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N-Dimensional FRW Barotropic Fluid Cosmological Model with Time Dependent Λ(t) in Creation Field Theory of Gravitation

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Abstract: N-dimensional FRW cosmological model in Hoyle-Narlikar's creation field theory of gravitation has been studied, when the universe is filled with barotropic fluid. To get the deterministic solution, the cosmological constant $\Lambda(t)$ is considered as a time dependent function of cosmic time t *i.e.* $\Lambda = \frac{1}{R^2}$, where R is a scale factor. The physical aspects of the model are also studied.

Keywords: Creation field theory of gravitation, N-dimensional FRW space time, Barotropic fluid, Varying cosmological constant $\Lambda(t)$.

1. Introduction:

Barotropic fluid is a fluid whose density is function of pressure only. Barotropic fluid model is useful for finding behavior in a wide variety of scientific field, from meteorology to astrophysics. In astrophysics, a barotropic fluid plays an important role in the study of stellar interiors or of the interstellar medium. The equation of state related to pressure and density $p = \gamma\rho$, where $0 \le \gamma \le 1$ is a special type of barotropic fluid. This mathematical form of equation of states describes particular cases *viz.* dust $(\gamma = 0)$, disordered radiation $(\gamma = 1/3)$ and stiff fluid $(\gamma = 1)$. Therefore, barotropic fluid is useful for describing matter content of the universe.

The three important observations in astronomy *viz.* the phenomenon of expanding universe, primordial nucleo-synthesis and the observed isotropy of the Cosmic Microwave Background Radiations (CMBR) are explained by the big bang cosmological model. These models are based on Einstein's field equations. However Smoot *et al.* [1] revealed that the earlier predictions of FRW type of models do not always exactly meet our expectations. The theoretical explanations given from big-bang type model were contradicted by some puzzling results regarding the red-shifts from extra-galactic objects. Also CMBR discovery did not prove it to be an outcome of big-bang theory. The possibilities of non-relic interpretation of CMBR have been proved by Narlikar *et al.* [2]. Alternative theories of gravitation have been proposed by cosmologists to explain such phenomenon. Bondi and Gold [3] proposed the steady state theory in which the universe does not have singular beginning or an end on the cosmic time scale. Moreover, they state that the statistical properties of the large scale features of the universe do not change. Further, the constancy of matter density has been accounted by continuous creation of matter going on in contrast to one time infinite and explosive creation of matter at $t = 0$ as in earlier standard model. But the principle of conservation of matter was violated in this formalism. This difficulty was overcome by Hoyle and Narlikar [4-6] by adopting a field theoretical approach and introducing a massless & chargeless scalar field C in the Einstein-Hilbert action to explain creation of matter. In Hoyle-Narlikar C-field theory there is no big-bang type singularity as in steady state theory by Bondi and Gold. Narlikar [7] has shown that matter creation is accomplished at the expense of

negative energy *C*-field in which he solves horizon and flatness problem faced by big-bang model. Narlikar and Padmanabhan [8] have obtained a solution of Einstein field equations admitting radiation with a negative energy massless scalar field *C*. Chatterjee and Banerjee [9] have investigated higher dimensional cosmology in *C*-field theory of gravitation. Singh and Chaubey [10] have studied Bianchi type I, III, V, VI_0 and Kantowski-Sachs universes in creation field cosmology. Many authors *viz.* Adhav *et al.* [11], Bali & Kumawat [12], Katore [13], Bali & Saraf [14], Chaubey *et. al*. [15], Ghate *et.al*.[16-17], Aygun *et. al*. [18] and Yadav *et. al*. [19] and Tyagi & Parikh [20] have investigated cosmological models in Creation-field theory of gravitation. Higher dimensional FRW solutions in Creation field cosmology have been studied by Çağlar and Aygün [21]. Also, Chundawat & Mehta [22] have investigated LRS Bianchi Type-II cosmological model with barotropic perfect fluid with time-dependent term- Λ in Creation field theory of gravitation.
In cosmology, the cosmological constant is considered as one of the most unsolved problem [23]. Einstein introduced cosmological constant as the universal repulsion to make the universe static in accordance with generally accepted picture of that time. In 1927, Hubble observed the general expansion of the universe. The smallness of the recently observed effective cosmological constant constitutes one of the most difficult problems involving cosmology and elementary particle physics theory. Cosmological model with positive cosmological constant which leads to de-Sitter space time asymptotically have been studied by Gibbon and Hawking [24]. Therefore, the cosmological models linking the variations of cosmological constant are having the form of Einstein's field equations unchanged and preserving the energy-momentum tensor of matter content. Bertolami [25] was the first who consider cosmological models with a variable cosmological constant of the form $\Lambda \sim t^{-2}$. Chen and Wu [26] have also solved the problem by considering $\Lambda \sim R^{-2}$, where R is the scale factor in the Robertson-Walker space time. Overduin [27] considered the assumptions $\Lambda \sim t^{-2}$ & $\Lambda \sim H^{-2}$ in FRW models and shows there compability with various observations. Several attempts have made by many researchers *viz.* Lui & Wesson [28], Carneiro *et al.* [29], Xu *et. al*. [30], Ghate *et al*. [31-32], Chirenti & Rodrigues [33] Chaubey *et. al*. [34], Ghate & Salve [35], Chauhan *et. al*. [36] and Gupta [37] in the favor of time dependent $\Lambda \sim t^{-2}$ in different contexts. Recently, Aich [38] has studied Phenomenological dark energy model with hybrid dynamic cosmological constant.
In this paper, we have investigated N-Dimensional FRW space-time, when the universe is filled with barotropic fluid. To get the deterministic solution, the cosmological constant $\Lambda(t)$ is considered as a time dependent function of cosmic time t *i.e.* $\Lambda = \frac{1}{R^2}$, where R is a scale factor. This work is organized as follows. In Section 2, the model and field equations have been presented. The solution of field equations has been discussed in Section 3. Then in Section 4, the physical aspects of the model have been discussed. In the last Section 5 concluding remarks have been expressed.

