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Bianchi type II Cosmological model in Saez Ballester theory of Gravitation with Macroscopic Body

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Abstract: In this paper, we have investigated Bianchi type-II cosmological models in Saez Ballester theory formulated by Saez Ballester under the influence of macroscopic body. Exact cosmological model is obtained with the help of power law relation between metric coefficient, and variation of Hubble parameter. Also, we discuss some observational parameters, kinematical properties and their graphical illustration of the obtained models.

Keyword: Bianchi type-II, Saez Ballester theory, macroscopic body.

1. Introduction

General theory of relativity has all the potential to explain the gravitational phenomenon, but it has been criticized due to lack of certain desirable features. Such as Mach's principle is not verified by general relativity. Hence to overcome such drawbacks, some scalar-tensor theories of gravitation were investigated. Scalar-Tensor Theories of Gravitation are considered to be the most natural generalization of general relativity. One of them, Saez Ballesster [1] constructed a scalar tensor theory of gravitation in which the metric is coupled with dimensionless scalar field in a simple manner .This coupling gives a satisfactory description of the weak field.

Saez Ballester theory of gravitation commits the field equations as

$$G_{ij} - \omega\phi^n\left(\phi_{,i}\,\phi_{,j} - \frac{1}{2}g_{ij}\phi_{,k}\,\phi^{,k}\right) = -8\pi T_{ij} \tag{1}$$

$$2\phi^n\phi^i_{;i} + n\phi^{n-1}\phi_{,k}\,\phi^{,k} = 0 \tag{2}$$

Where $G_{ij} = R_{ij} - \frac{1}{2} R g_{ij}$ is the Einstein tensor, R_{ij} is the Ricci tensor, R is the scalar curvature, n is arbitrary constant, ω is dimensionless coupling constant and T_{ij} is energy –momentum tensor. Here comma and semicolon denote partial and covariant differentiation respectively. Saez[2], Singh et al [3], Ram et al [4,5], Reddy et al [6], Rao et al [7], Sharma et al [8] , R.L. Naidu et al [9], Santhi et al [10], Mishra et al [11,12], Pradhan et al [13] are some of the authors who investigated the cosmological model in Saez –Ballester theory of gravitation.

The energy momentum tensor of the macroscopic body formulated by Landau L.D and Lifshitz E.M [14]. Macroscopic body is considered as a system, consisting of an infinite number of quantum particle. The pressure of the macroscopic body is less than one third of energy density. Nimkar et al [15,16] has studied macroscopic body cosmological models in scalar tensor theories of gravitation.

The purpose of the present work is to obtain a Bianchi type-II Cosmological model in presence of a macroscopic body. Our paper is organized as follows. In section 2, we derive the Metric and field Equations in seaz ballester theory of gravitation. Section 3, Solutions of macroscopic body. Section 4 is mainly concerned with the physical and Kinematical properties of the model and Section 5 contains the observational parameters of all the models. The last section contains conclusion.

2. Metric and Field Equations

We consider the Bianchi type II space time in the form,

$$ds^2 = dt^2 - A^2(dx - zdy)^2 - B^2 dy^2 - C^2 dz^2 \quad (3)$$

Where A, B, C are function of t only.

The field equation in Seaz Ballester theory are given by

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{B_4 C_4}{BC} - \frac{3A^2}{4B^2C^2} - \frac{\omega}{2}\phi^n \phi_4^2 = 8\pi T_1^1 \quad (4)$$

$$\frac{A_{44}}{A} + \frac{C_{44}}{C} + \frac{A_4 C_4}{AC} + \frac{A^2}{4B^2C^2} - \frac{\omega}{2}\phi^n \phi_4^2 = 8\pi T_2^2 \quad (5)$$

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4 B_4}{AB} + \frac{A^2}{4B^2C^2} - \frac{\omega}{2}\phi^n \phi_4^2 = 8\pi T_3^3 \quad (6)$$

$$\frac{A_4 B_4}{AB} + \frac{B_4 C_4}{BC} + \frac{A_4 C_4}{AC} - \frac{A^2}{4B^2C^2} - \frac{\omega}{2}\phi^n \phi_4^2 = 8\pi T_4^4 \quad (7)$$

$$\phi_{44} + \left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right)\phi_4 + \frac{n\phi_4^2}{2\phi} = 0 \quad (8)$$

Where the suffix '4' on the following unknown represent ordinary differentiation w. r. to 't' only.

