95	1.05
12.5	0.06	1.25	0.1	12.5	0.000005	0.006	0.004802562	2500	1000	1.2	1.3
50	0.06	5	0.1	50	0.000005	0.006	0.00120004	10000	1000	4.95	5.05

Table 3: Stress Comparison

$\frac{R}{h}$	Bending stress in inner fibre	Bending stress in outer fibre	Bending stress according to straight beam theory	Ratio of stress in outer fibre according to curved and straight beam theory	Ratio of stress in inner fibre according to curved and straight beam theory
	Pa	Pa	Pa		
1.25	3592018.744	-1777531.843	2500000	1.436807497	0.711012737
2.5	5932633.233	-4232866.599	5000000	1.186526647	0.84657332
5	10877701.2	-9202967.645	10000000	1.08777012	0.920296764
7.5	15861882.94	-14191647.58	15000000	1.057458863	0.946109838
10	20854376.93	-19185706.11	20000000	1.042718847	0.959285306
12.5	25849995.42	-24182047.06	25000000	1.033999817	0.967281882
50	100837373.7	-99170627.01	100000000	1.008373737	0.99170627

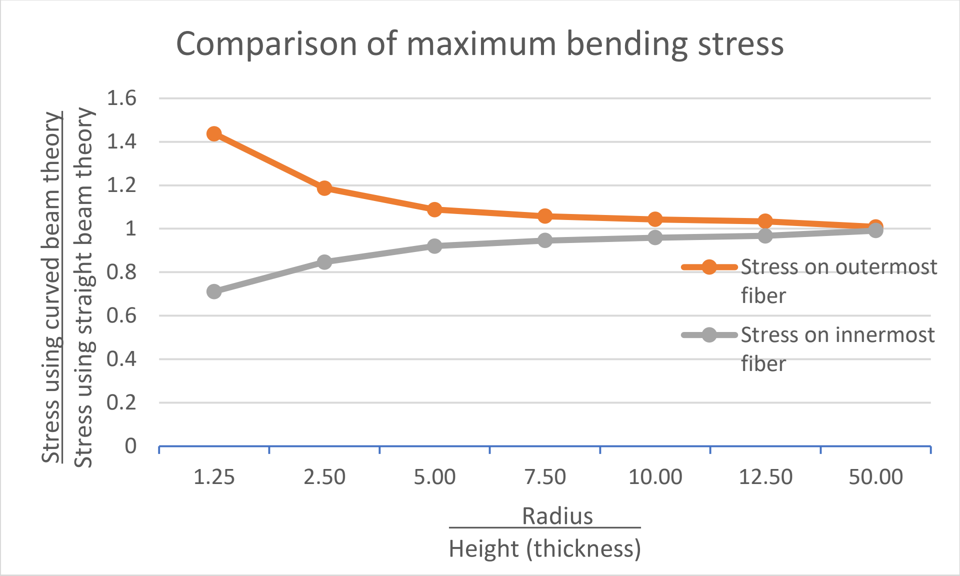

Figure 2: Comparison of stresses

From the comparison, it is observed that as the ratio of radius to depth of beam increases, the difference in stress calculated by straight beam theory approaches the value determined by curved beam theory. When the R/h ratio is greater than 5, difference between stress calculated from curved beam theory and straight beam theory is less than 10%. Further increase in the ratio gives similar results for both theories.

References:

[1] A. P. Boresi, R. J. Schmidt, O. M. Sidebottom, and others, *Advanced mechanics of materials*, vol. 6. Wiley New York, 1985.

[2] "Wiley: Elasticity in Engineering Mechanics, 3rd Edition - Arthur P. Boresi, Ken Chong, James D. Lee." http://as.wiley.com/WileyCDA/WileyTitle/productCd-0470402555.html (accessed Jul. 31, 2015).

[3] M. Levinson, "A new rectangular beam theory," *J. Sound Vib.*, vol. 74, no. 1, pp. 81–87, 1981, doi: 10.1016/0022-460X(81)90493-4.

[4] J. N. Reddy, C. M. Wang, and K. H. Lee, "Relationships between bending solutions of classical and shear deformation beam theories," *Int. J. Solids Struct.*, vol. 34, no. 26, pp. 3373–3384, 1997.