



# FIFTH ORDER DIFFERENCE SQUEEZING OF THE FIELD AMPLITUDE IN C-MODE UNDER DEGENERATE SIX-WAVE INTERACTION PROCESS

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**Abstract :** We studied fifth order difference squeezing of the field amplitude in three-photon absorption in C-mode (Signal Mode) degenerate six-wave interaction process is investigated. It is established that the Fifth Order squeezing of the fundamental mode directly convert into the normal squeezing of the signal mode. Detection of difference of the fields and fifth order squeezing in this process is also studied. Squeezing is found to be greater i.e. lowers the depth of classicality of the field amplitude in stimulated process than the corresponding squeezing in spontaneous interaction in signal mode of the process

**Keywords:** Squeezed States, Six-Wave Interaction Process, Difference Squeezing, Photon Number Operator, fifth order Squeezing

## I. INTRODUCTION

Squeezed light [1] is a purely quantum mechanical phenomenon [2]. Over the past years it has attracted considerable attention due to its low quantum fluctuation in quantum state [3, 4]. It has been focused on theoretical as well as experimental interpretation in several nonlinear optical processes, such as parametric amplification [5, 6], harmonic generation [7-9], multi-photon processes [10-13], Raman process [14], hyper-Raman process [15]. Hong and Mandel [16] and Hillery [17] have introduced the notion of higher-order squeezing of quantized electromagnetic field. Garcia Fernandez et al [18] have worked on higher-order squeezing in single mode multiphoton absorption process. Higher-order squeezing has also been studied in some nonlinear optical processes [19-20]. Further, sum and difference squeezing (higher-order squeezing) were proposed by Hillery [21] for the two modes and these concepts have been generalized in three modes [22] as well as an arbitrary number of modes [23-25]. In a recent publication [26, 27] we have investigated squeezing in the fundamental mode in six-wave mixing process. Further recently Prakash et al.[28], Arjun Mukherjee et al. [29] and Kaushik Mukherjee et al.[30] have reported an another type of higher -order squeezing like sum and difference squeezing in different nonlinear optical process.

The aim of this paper is to study one of the types of fifth order (higher-order) squeezing i.e. difference squeezing of the amplitude in three-photon absorption in degenerate six-wave interaction process. The paper is organized as follows: Section 1 gives the definition of higher-order squeezing in Signal mode (C-mode). We establish the analytic expression of difference squeezing in degenerate six-wave difference-frequency generation in section 2. Section 3 incorporates results and discussion. Finally, we conclude this paper in section 4.

## 2. DEFINITION OF FIFTH-ORDER SQUEEZING

Higher-order squeezing is a special class of minimum uncertainty states. It is the higher powers of the field amplitude [17], which are characterized by reduced quantum noise in one quadrature of the field at the expense of the increased noise in the other one.

### 2.1 Fifth order squeezing of single mode

For a single mode of the electromagnetic field with frequency  $\omega$  and creation (annihilation) operators  $a^\dagger$  ( $a$ ), the amplitude-cubed squeezing may be characterized by its real and imaginary parts as

$$Z_1 = (1/2)(A^5 + A^{\dagger 5}) \tag{1}$$

and  $Z_2 = (1/2i)(A^5 - A^{\dagger 5})$  (2)

where  $A$  and  $A^\dagger$  are the gradually varying operators., they are given by

$$A = a \exp(i\omega t) \text{ and } A^\dagger = a^\dagger \exp(-i\omega t) \tag{3}$$

Equations (1) and (2) obey the commutation relation

$$[Z_1, Z_2] = \frac{i}{2} (25N_A^2 + 25N_A + 10) \tag{4}$$

where  $A^\dagger A = N_A$  is the number operator.

The equation (4) leads to the uncertainty relation ( $\hbar = 1$ )

$$\Delta Z_1 \Delta Z_2 \geq \frac{1}{4} \langle 25N_A^2 + 25N_A + 10 \rangle \tag{5}$$

where  $\Delta Z_1$  and  $\Delta Z_2$  are the uncertainties in the quadrature.