3. Solution and model of macroscopic body

The energy momentum tensor of macroscopic body (Landue L.D and Lifshitz E.M) [14] is given by,

$$T^{ij} = (\varepsilon + p)u^i u^j - p g^{ij} \tag{9}$$

Here p is pressure, ε is energy density and u_i represents the four velocity vectors of the distribution respectively.

$$T_1^1 = T_2^2 = T_3^3 = -p, \qquad T_4^4 = \varepsilon \tag{10}$$

The equation (5)-(8) contain seven unknown $A, B, C, \phi, \rho, p, \omega$. To find the deterministic solution use power relation $A = B^n, C = B^m$ and proper volume v and average scale factor a(t) for Bianchi type II is

$$a(t) = (ABC)^{\frac{1}{3}} \tag{11}$$

$$v = [a(t)]^3 \tag{12}$$

The physical quantities of cosmological model are expansion scalar θ, the mean anisotropy parameter A_m and shear scalar σ are defined as

$$\theta = \frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C} \tag{13}$$

$$H = \frac{1}{3}\theta \tag{14}$$

$$Am = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{\Delta Hi}{H}\right)^2 \tag{15}$$

$$\sigma^2 = \frac{3}{2} Am\, H^2 \tag{16}$$

The variation of Hubble's parameter suggested by Berman [17] that yields constant deceleration parameter of the model is given by

$$q = \frac{a a_{44}}{a_4^2} \tag{17}$$

Solving (11) and (17) we get

$$A = (C_3 t + C_4)^{\frac{3n}{(q+1)(m+n+1)}} \tag{18}$$

$$B = (C_3 t + C_4)^{\frac{3}{(q+1)(m+n+1)}} \tag{19}$$

$$C=(C_3t+C_4)^{\frac{3m}{(q+1)(m+n+1)}} \tag{20}$$

From equation (18), (19) and (20) cosmological model of equation (3) can be written as

$$ds^2=dt^2-(C_3t+C_4)^{\frac{6n}{(q+1)(m+n+1)}}(dx-zdy)^2-(C_3+C_4)^{\frac{6}{(q+1)(m+n+1)}}dy^2-(C_3t+C_4)^{\frac{6m}{(q+1)(m+n+1)}}dz^2 \tag{21}$$

Scalar field, pressure and energy density is given by,

$$\phi=C_6\,(C_3t+C_4)^{\frac{2(q-2)}{(q+1)(n+2)}} \tag{22}$$

Where $C_6=\left[\frac{(q+1)2C_5}{(q-2)(n+2)}\right]^{\frac{2}{n+2}}$ (23)

$$P=\frac{-3C_3^2}{8\pi(C_3t+C_4)^2}\left[\frac{C_7}{(q+1)^2(m+n+1)^2}-\frac{1}{4C_3^2(C_3t+C_4)^{\frac{6(m-n+1)}{(q+1)(m+n+1)}-2}}-\frac{\omega C_5^2}{6C_3^2(C_3t+C_4)^{\frac{6}{q+1)(m+n+1)}-2}}\right] \tag{24}$$

$$\varepsilon=\frac{C_3^2}{8\pi(C_3t+C_4)^2}\left[\frac{9(m+n+mn)}{(q+1)^2(m+n+1)^2}-\frac{1}{4C_3^2(C_3t+C_4)^{\frac{6(m-n+1)}{(q+1)(m+n+1)}-2}}+\frac{\omega C_5^2}{2C_3^2(C_3t+C_4)^{\frac{6}{q+1)(m+n+1)}-2}}\right] \tag{25}$$

4. Physical and kinematical Properties

Spatial volume $V=(C_3t+C_4)^{\frac{3}{q+1}}$ (26)

Hubble parameter $H=\frac{C_3}{(q+1)(C_3t+C_4)}$ (27)

Expansion scalar $\theta=\frac{3C_3}{(q+1)(C_3t+C_4)}$ (28)

Average scale factor $a(t)=(C_3t+C_4)^{\frac{1}{q+1}}$ (29)

Shear scalar $\sigma^2=\frac{3C_3^2\,(m^2+n^2-1-mn-m-n)}{(m+n+1)^2(q+1)(C_3t+C_4)^2}$ (30)

