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Applications of Graph Theory in Geosciences, Economics and Image Processing

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Abstract:

The field of mathematics serves a critical role across various domains, with Graph Theory emerging as a particularly significant area. Graphs are highly regarded as effective modeling tools, capable of representing diverse types of relationships in any given situation. This paper offers an overview of Graph Theory's applications across heterogeneous fields, with a primary emphasis on its utilization in Geosciences, particularly within quantitative geography and landscape ecology. Additionally, it explores how Graph Theory contributes to Image Processing by facilitating the segmentation of digital images into distinct regions, thereby aiding in image analysis. Furthermore, Graph Theory plays a crucial role in economics, where it helps in comprehensively understanding complex systems and their interdependencies. Through the examination of various papers on these topics, this paper provides a comprehensive overview of Graph Theory's applications in these diverse fields.

Key Words: Graphs, image processing, cut-vertex, bi-partite graph, Connectivity, Constraints, Walk.

I. Introduction:

Graph Theory is abranch of Discrete Mathematics. It is becoming significant tool applied widely in the numerous research areas of Maths, Science, Research and Technology. Graph Theory is the study of graphs which are Mathematical structures used to model pair wise relations between objects. There is wide use of graphs in providing problem solving techniques, because it gives an intuitive manner prior to presenting formal definition.

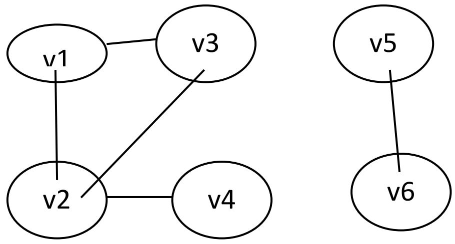
Graph theoretical ideas are highly utilized by computer science applications. Especially in research areas of computer science such data mining, image segmentation, clustering, image capturing, networking genomics etc., for example a data structure can be designed in the form of tree which in turn utilized vertices and edges. Similarly, modelling of network topologies can be done using graph concepts. It is also considered as a most powerful tool in order to explain the algorithms based on image processing, and the theoretical result of graph theory helps to analyse the methods [1]. In order to segment the image based representation such as graph, the image is represented as vertex labelling and graph cut. It categorizes the image elements mathematically through a structure that is well defined with the formulation for creating an image segmentation problem and the computation is found to be more effective.

Economic policy development is crucial in shaping nation’s growth, stability and prosperity. Traditionally, economic analysis has primarily focussed on the behaviour of individual economic agents and macroeconomic aggregates, often overlooking the intricate web of interdependencies. Economic systems are inherently interconnected with entities and sectors influencing one another through diverse channels.This Leads to The Development of NewAlgorithms And New Theorems That Can Be Used In Tremendous Applications.

II. GRAPH THEORY

Graph theory is the study of *graphs*, which are mathematical structures used to model pairwise relations between objects.

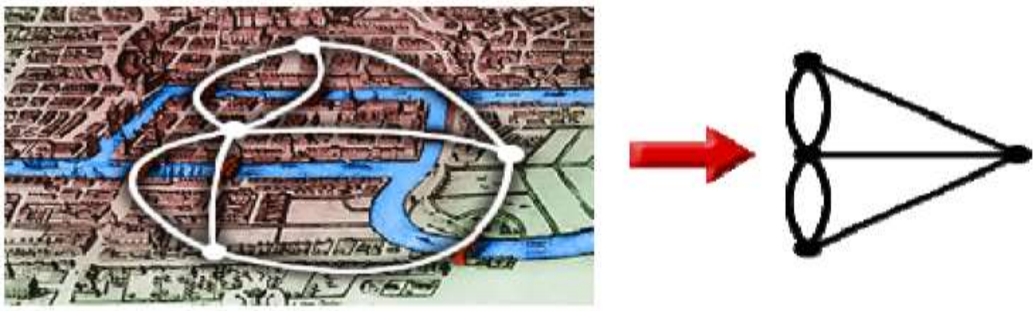
A graph G can be represented as a plane figure bydrawing a line (or a curve) between the points u and$v$ (representing vertices) if $e = uv$is an edge of G.The figure on the right is a geometric representationof the graph G with $VG = \{v1, v2, v3, v4, v5, v6\}$ and
$EG = \{v1v2, v1v3, v2v3, v2v4, v5v6\}$.



Depending on the specific context, limitations are placed on the types of edges permissible. In certain scenarios, directed edges are utilized, while in others, undirected edges connect vertices. Directed edges indicate a one-way relationship or flow of information, while undirected edges represent symmetric or bidirectional connections between vertices. These distinctions are crucial for accurately modeling various problems and understanding the underlying relationships within a graph or network.

