NOTE ON TIME FRACTIONAL ORDER THERMAL DEFLECTION IN CIRCULAR PLATE

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Abstract : The present article involve the study of time fractional order heat conduction problem of circular plate solved by using integral transform like Marchi-Zgrablich, finite Fourier and Laplace transform. The solutions are obtained in the form of infinite series and converge to finite values. This investigates the thermal deflection of circular plate.

IndexTerms **–Time fractional heat conduction, thermal deflection and circular plate.**

I. INTRODUCTION

The problems of thermo elasticity consist of determination of temperature distribution and thermal deflection of solids when the conditions of temperature and deflection are known at the some points of the solid under consideration. Raslan [01] exposed application of fractional-order theory of thermo elasticity in a thick plate under axisymmetric temperature distribution. The fractional-order theory of thermo elasticity was derived by Sherief et al. [02]. Warbhe et al. [03-04] studied a fractional heat conduction problem in a thin circular plate with constant temperature distribution and associated thermal stresses within the context of quasi-static theory.

Povstenko [05-08] investigated the time Fractional heat conduction equation, studied the problems of fractional order thermoelasticity in many aspects and associated thermal stresses. Khobragade et al.[09] discuss an inverse steady state and transient thermoplastic problem of thin circular plate and annular disc in Marchi-Fasulo transform domain. Deshmukh et al. [10] investigated inverse heat conduction problem of semi-infinite, clamped thin circular plate and their thermal deflection by quasi-static approach. Ghonge and Ghadle [11]-[15] investigated problems of thermoelastic behavior in different solids by integral transform methods. Further Ghonge [16]-[18] derived the analytical solution and investigated the deflection, thermal stresses of circular plates for different conditions by using Marchi-Fasulo, Marchi-Zgrablich and Laplace integral transform.

Here the time fractional order heat conduction problem is studied and corresponding thermal deflection of circular plate is obtained.

II. MATHEMATICAL FORMULATION

A thin wall work piece under lathe machine is modeled a circular plate occupying the space $D: 0 \le r \le a, 0 \le \theta \le 2\pi, 0 \le z \le h$ in terms of cylindrical coordinates. As the work piece is rotated with outer edge is clamped and machined by tool moving along the horizontal radius. The zero heat flux is applied on the upper surface of plate and known temperature Q_0 is provided at lower surface plate is insulated at curved surface. Under this more realistic prescribed conditions, the temperature distribution and quasi-static thermal deflection due to temperature is required to determine. The differential equation satisfying the deflection function as in [12-14] is given as

$$\nabla^4 w = \frac{-1}{(1-\nu)D} \nabla^2 M_T \tag{1}$$

Where, the operator ∇^2 is defined by

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} \tag{2}$$

M_T is the thermal moment of the plate defined as

$$M_T = \beta E \int_{-h}^{h} z T(r,z,t) dz \tag{3}$$

and D is the flexural rigidity of the plate denoted as

$$D = \frac{Eh^3}{12(1-\nu^2)} \tag{4}$$

β, E and ν are the coefficients of the linear thermal expansion, the Young's modulus and the Poisson's ration of the plate material respectively.

Since the edge of the circular plate is fixed and clamped;

$$w = \frac{\partial w}{\partial r} = 0 \qquad at \;\; r = a \tag{5}$$

$$w = 0 \qquad at \;\; t = 0 \tag{6}$$

Mathematical model is prepared considering nonlocal Caputo type time fractional heat conduction equation of order α as in [5]. The governing heat conduction equation in the context of fractional-order theory subjected to a time dependent heat flux for a circular plate satisfies the differential equation as in [3-4]

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} = \frac{1}{k}\frac{\partial^\alpha T}{\partial t^\alpha} \quad in\; 0 \le r \le a,\; -h \le z \le h, t > 0 \tag{7}$$

with initial condition

$$T(t,r,z) = 0 \qquad at\; t = 0,\; 0 \le r \le a,\; 0 \le z \le h \tag{8}$$

the boundary condition's

$$\left[T(t,r,z) + k_1 \frac{\partial T(t,r,z)}{\partial r}\right]_{r=0} = 0 \qquad 0 \le z \le h, t \ge 0 \tag{9}$$

$$\left[T(t,r,z) + k_2 \frac{\partial T(t,r,z)}{\partial r}\right]_{r=a} = 0 \qquad 0 \le z \le h, t \ge 0 \tag{10}$$

$$\left[T(t,r,z) + \frac{\partial T(t,r,z)}{\partial z}\right]_{z=0} = Q_0 \qquad 0 \le r \le a, t \ge 0 \tag{11}$$

$$\left[T(t,r,z) + \frac{\partial T(t,r,z)}{\partial z}\right]_{z=h} = 0 \qquad 0 \le r \le a, t \ge 0 \tag{12}$$

where k_1 and k_2 are the radiation constants on the two plane surfaces, k is the thermal diffusivity of the material of the circular plate. The equations (1) to (12) constitute the mathematical formulation of the inverse transient thermoelastic deflection problem of circular plate.