2. Metric and Field Equations:

We consider N-Dimensional FRW metric considered in the form

$$ds^2 = dt^2 - R^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\phi_{n-2})^2\right], \text{ where } k = 0, \pm 1, \tag{1}$$

and

$$(d\phi_{n-2})^2 = d\theta_1^{\,2} + \sin^2\theta_1 d\theta_2^{\,2} + \sin^2\theta_1 \sin^2\theta_2 d\theta_3^{\,2} + ----+ \sin^2\theta_1 \sin^2\theta_2 --- \sin^2\theta_{n-3} d\theta_{n-2}^{\,2}.$$

Here $R(t)$ represents the scale factor.
Einstein field equations in Creation-field theory of gravitation [4-6] with varying $\Lambda(t)$ are

$$R_i^j - \frac{1}{2} R g_i^j = -8\pi G\left(\underset{(m)}{T_i^j} + \underset{(c)}{T_i^j}\right) - \Lambda g_i^j . \tag{2}$$

The energy momentum tensor $\underset{(m)}{T_i^j}$ for perfect fluid and $\underset{(c)}{T_i^j}$ for creation field are given by

$$\underset{(m)}{T_i^j} = (\rho + p) v_i v^j - p g_i^j , \tag{3}$$

and
$$\underset{(c)}{T_i^j} = -f\left(c_i c^j - \frac{1}{2} g_i^j C^\alpha C_\alpha\right). \tag{4}$$

Here ρ is the energy density of massive particle and p is the pressure. v_i are co-moving four velocities which obeys the relation $v_i v^j = 1, v_\alpha = 0, \alpha = 1, 2, 3$. The coupling constant between matter and creation field is greater than zero. It is assumed that creation field C is a function of time only i.e. $C(x,t) = C(t)$.

With the help of equations (3) and (4), the field equations (2) for the metric (1) are

$$\frac{(n-1)(n-2)}{2}\frac{\dot{R}^2}{R^2} + \frac{(n-1)(n-2)}{2}\frac{k}{R^2} = 8\pi G\left(\rho - \frac{1}{2} f\dot{C}^2\right) + \Lambda, \tag{5}$$

$$(n-2)\frac{\ddot{R}}{R} + \frac{(n-2)(n-3)}{2}\frac{\dot{R}^2}{R^2} + \frac{(n-2)(n-3)}{2}\frac{k}{R^2} = 8\pi G\left(-p + \frac{1}{2} f\dot{C}^2\right) + \Lambda, \tag{6}$$

where overhead dot $(\dot{})$ denotes differentiation with respect to cosmic time t.

