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Accelerated expansion of the Universe With Wet Dark Fluid In $f(T)$ Theory of Gravity

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Abstract:

In this paper we have investigated spatially homogeneous Bianchi type-I (LRS) space-time filled with wet dark fluid (WDF), which is a candidate for dark energy in the framework of $f(T)$ gravity. The equation of state model on $p=\omega(\rho-\rho^{*})$ in the form of wet dark fluid for the dark energy component of the universe. Solutions to the corresponding field equations are obtained for power law. The geometrical and physical parameters of the derived model are studied.

Keywords:

Bianchi-I, Teleparallel gravity, WDF, dark energy

1. Introduction:

The $f(T)$ theory is an expansion of teleparallel theory of gravity, where T is torsion scalar. The $f(T)$ theory of gravity uses the weitzenbök connections which has no curvature but only torsion. Here torsion is answerable for accelerated expansion of the universe and is formed using four parallel vector fields. A significant advantage of this theory is that its field equations are second order and hence easy to handle as compare to fourth order equation of $f(R)$ theory. The Universe is undergoing accelerated expansion [1–6] as through an experiment evidenced by current remarks. Dark energy (DE) with a negative pressure is may be well-thought-out as the foremost cause of accelerated expansion behaviour of the universe. In interpretation of the late time acceleration of the Universe and the presence of dark energy and dark matter, numerous modified theories of gravity have been established and considered. The new theories of gravity $f(R)$, $f(R,T)$, $f(G)$ and $f(T)$ are recently developed. $f(T)$ theory has motivating features as it enlightens the current acceleration without linking dark energy. Ferraro & Fiorini [7] examined models based on modified TG to inflation. In $f(T)$ gravity, the Teleparallel Lagrangian density described by the function of torsion scalar T in order to account for the late time cosmic acceleration [8-11]. Jamil and his collaborators [12] explored the Dark Matter (DM) problem in $f(T)$ gravity. In Ref. [13], the authors inspected particle creation in the context of $f(T)$ gravity. $f(T)$ gravity has been broadly considered in the literature by several eminent researchers [14-21]. Motivated by above investigations, the paper deals with the investigations of Wet dark fluid for a linearly varying deceleration parameter of $f(T)$ model by using spatially homogeneous and anisotropic Bianchi type-I (LRS) space-time.

2. Wet Dark Fluid:

Wet dark fluid is a new candidate for dark energy in the spirit of generalized chaplygin gas, where a physically motivated equation of state is offered with the properties relevant for a dark energy problem. The equation of state for a wet dark fluid is $\frac{p_{WDF}}{\omega}+\rho^* = \rho_{WDF}$. The parameter ω and ρ^* are taken to be positive and we restrict ourselves $0 \le \omega \le 1$. Note that if c_s denotes the adiabatic sound speed in wet dark fluid, then $c_s^2 = \partial p/\partial\rho \ge 0$.

The energy conservation equation $\dot{\rho}_{WDF} + 3H\left(p_{WDF}+\rho_{WDF}\right)=0$. Using $3H=\frac{\dot{V}}{V}$, we get $p_{WDF}=\left(\frac{\omega}{1+\omega}\right)\rho^* + \frac{c}{V^{(1+\omega)}}$, where c is the constant of integration and V is the volume expansion. If we take $c>0$, this fluid will not violate the strong energy condition $p+\rho \ge 0$. Thus, we get $p_{WDF}+\rho_{WDF} = (1+\omega)\rho_{WDF} - \omega\rho^* = (1+\omega)\left(\frac{c}{V^{(1+\omega)}}\right) \ge 0$.

The wet dark fluid has been used as dark energy in the homogeneous, isotropic FRW case by Holman and Naidu [22]. Singh and Chaubey [23] examined Bianchi type-I universe filled with dark energy in the form of wet dark fluid . The author in Ref. [24] explored the panoramic scenario of wet dark fluid. Several Relativists [25-37] considered cosmological models with WDF in General Relativity and theories of gravitations.