Graphical representation of spatial Volume, Shear scalar, Hubble Parameter and Expansion Scalar with time of macroscopic body are as follows-

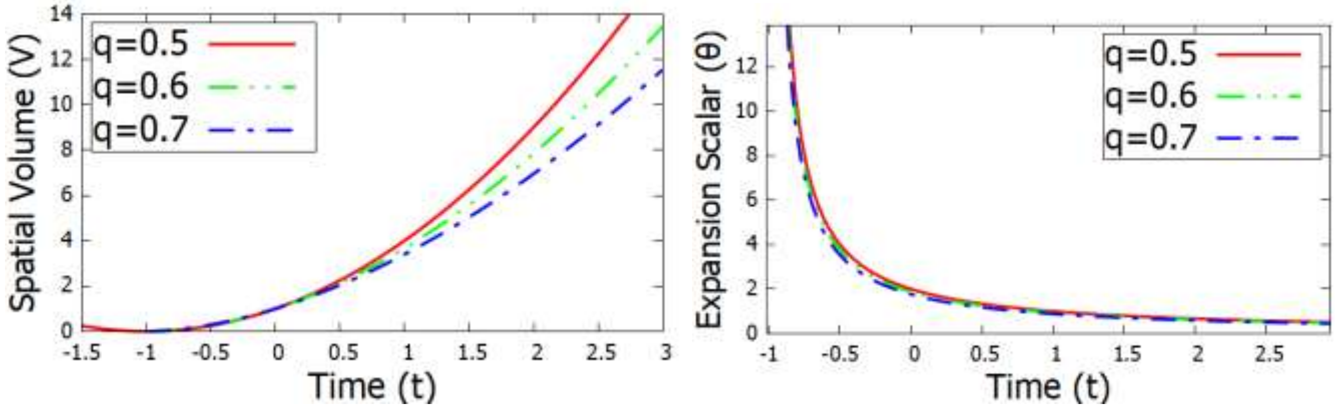

Fig.1 Spatial volume and Expansion Scalar versus time

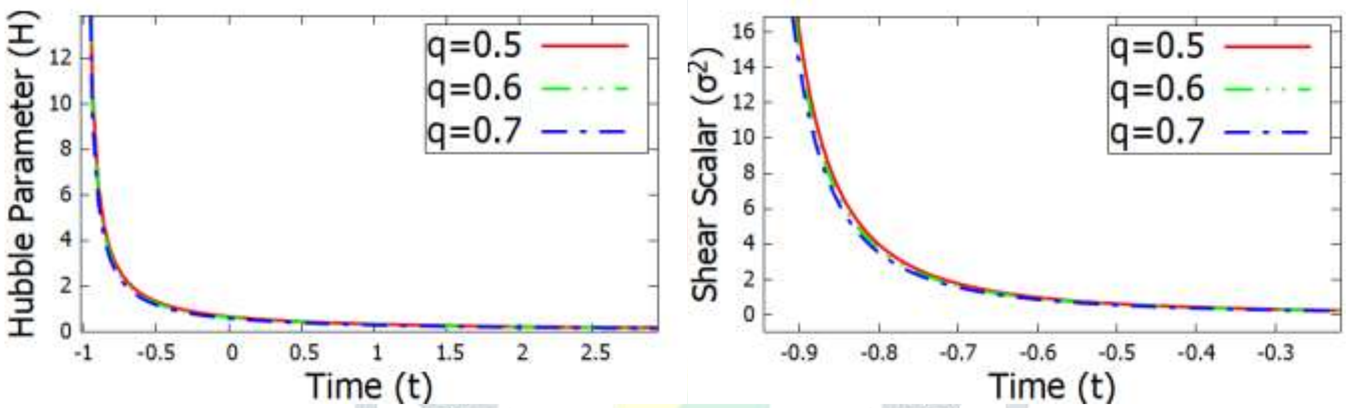

Fig.2 Hubble Parameter and Shear Scalar versus time

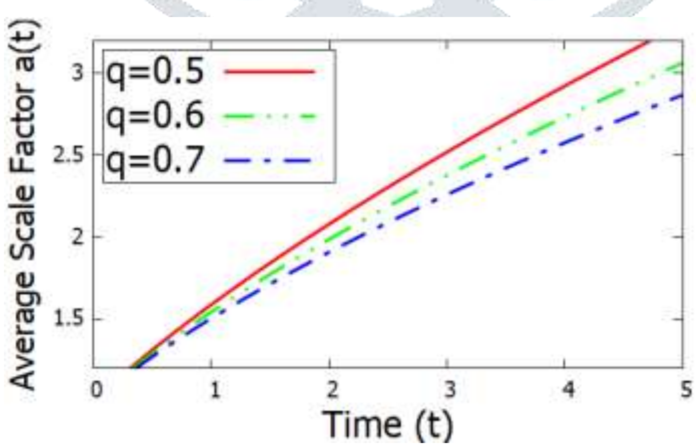

Fig.3 Average scale factor versus time

Also, the expression for the energy density W, energy flow vector S and stress tensor $\sigma_{\alpha\beta}$ are

$$W=\left(\frac{1+\frac{\upsilon^2}{3c^2}}{1-\frac{\upsilon^2}{c^2}}\right)\frac{C_3^2}{8\pi(C_3t+C_4)^2}\left[\frac{9(m+n+mn)}{(q+1)^2(m+n+1)^2}-\frac{1}{4C_3^2(C_3t+C_4)^{\frac{6(m-n+1)}{(q+1)(m+n+1)}-2}}+\frac{\omega C_5^2}{2C_3^2(C_3t+C_4)^{\frac{6}{(q+1)(m+n+1)}-2}}\right] \tag{31}$$

$$S=\left(\frac{4\upsilon}{3\left(1-\frac{\upsilon^2}{c^2}\right)}\right)\frac{C_3^2}{8\pi(C_3t+C_4)^2}\left[\frac{9(m+n+mn)}{(q+1)^2(m+n+1)^2}-\frac{1}{4C_3^2(C_3t+C_4)^{\frac{6(m-n+1)}{(q+1)(m+n+1)}-2}}+\frac{\omega C_5^2}{2C_3^2(C_3t+C_4)^{\frac{6}{(q+1)(m+n+1)}-2}}\right] \tag{32}$$