Amplitude-cubed squeezed state in  $Z_i$  exists if

$$(\Delta Z_i)^2 < \frac{1}{4} \langle 20N_A^2 + 25N_A + 10 \rangle \quad \text{where } i = 1 \text{ or } 2 \tag{6}$$

These states have purely quantum mechanical nature. Expressing the variance  $(\Delta Z_i)^2$  classically in the P-representation of the state one can show

$$(\Delta Z_i)^2 = \frac{1}{4} \langle 25N_A^2 + 25N_A + 10 \rangle + (1/4) \int d^2\alpha P(\alpha) [ \exp(-5i\omega t)\alpha^{*5} + \exp(5i\omega t)\alpha^5 - \langle A^5 + A^{\dagger 5} \rangle ]^2 \tag{7}$$

where  $P(\alpha)$  is the coherent-state quasi-probability function. A classical state, whose P-representation is non-negative definite, satisfy the relation from equation (7) as

$$(\Delta Z_i)^2 \geq \frac{1}{4} \langle 25N_A^2 + 25N_A + 10 \rangle \tag{8}$$

This means that a state which satisfies equation (6) exhibits non-classical features. A coherent state is that for which the variances of field quadratures satisfy the equation

$$(\Delta Z_i)^2 = \frac{1}{4} \langle 25N_A^2 + 25N_A + 10 \rangle \tag{9}$$

### 3. Difference squeezing effect in C-mode under degenerate six-wave interaction process

From figure 1, the corresponding Hamiltonian is

$$H = \omega_1 a^\dagger a + \omega_2 b^\dagger b + \omega_3 c^\dagger c + g (a^3 b^{\dagger 2} c^\dagger + a^{\dagger 3} b^2 c) \tag{10}$$

where  $a^\dagger$  ( $a$ ),  $b^\dagger$  ( $b$ ) and  $c^\dagger$  ( $c$ ) are the creation (annihilation) operators,  $g$  is the coupling constant per second and  $A = a \exp(i\omega_1 t)$ ,  $B = b \exp(i\omega_2 t)$  and  $C = c \exp(i\omega_3 t)$  with the relation  $3\omega_1 = 2\omega_2 + \omega_3$

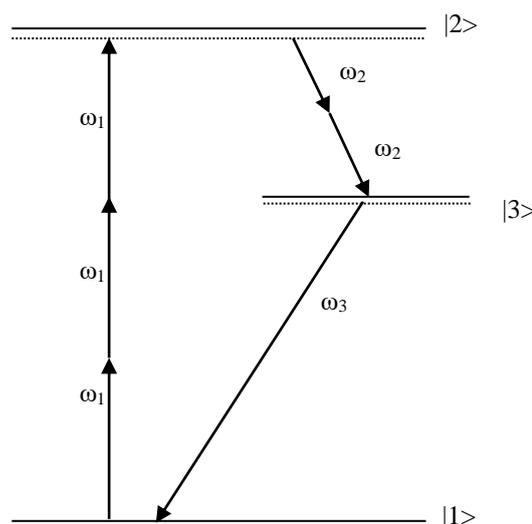


Figure 1: Degenerate six-wave interaction model

Using equation (16) in coupled Heisenberg equation of motion

$$\dot{A} = \frac{\partial A}{\partial t} + i [H, A] \quad (\hbar = 1) \quad (11)$$

we obtain  $\dot{A} = -3igA^{\dagger 2}B^2C$  (12)

Similarly  $\dot{B} = -2igA^3B^{\dagger}C^{\dagger}$  (13)

and  $\dot{C} = -igA^3B^{\dagger 2}$  (14)

In this section we investigate the dependence of squeezing in the C- mode on the difference squeezing of the fundamental mode and stokes mode under

Using equations (12) and (13) equation (14) we obtain

$$\ddot{C} = -|g|^2 \left[ 9N_A^2 N_B^2 + 18N_A N_B^2 - 2N_A^3 + 6N_B^2 - 4N_A^3 N_B \right] C \quad (15)$$

In the interaction Hamiltonian the coupling constant  $g$  is in general complex. However we have used  $|g|^2$  in place of  $g^2$  as we are not considering the phase terms.

Using Taylor's expansion with short- interaction time ( $\approx 10^{-10}$ sec) and keep terms up to second-order in ' $gt$ ' ( $gt \ll 1$ ), we get

We again study the squeezing effect in C-mode and assume a constant stokes mode so that the change in the B mode is negligible. We associate a constant term  $m$  for B and  $B^{\dagger}$  in the signal mode.