3. $f(T)$ gravity , field equations and some physical quantities

We define the action by generalizing the TG i.e. $f(T)$ theory as

$$S=\int [T+f(T)+L_{matter}]e\, d^4x \quad . \tag{1}$$

Here, $f(T)$ denotes an algebraic function of the torsion scalar T. Making the functional variation of the action (1) with respect to the tetrads, we get the following equations of motion

$$S_{\mu}^{\nu\rho}\partial_{\rho}Tf_{TT} + \left[e^{-1}e_{\mu}^{i}\partial_{\rho}\left(ee_{i}^{\alpha}S_{\alpha}^{\nu\rho}\right)+T^{\alpha}{}_{\lambda\mu}S_{\alpha}^{\nu\lambda}\right](1+f_T)+\frac{1}{4}\delta_{\mu}^{\nu}(T+f)=4\pi(T_{\mu}^{\nu}+\bar{T}_{\mu}^{\nu}), \tag{2}$$

where T_μ^ν is the energy momentum tensor, $f_T = df(T)/dT$ and by setting $f(T) = a_0 =$constant this is dynamically equivalent to the GR. We consider spatially homogeneous and anisotropic Bianchi type-I (LRS) space-time of the form

$$ds^2 = dt^2 - A^2(t)dx^2 - B^2(t)\left[dy^2 + dz^2\right], \quad (3)$$

where A and B be the metric potential which is the functions of cosmic time t only.

The corresponding Torsion scalar is given by $T = -2\left(2\frac{\dot{A}}{A}\frac{\dot{B}}{B} + \frac{\dot{B}^2}{B^2}\right)$. Let us consider that the matter content is dark energy in the form of wet dark fluid such that the energy momentum tensor T_μ^ν given by $T_\mu^\nu = (\rho_{wdf} + p_{wdf})\, u^\nu u_\mu - p_{wdf}\, g_\mu^\nu$, together with commoving coordinates

$u^\nu = (0,0,0,1)$ and $u^\nu u_\nu = 1$, where u^ν is the four-velocity vector of the fluid, p and ρ be the pressure and energy density of the fluid respectively. The field equations can be written as

$$(T + f) + 4(1 + f_T)\left\{\frac{\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} + \frac{\dot{A}}{A}\frac{\dot{B}}{B}\right\} + 4\frac{\dot{B}}{B}\dot{T}\, f_{TT} = k^2(-p_{wdf}), \quad (4)$$

$$(T + f) + 2(1 + f_T)\left\{\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} + 3\frac{\dot{A}}{A}\frac{\dot{B}}{B}\right\} + 2\left\{\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\right\}\dot{T}\, f_{TT} = k^2(-p_{wdf}), \quad (5)$$

$$(T + f) + 4(1 + f_T)\left\{\frac{\dot{B}^2}{B^2} + 2\frac{\dot{A}}{A}\frac{\dot{B}}{B}\right\} = k^2(\rho_{wdf}). \quad (6)$$

where the dot $(\cdot)$ denotes the derivative with respect to time t. Here we have three differential equations with five unknowns namely $A, B, f, p_{wdf}, \rho_{wdf}$. The relation between average scale factor and volume is given by $V = a^3 = \sqrt{AB^2}$. The deceleration parameter is $q = -1 + \frac{d}{dt}\left(\frac{1}{H}\right)$, for $-1 \le q < 0$, $q > 0$ and $q = 0$ the universe exhibit accelerating volumetric expansion, decelerating volumetric expansion and expansion with constant-rate respectively. The mean Hubble parameter, which expresses the volumetric expansion rate of the universe, given as

$H = \frac{1}{3}(H_1 + H_2 + H_3)$, where H_1, H_2, H_3 are the directional Hubble parameter in the direction of x, y and z-axis respectively. Thus , we have $H = \frac{1}{3}\frac{\dot{V}}{V} = \frac{1}{3}(H_1 + H_2 + H_3) = \frac{\dot{a}}{a}$. We define an anisotropy parameter , expansion scalar and shear scalar as $A_m = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)^2$,

$$\theta = u^\mu_{;\mu} = \frac{\dot{A}}{A} + 2\frac{\dot{B}}{B}, \sigma^2 = \frac{3}{2}H^2 A_m.$$

4. Law of Variation for Hubble's parameter and solution of field equations:

Akarsu and Dereliwe [38] proposed a linearly varying deceleration parameter of the form $q=-\frac{a\ddot{a}}{\dot{a}^2}=-kt+m-1$, (7)

where $k\geq 0$ and $m\geq 0$ are constant. Solving (7) one obtains different form of solutions for the scale factor:

$$a=a_1(mt+m_1)^{\frac{1}{m}}, \quad \text{for} \quad k=0 \text{ and } m>0, \tag{8}$$

$$a=a_2e^{m_2t}, \quad \text{for} \quad k=0 \text{ and } m=0, \tag{9}$$

where a_1, a_2, m_1, m_2 are the constants of integration.