$$\sigma_{\alpha\beta}=\left(\frac{4\upsilon_\alpha\upsilon_\beta}{c^2\left(1-\frac{\upsilon^2}{c^2}\right)}+\delta_{\alpha\beta}\right)\frac{-3C_3^2}{8\pi(C_3t+C_4)^2}\left[\frac{C_7}{(q+1)^2(m+n+1)^2}-\frac{1}{4C_3^2(C_3t+C_4)^{\frac{6(m-n+1)}{(q+1)(m+n+1)}-2}}-\frac{\omega C_5^2}{6C_3^2(C_3t+C_4)^{\frac{6}{(q+1)(m+n+1)}-2}}\right] \tag{33}$$

If the velocity of the macroscopic motion is small compared with the velocity of the light, then we have approximately $S=(p+\varepsilon)v$. Since $\frac{S}{c^2}$ is the momentum density and $\frac{(p+\varepsilon)}{c^2}$ plays the role the mass density of the body. From the expression (10), we get

$$T_i^j=\varepsilon-3p \tag{34}$$

But, $$T_i^j=\sum_a m_a c^2\sqrt{1-\frac{v_a^2}{c^2}}\,\delta(r-r_0) \tag{35}$$

Compare the relation (34) with the general formula (35) which we saw was valid for an arbitrary system. Since we are at present considering a macroscopic body, the expression (35) must be averaged over all the value of r in unit volume .We obtained the result

$$\varepsilon-3p=\sum_a m_a c^2\sqrt{1-\frac{v_a^2}{c^2}} \tag{36}$$

Here the summation extends over all particles in the unit volume

The right side of this equation tends to zero in the ultra-relativistic limit, so in this the equation of state of matter is $p=\frac{\varepsilon}{3}$.