The conservation equation

$$\left(8\pi G T_i^j + \Lambda g_i^j\right)_{;j} = 0 \tag{7}$$

leads to

$$8\pi\dot{G}\left[\rho - \frac{1}{2} f\dot{C}^2\right] + 8\pi G\left[\dot{\rho} - f\dot{C}\ddot{C} + (n-1)\rho\frac{\dot{R}}{R} - (n-1) f\dot{C}^2\frac{\dot{R}}{R} + (n-1)p\frac{\dot{R}}{R}\right] + \dot{\Lambda} = 0. \tag{8}$$

Using G = constant, equation (8) leads to

$$8\pi G\left[\dot{\rho} - f\dot{C}\ddot{C} + (n-1)\rho\frac{\dot{R}}{R} - (n-1) f\dot{C}^2\frac{\dot{R}}{R} + (n-1)p\frac{\dot{R}}{R}\right] + \dot{\Lambda} = 0. \tag{9}$$

3. Solution of the Field Equations:

Following Hoyle and Narlikar, the source equation of C-field: $C^i_{;i} = n/f$ leads to $C = t$, for large values of r. Thus $\dot{C} = 1$.

Now using $\dot{C} = 1$ and barotropic condition $p = \gamma\rho$ in equation (6), we get

$$-8\pi G\gamma\rho = (n-2)\frac{\ddot{R}}{R} + \frac{(n-2)(n-3)}{2}\frac{\dot{R}^2}{R^2} + \frac{(n-2)(n-3)}{2}\frac{k}{R^2} - 4\pi G f - \Lambda. \tag{10}$$

Where $0 \le \gamma \le 1$.

Using $\dot{C} = 1$ in equation (5) therein, we obtained

$$8\pi G\rho = \frac{(n-1)(n-2)}{2}\frac{\dot{R}^2}{R^2} + \frac{(n-1)(n-2)}{2}\frac{k}{R^2} + 4\pi G f - \Lambda, \tag{11}$$

Solving equations (10) and (11), we get

$$(n-2)\frac{\ddot{R}}{R} + \left(\frac{(n-2)(n-3)}{2} + \frac{(n-1)(n-2)}{2}\gamma\right)\frac{\dot{R}^2}{R^2} = (1-\gamma)4\pi G f - \left(\frac{(n-2)(n-3)}{2} + \frac{(n-1)(n-2)}{2}\gamma\right)\frac{k}{R^2} + (\gamma+1)\Lambda \tag{12}$$

For getting deterministic solution in terms of cosmic time t, we assume $\Lambda = \frac{1}{R^2}$ [26].

Using $\Lambda = \frac{1}{R^2}$ in equation (12), we get

$$(n-2)\ddot{R} + \left(\frac{(n-2)(n-3)}{2} + \frac{(n-1)(n-2)}{2}\gamma\right)\frac{\dot{R}^2}{R} = (1-\gamma)4\pi G f R + \frac{1}{R}\left((\gamma+1) - \left(\frac{(n-2)(n-3)}{2} + \frac{(n-1)(n-2)}{2}\gamma\right)k\right) \tag{13}$$

To find the solution of equation (13), let $\dot{R} = F(R)$

which implies $\ddot{R} = FF'$, where $F' = \frac{dF}{dR}$. (14)

Using equation (14), equation (13) leads to

$$\frac{dF^2}{dR}+\left((n-3)+(n-1)\gamma\right)\frac{F^2}{R}=\frac{8\pi Gf\,(1-\gamma)}{(n-2)}R+\frac{1}{R}\left(\frac{2(\gamma+1)}{(n-2)}-\{(n-3)+(n-1)\gamma\}k\right) \tag{15}$$

On integration, equation (15) simplifies to

$$F^2=\frac{8\pi Gf\,(1-\gamma)}{(n-1)(n-2)(1+\gamma)}R^2+\left[\frac{2(1+\gamma)}{(n-2)\{(n-3)+(n-1)\gamma\}}-k\right]. \tag{16}$$

The integration constant taken to be zero for simplicity.
which reduces to

$$\frac{dR}{\sqrt{\alpha R^2+\beta}}=dt, \tag{17}$$

where $\alpha=\frac{8\pi Gf\,(1-\gamma)}{(n-1)(n-2)(1+\gamma)}$, $\beta=\frac{2(1+\gamma)}{(n-2)\{(n-3)+(n-1)\gamma\}}-k.$ (18)

Equation (17) on integration gives

$$R=\sqrt{\frac{\beta}{\alpha}}\sinh\sqrt{\alpha}t. \tag{19}$$

Thus, we have

$$\Lambda=\frac{1}{R^2}=\frac{\alpha}{\beta}\text{cosech}^2\sqrt{\alpha}t. \tag{20}$$

Using equation (19), (20) in equation (11), we have

$$\rho=\frac{1}{8\pi G}\left[\left(\text{cosech}^2\sqrt{\alpha}t\right)\left\{\frac{(n-1)(n-2)}{2}\alpha+\left(\frac{(n-1)(n-2)}{2}k-1\right)\frac{\alpha}{\beta}\right\}+\frac{8\pi Gf}{1+\gamma}\right]. \tag{21}$$