Using equations (4) and (5) , we get $\frac{d}{dt}\left(\frac{\dot{A}}{A}-\frac{\dot{B}}{B}\right)+\left(\frac{\dot{A}}{A}-\frac{\dot{B}}{B}\right)\frac{\dot{V}}{V}=0$, which on integration gives

$$\frac{A}{B}=k_2\exp\left[k_1\int\frac{dt}{V}\right], \tag{10}$$

where k_1 and k_2 are constants of integration. In view of $V=AB^2=a^3$, we write A and B in the explicit form as

$$A=D_1V^{\frac{1}{3}}\exp\left(\chi_1\int\frac{1}{V}dt\right), \tag{11}$$

$$B=D_2V^{\frac{1}{3}}\exp\left(\chi_2\int\frac{1}{V}dt\right), \tag{12}$$

where $D_i(i=1,2)$ and $\chi_i(i=1,2)$ satisfy the relation $D_1D_2^2=1$ and $\chi_1+2\chi_2=0$.

5. Model for $k=0$ and $m>0$

We get the following expressions for metric potentials:

$$A=D_1a_1(mt+m_1)^{\frac{1}{m}}\exp\left(\frac{\chi_1}{a_1^3}\int\frac{1}{(mt+m_1)^{\frac{3}{m}}}dt\right), \tag{13}$$

$$B=D_2a_1(mt+m_1)^{\frac{1}{m}}\exp\left(\frac{\chi_2}{a_1^3}\int\frac{1}{(mt+m_1)^{\frac{3}{m}}}dt\right), \tag{14}$$

Using metric potentials, the model can be written as

$$ds^2=dt^2-(D_1a_1)^2(mt+m_1)^{\frac{2}{m}}\exp\left(\frac{2\chi_1(mt+m_1)^{\frac{m-3}{m}}}{ma_1(m-3)}\right)dx^2 -(D_2a_1)^2(mt+m_1)^{\frac{2}{m}}\exp\left(\frac{2\chi_2(mt+m_1)^{\frac{m-3}{m}}}{ma_1(m-3)}\right)\left[dy^2+dz^2\right] \tag{15}$$

We obtain the expressions for pressure and energy density as

$$\rho_{WDF} = \left(\frac{\omega}{1+\omega}\right)\rho^{*} + \frac{c}{a_1^3\left(mt+m_1\right)^{\frac{3(1+\omega)}{m}}}, \quad (16)$$

and

$$p_{WDF} = \frac{c\omega}{a_1^3\left(mt+m_1\right)^{\frac{3(1+\omega)}{m}}} - \left(\frac{\omega}{1+\omega}\right)\rho^{*}. \quad (17)$$

The scalar expansion, shear scalar, the average anisotropy parameter and deceleration parameter are obtained as

$$\theta = \frac{3}{(mt+m_1)}, \sigma^2 = \frac{\left(\chi_1^2 + 2\chi_2^2\right)}{18a_1^2}(mt+m_1)^{\frac{2}{3}(m-6)}, \ A_m = \frac{\left(\chi_1^2 + 2\chi_2^2\right)}{27a_1^2}(mt+m_1)^{\frac{2}{3}(m-3)}.$$

The average generalized Hubble's parameter and deceleration parameter are given by

$$H = \frac{1}{(mt+m_1)}$$

$$q = -1+m$$

6. Discussion and Concluding Remarks:

In this paper we have obtained some exact Bianchi type-I space-time in $f(T)$ theory of gravitation with wet dark fluid. It can be seen that the spatial volume is constant at $t=0$ if $m_1 > 0$ and zero if $m_1 = 0$ and expands with cosmic time. In derived model, the energy density and pressure tend to infinity at $t=0$. The energy density and pressure become zero as $t \to \infty$ [39-40]. The model has the point-type singularity at $t=0$. The rate of the expansion, the mean anisotropy parameter for $m<3$ and shear scalar all diverse at $t=0$. As $t \to \infty$, the scale factors A and B tend to infinity. The expansion scalar, mean anisotropy parameter and shear scalar all tend to zero as $t \to \infty$ (for $m<3$). The isotropy condition $\frac{\sigma^2}{\theta^2} \to \text{constant}$ satisfied at infinite expansion, this indicates that after a large time the expansion will stop completely and approaches to isotropy in singular model. The model represents an accelerated universe. Therefore, the model is consistent with the cosmological observations [41-42].

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