Equation (20) gives

$$\dot{C} = -igA^3m^2 \quad (16)$$

then  $\ddot{C} = -3|g|^2 m^4 (5N_A^2 + 5N_A + 2)C = -|g|^2 m^4 (25N_A^2 + 25N_A + 10)C$  (17)

Hence, the corresponding results in the amplitude signal mode are

$$C(t) = C - i|g|tA^3 m^2 - \left( \frac{|g|^2 t^2 m^4}{2} \right) (25N_A^2 + 25N_A + 10)C \quad (18)$$

and  $C^{\dagger}(t) = C^{\dagger} + i|g|tA^{\dagger 3} m^2 - \left( \frac{|g|^2 t^2 m^4}{2} \right) (25N_A^2 + 25N_A + 10)C^{\dagger}$  (19)

We define the amplitude quadrature components in the signal mode as

$$X_{1C}(t) = (1/2) [C(t) + C^{\dagger}(t)] \quad (20)$$

$$\text{and } X_{2C}(t) = (1/2i) [C(t) - C^{\dagger}(t)] \quad (21)$$

Use of equations (38) and (39) in equations (40) and (41), we get

$$X_{1C}(t) = X_{1C} + |g| m^2 t Z_{2A} - \left( \frac{|g|^2 t^2 m^4}{2} \right) (25N_A^2 + 25N_A + 10)X_{1C} \quad (22)$$

and  $X_{2C}(t) = X_{2C} - |g| m^2 t Z_{1A} - \left( \frac{|g|^2 t^2 m^4}{2} \right) (25N_A^2 + 25N_A + 10)X_{2C}$  (23)

For uncorrelated modes at  $t = 0$ , we get

$$[\Delta X_{1C}(t)]^2 = (\Delta X_{1C})^2 + |g|^2 m^4 t^2 [(\Delta Z_{2A})^2 - \langle 25N_A^2 + 25N_A + 10 \rangle] (\Delta X_{1C})^2 \quad (24)$$

$$\text{and } [\Delta X_{2C}(t)]^2 = (\Delta X_{2C})^2 + |g|^2 m^4 t^2 [(\Delta Z_{1A})^2 - \langle 25N_A^2 + 25N_A + 10 \rangle] (\Delta X_{2C})^2 \quad (25)$$

If C mode is initially in a coherent state, then

$$(\Delta X_{1C})^2 = (\Delta X_{2C})^2 = 1/4 \quad (26)$$

We obtain

$$[\Delta X_{1C}(t)]^2 - 1/4 = |g|^2 m^4 t^2 [(\Delta Z_{2A})^2 - \frac{1}{4} \langle 25N_A^2 + 25N_A + 10 \rangle] \quad (27)$$

and  $[\Delta X_{2C}(t)]^2 - 1/4 = |g|^2 m^4 t^2 [(\Delta Z_{1A})^2 - \frac{1}{4} \langle 25N_A^2 + 25N_A + 10 \rangle]$  (28)

Equations (27) and (28) show that the signal mode is squeezed in the  $X_{1C}$  direction if the A mode is fifth -order squeezing in the  $Z_{2A}$  direction and the signal mode is squeezed in the  $X_{2C}$  direction if the A mode is amplitude cubed squeezing in the  $Z_{1A}$  direction. That is, if a fundamental mode with fifth -order squeezing propagates through a nonlinear medium a squeezed signal mode (normal squeezing) is generated. The result suggests a method for detection of fifth order squeezing in degenerate six-wave interaction process.

In earlier publication [26], the results of third-order squeezing in fundamental mode in the spontaneous and stimulated interaction under the short-time scale in six-wave mixing process are quoted below

$$[(\Delta Z_{1A})^2 - \frac{1}{4} \langle 25N_A^2 + 25N_A + 10 \rangle] = -27 |g|^2 t^2 \left( |\alpha|^8 + 2|\alpha|^6 \right) \cos 6\theta \tag{29}$$

and  $[(\Delta Z_{1A})^2 - \frac{1}{4} \langle 25N_A^2 + 25N_A + 10 \rangle] = -\frac{27}{2} |g|^2 t^2 (|\beta|^4 + 4|\beta|^2 + 2) \left( |\alpha|^8 + 2|\alpha|^6 \right) \cos 6\theta \tag{30}$

where  $|\alpha|^2 = \langle A^\dagger A \rangle$ ,  $|\beta|^2 = \langle B^\dagger B \rangle$  and  $\theta$  is the phase angle.