III. History of Graph Theory

This story involves mathematician Leonhard Euler contemplating the challenge of traversing Königsberg's seven bridges exactly once. Despite attempting various routes, Euler discerned the impossibility due to unavoidable revisits. Leveraging his solution to the Drawing Graph problem, Euler converted Königsberg's map into a graph for analysis. Every island is represented by a vertex, and every bridge connecting two islands is represented by an edge connecting the two points.



A.F Mobius gave the idea of CompleteGraph and Bipartite Graph and Kuratowski proved that they are planar by means of recreational problems. The concept of Tree, (A Connected Graph Without Cycles) was implemented by Gustav Kirchhoff in 1845, and he employed Graph Theoretical Ideas in the calculation of currents in Electrical Networks or Circuits. In 1852, Thomas Gutherie found the famous Four Color Problem. Then In 1856, Thomas. P. Kirkman and William R.Hamilton studied Cycles on Polyhydra and invented the concept called Hamiltonian Graph by studying trips that visited certain sites exactly Once. In 1913, H.Dudeney mentioned a puzzle problem.Even though The Four Color Problem was invented it was solved only after a century by Kenneth Appeland Wolfgang Haken. This time is considered as the birth of Graph Theory.

IV.APPLICATION OF GRAPH THEORY IN IMAGE PROCESSING

Graph theory is considered as a most powerful tool in order to explain the algorithms based on image processing, and the theoretical result of graph theory helps to analyze the methods.Image Analysis is the methodology by which information from images is extracted. Image analysis is mainly performed by digital image processing techniques. The image processing techniques can be improved using a graph theoretic approach. The applications of graphs in image processing are:

- To calculate the alignment of the picture as well as find edge boundaries.
- Finding mathematical constraints such as entropy by using minimum spanning tree.

- Finding distance transforms of the pixels and calculates the distance between the interior pixels by using shortest path algorithms.

Image segmentation serves as a significant topic within the realm of image processing. Below, I've provided an overview of how graph theory is employed in image segmentation.

Image segmentation is a process of subdividing a digital image into its synthesized regions or objects which are useful for image analysis and has a wide variety of applications in security, forensic, medical and so on.Segmentation subdivides an image into its constitute objects. The level of subdivision depends on the type of problem solved.

Graph Based Segmentation involves the application of a graph theory to construct a representation of an image in the form of a graph. In this approach, each image pixel is represented as a node, while the edges connecting the nodes represent the degree of similarity between the corresponding pixels.

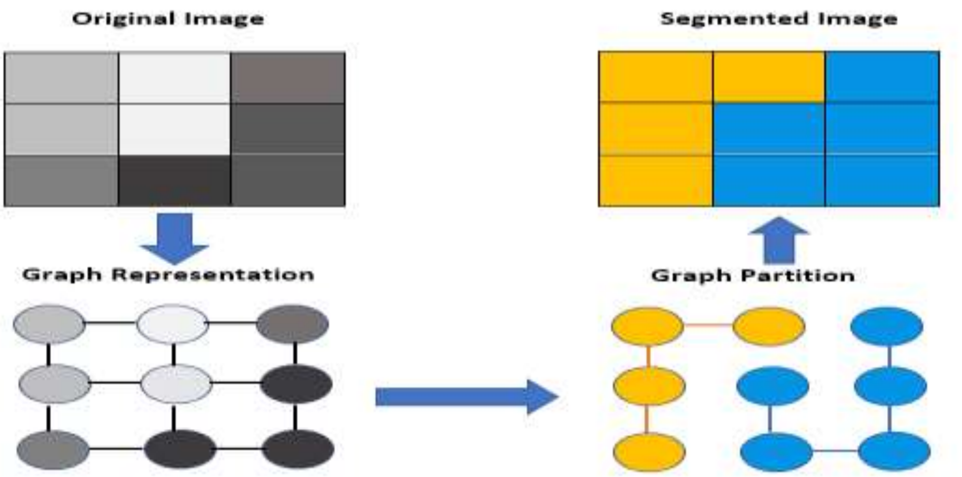
The primary aim of Graph Based Segmentation is to partition an image into distinct regions, with each region representing a segment within the image.Additionally, GBS utilizes graph partitioning algorithms with the objective of minimizing the expense associated with segmenting the image by reducing the cumulative weight of the edges that require cutting.