Using equation (19) in metric (1), we get

$$ds^2=dt^2-\left(\frac{\beta}{\alpha}\sinh^2\sqrt{\alpha}t\right)\left[\frac{dr^2}{1-kr^2}+r^2(d\phi_{n-2})^2\right]. \tag{22}$$

Using $p=\gamma\rho$, equation (9) leads to

$$8\pi G\left[\dot{\rho}-f\dot{C}\ddot{C}+(n-1)\rho\frac{\dot{R}}{R}-(n-1)f\dot{C}^2\frac{\dot{R}}{R}+(n-1)\gamma\rho\frac{\dot{R}}{R}\right]+\dot{\Lambda}=0. \tag{23}$$

Substituting equations (19), (20) and (21) into equation (23), we get

$$\frac{d\dot{C}^2}{dt}+2(n-1)\sqrt{\alpha}\coth\sqrt{\alpha}t\dot{C}^2=$$

$$2(n-1)\sqrt{\alpha}\coth\sqrt{\alpha}t\frac{(1+\gamma)}{8\pi Gf}\left[\left(\text{cosech}^2\sqrt{\alpha}t\right)\left\{\frac{(n-1)(n-2)}{2}\alpha+\left(\frac{(n-1)(n-2)}{2}k-1\right)\frac{\alpha}{\beta}\right\}+\frac{8\pi Gf}{1+\gamma}\right]$$

$$+\frac{2}{8\pi Gf}\left[\left\{\frac{(n-1)(n-2)}{2}\alpha+\left(\frac{(n-1)(n-2)}{2}k-1\right)\frac{\alpha}{\beta}\right\}\left(-2\sqrt{\alpha}\coth\sqrt{\alpha}t\,\text{cosech}^2\sqrt{\alpha}t\right)\right]$$

$$-\frac{2\alpha\sqrt{\alpha}}{4\pi Gf\beta}\coth\sqrt{\alpha}t\,\text{cosech}^2\sqrt{\alpha}t \tag{24}$$

To get deterministic value of $\dot{C}$, we assume $\alpha=1$.
Thus equation (24) leads to

$$\frac{d\dot{C}^2}{dt}+2(n-1)\coth t.\dot{C}^2=2(n-1)\coth t. \tag{25}$$

From equation (25), we get

$$\dot{C}^2(\sinh t)^{2(n-1)}=2(n-1)\int\coth t(\sinh t)^{2(n-1)}dt. \tag{26}$$

On simplification equation (26) reduces to

$$\dot{C}=1, \tag{27}$$

which again leads to

$$C=t. \tag{28}$$

We find $\dot{C}=1$, which agrees with the value used in source equation. Thus creation field C is proportional to time t and the metric (1) for constraints mentioned above, leads to

$$ds^2 = dt^2 - (\beta \sinh^2 t)\left[\frac{dr^2}{1-kr^2} + r^2 (d\phi_{n-2})^2\right], \tag{29}$$

4. Physical and Geometrical Aspects:

The Energy mass density (ρ), the cosmological constant (Λ) and the deceleration parameter (q) for above cosmological model (29) obtained as

$$\rho = \left\{\frac{(n-1)(n-2)}{2} + \left(\frac{(n-1)(n-2)}{2}k - 1\right)\left(\frac{(n-2)\{(n-3)+(n-1)\gamma\}}{2(1+\gamma)} - \frac{1}{k}\right)\right\}(\mathrm{cosech}^2 t) + 4\pi Gf + 3 \tag{30}$$

$$\Lambda = \left(\frac{(n-2)\{(n-3)+(n-1)\gamma\}}{2(1+\gamma)} - \frac{1}{k}\right)\mathrm{cosech}^2 t, \tag{31}$$

$$R = \sqrt{\frac{2(1+\gamma)}{(n-2)\{(n-3)+(n-1)\gamma\}} - k}\ \sinh t, \tag{32}$$

$$q = -\tanh^2 t, \tag{33}$$

where $k = 0, -1, 1$.

5. Conclusion:

N-dimensional FRW cosmological model has been obtained with time dependent Λ in Creation field theory of gravitation when the universe is filled with baratropic fluid. Our model is the extension work of Bali & Saraf [14] for FRW space times. In this model, the spatial volume V increases with the cosmic time t. The deceleration parameter $q<0$ representing accelerating phase of the universe which matches with the recent SNe Ia observations. Also, the energy density (ρ) is positive. The creation field C is directly proportional to time t. Hence the creation of matter increases as time increases which follows the results as obtained by Hoyle and Narlikar.

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