III. Analysis of the problem

Now making use of finite Marchi-Zgrablich transform to equations (7)-(12), as in [14], then applying Finite sine transform as in [22] and Laplace transform [3] and using transformed initial and boundary conditions, once applying successive inversion of each transforms, ones obtain the temperature distribution function as

$$T(t,r,z) = \sum_{n=0}^{\infty}\sum_{m=1}^{\infty} Q_0 \frac{S_0(k_1,k_2,\mu_m r)}{C_m} K(\lambda_n, z) E_\alpha\left[-k\left(\lambda_n^2 + \mu_m^2\right)t^\alpha\right] \tag{13}$$

where

$$C_m = \frac{a^2}{2}\left\{S_0^2(k_1,k_2,\mu_m a) - S_{-1}(k_1,k_2,\mu_m a) S_1(k_1,k_2,\mu_m a)\right\}$$

$$S_0(k_1,k_2,\mu_m r) = J_0(\mu_m r)\{G_0(k_1,0) + G_0(k_2,\mu_m a)\} - G_0(\mu_m r)\{J_0(k_1,0) + J_0(k_2,\mu_m a)\}$$

and μ_m are the positive roots of equation $J_0(k_1,0)G_0(k_2,\mu a) - J_0(k_2,\mu a)G_0(k_1,0) = 0$

Also $K(\lambda_n, z) = \sqrt{\frac{2}{h}}\sin(\lambda_n z)$ and $\lambda_1, \lambda_2, \lambda_3, ...$ are positive roots of transcendental equation $\sin(nh) = 0, \quad n = 1,2,3,.......$

And $E_\alpha[....]$ represents the Mittag-Leffler function.

Using equation (13) in equation (3), we obtain

$$M_T = \beta E \sqrt{\frac{2}{h}} \sum_{n=0}^{\infty}\sum_{m=1}^{\infty} Q_0 \frac{S_0(k_1,k_2,\mu_m r)}{C_m}\left[\frac{\sin(\lambda_n h)}{\lambda_n^2} - \frac{h\cos(\lambda_n h)}{\lambda_n}\right] E_\alpha\left[-k\left(\lambda_n^2 + \mu_m^2\right)t^\alpha\right] \tag{14}$$

Assume the solution of (1) satisfying the (5) as

$$w(t,r) = \sum_{n=1}^{\infty} A_n(t)\left[J_0(\lambda_n r) - J_0(\lambda_n a)\right] \tag{15}$$

Using the (14), (15) and the result

$$\left[\frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr}\right] J_0(\lambda_n r) = -\lambda_n^2 J_0(\lambda_n r)$$

$$\left[\frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr}\right] S_0(k_1,k_2,\mu_m r) = -\mu_m^2 S_0(k_1,k_2,\mu_m r)$$

in (1), once we obtain the expression for $A_n(t)$ as

$$A_n(t) = \frac{\beta E}{D(1-\nu)}\sqrt{\frac{2}{h}} \sum_{n=0}^{\infty}\sum_{m=1}^{\infty} Q_0 \frac{S_0(k_1,k_2,\mu_m r)}{C_m \mu_m^2 J_0(\mu_m r)}\left[\frac{\sin(\lambda_n h)}{\lambda_n^2} - \frac{h\cos(\lambda_n h)}{\lambda_n}\right] E_\alpha\left[-k\left(\lambda_n^2 + \mu_m^2\right)t^\alpha\right] \tag{16}$$

Substituting the equation (16) in the equation (15), once obtain the expression for thermal deflection function as

$$\begin{aligned} w(r,t) = \frac{\beta E}{D(1-\nu)} \sqrt{\frac{2}{h}} \sum_{n=0}^{\infty} \sum_{m=1}^{\infty} Q_0 \left\{ \frac{\left[J_0(\mu_m r) - J_0(\mu_m a) \right] S_0(k_1, k_2, \mu_m r)}{C_m \mu_m^2 J_0(\mu_m r)} \right\} \\ \times \left[\frac{\sin(\lambda_n h)}{\lambda_n^2} - \frac{h \cos(\lambda_n h)}{\lambda_n} \right] E_\alpha \left[-k\left(\lambda_n^2 + \mu_m^2\right) t^\alpha \right] \end{aligned} \tag{17}$$

IV. PARTICULAR CASE AND NUMERICAL OUTCOMES

Any special case can be carried out and examine the numerical calculation of fractional order thermal behaviour of a circular plate, we consider the following functions and parameters:

- Radius of a circular plate $a = 1$ m.
- Thickness of a circular plate $h = 0.2$ m.
- For our convenience setting
- $A = Q_0$ and $B = Q_0 \dfrac{\beta E}{D(1-\nu)}$

V. CONCLUSION

This article investigates the unknown temperature distribution and quasi-static thermal deflection at time fractional heat conduction effect. First, the mathematical model is constructed, and then the series solutions are obtained by using integral transform methods. As a special case and numerical results the functions and parameters are consider and the temperature and quasi-static thermal deflection on upper surface determine. This type of problems has the many applications in engineering such as main shaft of a lathe machine and aircraft structure. The results obtained here are mainly useful in the determination of state of strain in a circular plate.

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