Using equations (29) and (30) in equation (28), we obtain respectively as

$$[\Delta X_{2c}(t)]^2 - 1/4 = -27 |g|^4 t^4 m^4 \left( |\alpha|^8 + 2|\alpha|^6 \right) \cos 6\theta \tag{31}$$

$$\text{and } [\Delta X_{2c}(t)]^2 - 1/4 = -\frac{27}{2} |g|^4 t^4 m^4 (|\beta|^4 + 4|\beta|^2 + 2) \left( |\alpha|^8 + 2|\alpha|^6 \right) \cos 6\theta \tag{32}$$

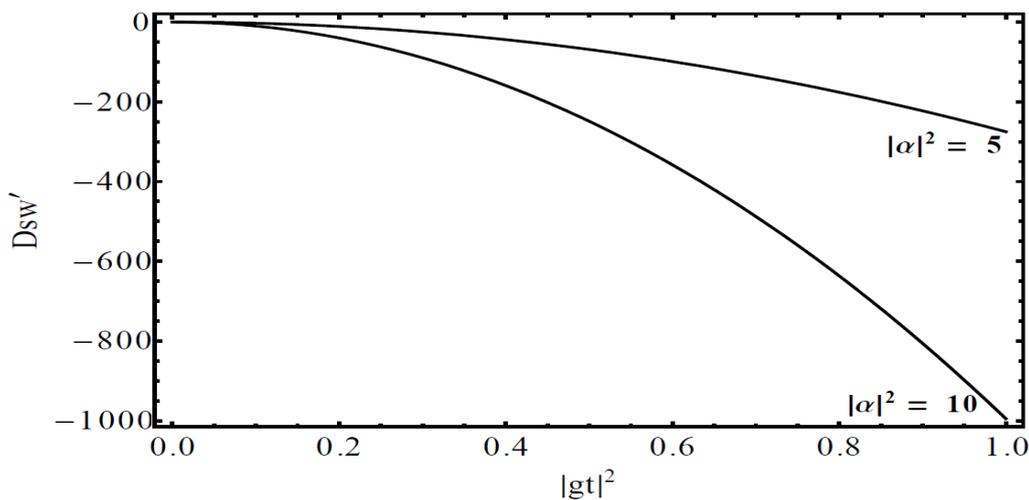
The nonlinear factor  $(|\beta|^4 + 4|\beta|^2 + 2)$  in equation (32) is due to the effect of stimulated interaction. This proves the existence of squeezing in the signal mode. The results also tell that in stimulated process the squeezing is greater than the corresponding squeezing in spontaneous process, It is observed that maximum squeezing will occur when  $\theta \rightarrow 0$  and minimum when  $\theta \rightarrow \pi/6$ .

An analysis of equations (49-52) shows that

if  $g^2 t^2 m^4 > 1$ , squeezing is greater in signal mode compared to fundamental mode.  
and, if  $g^2 t^2 m^4 < 1$ , corresponding squeezing is larger in fundamental mode.