5. Observational parameters of Macroscopic body model

Jark parameter $J = (2q+1)q$ (37)

Snap parameter $S = -q(2q+1)(3q+2)$ (38)

Lark parameter $L = q(2q+1)(3q+2)(4q+3)$ (39)

Look-back time redshift: Look –back time is defined as the difference between present age of universe t_0 and the age of the universe t when a particular light ray at redshift z was emitted. It depends on the dynamics of the universe.

$$t_L = t_0 - t \quad (40)$$

Where t_0 is present age of universe and z denotes redshift of light well measured quantity of a far distant object such as galaxies. The redshift of light is emitted due to expansion of universe. For given redshift z, the average scale factor of the universe $a(t)$ is related to the present scale factor of the universe $a_0(t)$ by

$$1+z = \frac{a_0(t)}{a(t)} \quad (41)$$

From equation (29), (40) and (41) gives

Look back time : $$t_L = \frac{C_3}{(q+1)H_0}\left[1 - \frac{1}{(1+z)^{\frac{1}{(q+1)}}}\right] \quad (42)$$

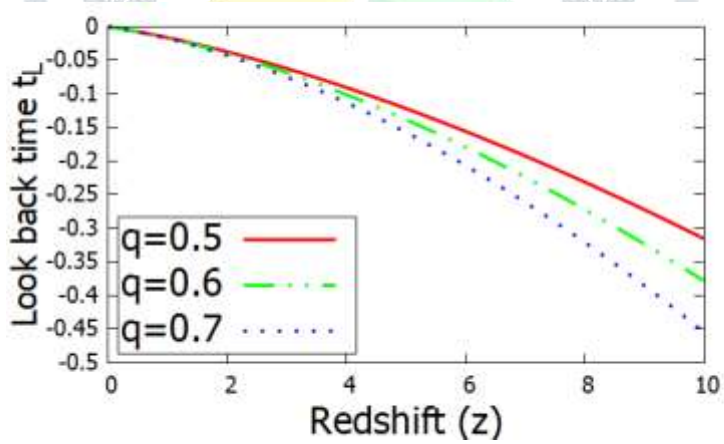

Fig.4 Look back time versus redshift

Luminosity Distance redshift: The Luminosity distance of light source is given by,

$$H_0 d_L = \frac{C_3}{q}\left[1 - (1+z)^{-q}\right](1+z) \quad (43)$$

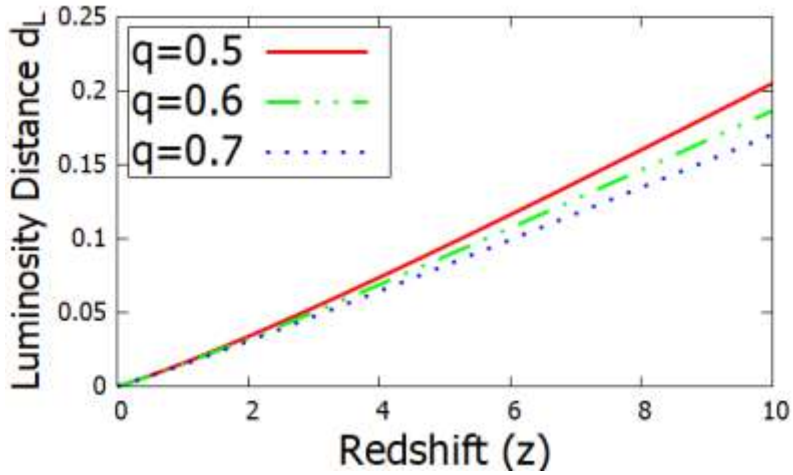

Fig.5 Luminosity distance versus redshift

Above figure 5 shows that the Luminosity distance increase faster with the redshift.

Conclusion

In brief, Energy momentum tensor and space-time associated with them have cosmological interest due to their important applications in structure formation of the universe. Also, it is well known that scalar fields have considerable effects in the early stages of the revolutionary universe. Here we have presented Bianchi type II cosmological models in Saez Ballester theory of gravitation proposed by Saez[1] with a macroscopic body. It is observed that the macroscopic body could exist in the early epoch of the universe by using law of variation of Hubble's parameter which yield constant deceleration parameter and average scale factor.The behavior of physical parameter of Bianchi type II macroscopic body cosmological model is similar to Rao et al [7].Also discussed some physical and kinematical properties and graphical illustration of macroscopic body. The model studied here will be useful for a better understanding of Saez Ballester cosmology and structure formation of the universe.

It may be observed that at initial moment,when t=0, the spatial volume will be zero while energy density and pressure diverge. When t tends to zero, then the expansion scalar , shear scalar and Hubble's parameters tends to infinity. For large value of t, we observe that expansion scalar, shear scalar and Hubble parameters become zero at late time

As we know the motivation behind the scalar tensor theories is to explain the accelerated expansion of universe.

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