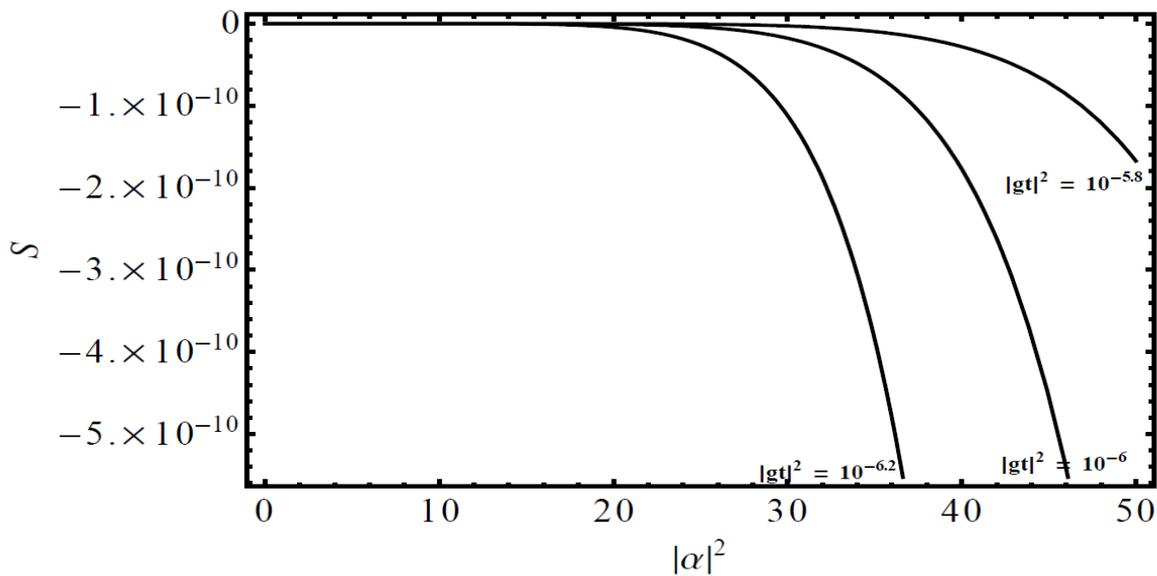
**4. Results and discussion**

To study higher-order squeezing, we denote the right hand side of equation (27) or (28) by  $D_{sw}'$  and plots with  $|gt|^2$  as shown in figure 3.



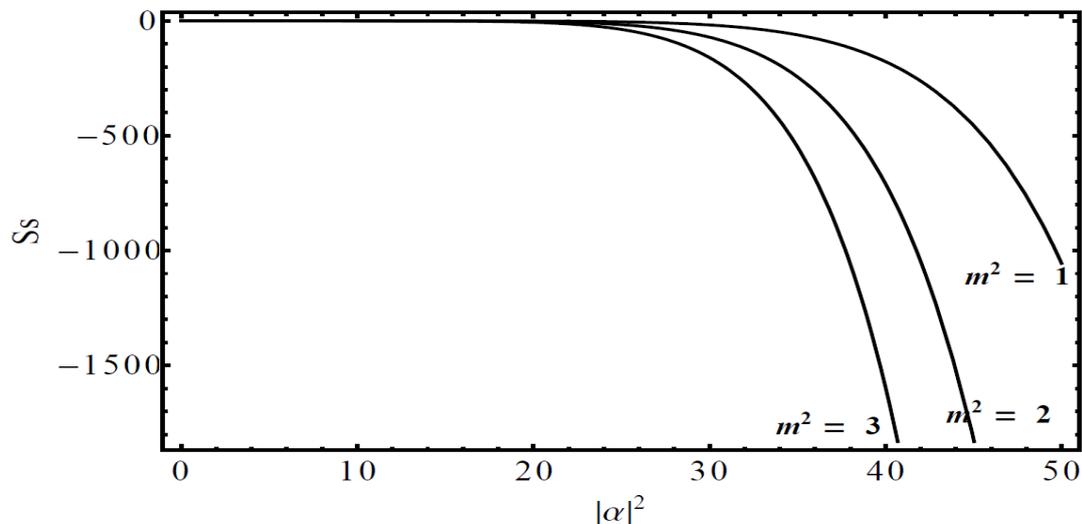
**Figure 1: Variation of the squeezing  $D_{sw}'$  with  $|gt|^2$  (when  $m^2 = 4 = \text{Constant}$ ) in degenerate six-wave interaction process**

The steady decrease of the curve infers that the difference squeezing response nonlinearly to the number of pump photons. It shows that squeezing increases with the increase of pump photon number.

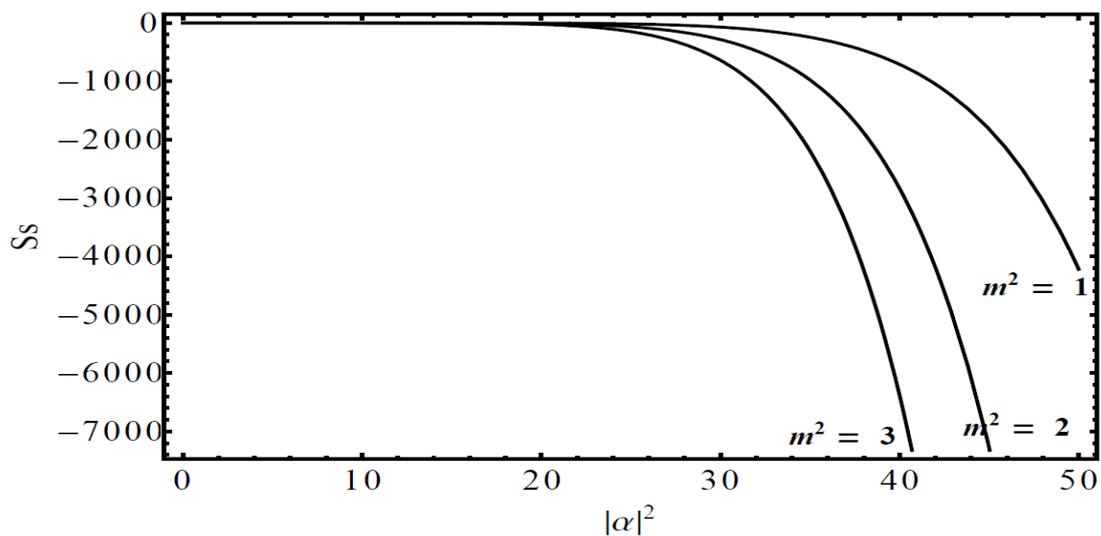


**Figure 2: Variation of the squeezing S with  $|\alpha|^2$  ( $\theta = 0$ ,  $|\beta|^2=0$  &  $m^2=1$ ) in degenerate six-wave interaction process** Comparing equations (29) and (31) and plot a graph between squeezing S and  $|\alpha|^2$  with different values of  $|gt|^2$  as shown in figure 2.

We denote the right hand side of equations (31) and (32) respectively by  $S_s$  and  $S'_s$ , and plot the curve with  $|\alpha|^2$  as shown in figures 5 and 6 for arbitrary constant values of  $m^2$ .



**Figure 3: Variation of the squeezing  $S_s$  in signal mode with  $|\alpha|^2$  ( $|\beta|^2=0$ ) in spontaneous degenerate six-wave interaction process (when  $|gt|^4 = 10^{-12}$  and  $\theta = 0$ )**



**Figure 4: Variation of the squeezing  $S'_s$  in signal mode with  $|\alpha|^2$  ( $|\beta|^2=4$ ) in stimulated degenerate six-wave interaction process (when  $|gt|^4 = 10^{-12}$  and  $\theta = 0$ )**

The steady fall of the curve (figures 3 and 4) show that the squeezing increases nonlinearly with  $|\alpha|^2$  which is directly dependent upon the number of photons. Further, in stimulated process, we observe that when higher the value of  $|\beta|^2$ , then the squeezing increases and it lowers the depth of classicality of the field amplitude. It shows that the degree of squeezing in signal mode

depends directly upon the photon number of the fundamental mode as well as of the stokes mode. It also confirmed that squeezing is more in stimulated interaction than the corresponding squeezing in spontaneous interaction, having the same number of photons.

#### 4. CONCLUSIONS

In this paper, It is found that when a fundamental mode with fifth order propagates through a nonlinear medium a squeezed signal mode (normal squeezing) will result. The nonlinear interaction (signal mode) converts higher-order squeezing into normal squeezing. It also suggests a method for detection of fifth -order squeezing in degenerate six-wave difference frequency generation.

The squeezing obtained in the present paper in degenerate signal mode is found to be greater than the corresponding squeezing in the fundamental mode [26]. Further, in stimulated process, we observed that when higher the value of  $|\beta|^2$ , then the squeezing increases and it lowers the depth of classicality of the field amplitude. It is observed that the existence of nonlinear multiplication factor  $(|\beta|^4 + 4|\beta|^2 + 2)$ , tells that squeezing in stimulated process is greater than corresponding squeezing in spontaneous process. It also confirmed that squeezing is more in stimulated interaction than the corresponding squeezing in spontaneous interaction, having the same number of photons. It shows that the degree of squeezing in signal mode depends directly upon the photon number of the fundamental mode as well as of the stokes mode. It is found that maximum squeezing will occur when  $\theta \rightarrow 0$  and minimum when  $\theta \rightarrow \pi/6$ .

The present paper establishes the fact that the process with higher-order non-linearity is more suitable for generation of squeezed light. These findings suggest and may help in selecting suitable process to generate optimum squeezing of the radiation and further can be useful as a resource to improve high quality optical telecommunication.

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