GBS algorithms follow a set of standard steps for segmenting images

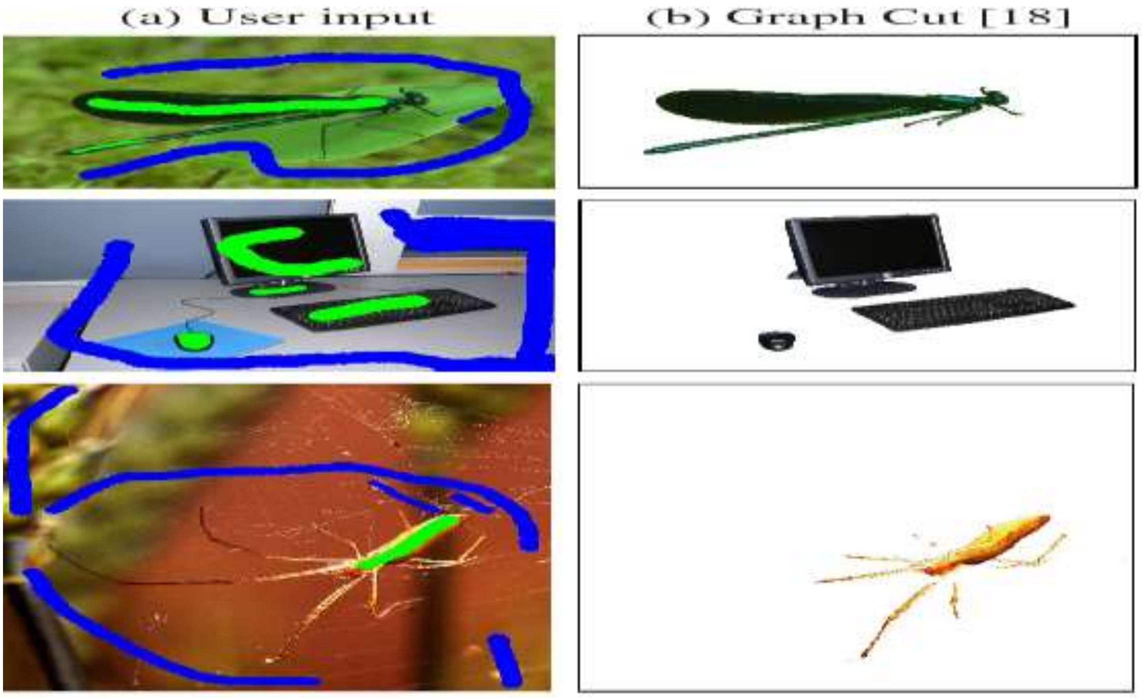
In the initial stage, a graph is constructed to represent the image, with each pixel acting as a node. The weights of the edges connecting the nodes are determined by factors such as the similarity or dissimilarity between the pixels, such as disparities in color or intensity values. Further , the graph is partitioned into distinct regions using a graph partitioning algorithm. The primary objective of this algorithm is to minimize the weights of the edges linking the segments.

Finally, the segments undergo refinement through processes like merging or splitting, guided by various criteria such as size, shape, or texture.

The subsequent figure illustrates the sequential steps of GBS and illustrates how an image undergoes transformation into a segmented image.



Displayed below are several examples of input images alongside their corresponding segmented images.



V. APPLICATION OF GRAPH THEORY IN ECONOMICS (Mutual Debt Compensation)

Mutual debt compensation is a process where people who owe each other money agree to offset their debts against each other. Financial research has long explored methods to settle debts and mitigate losses from unpaid debts. Two prominent approaches differ in their focus and methods. Rotemberg tackles debt repayment between lenders, borrowers, and intermediaries, emphasizing consecutive repayments limited by transaction frequency rather than direct debt compensation. He highlights the need for external liquidity to resolve cyclic debt structures represented by directed graphs. Verhoeff, on the other hand, employs graph representation to optimize debt settlement among multiple agents, aiming to minimize transaction links and total money transferred for an efficient solution. Boerner and Hatfield analyze the clearing process of debts between agents in the balanced (the debts vs. claims position is zero for each agent) and the unbalanced financial position. In the balanced case, the cycle removal mechanism that clears all debts is used, while in the case of unbalanced positions of agents, consecutive removal of debts in the chain is applied.

A subsidy centre faces the challenge of deciding which firm to subsidize without prior knowledge. They have limited resources for subsidies, and each firm has an upper limit on the subsidy it can receive. To address this, a linear programming model is formulated to maximize the total subsidies provided while respecting these constraints. However, this approach may not be efficient for solving complex graph problems.

To enhance efficiency, Klein's cycle-cancelling method is proposed. This method involves modifying the debt graph by introducing a subsidy vertex and an auxiliary vertex. The subsidy vertex represents potential support from the subsidy centre, while the auxiliary vertex helps balance the subsidy centre. With these modifications, the graph is updated to enable the application of the cycle-cancelling algorithm for optimizing subsidy circulation.

Overall, the process involves formulating linear programming models to allocate subsidies efficiently, and then adapting Klein's cycle-cancelling method to enhance the optimization process within the context of the subsidy centre's decision-making.

The Ministry of Finance aims to minimize the receivables (debts owed to it) from a particular subject within a complex debt structure involving multiple firms. The Ministry collects income from firms (e.g., taxes) and spends its budget by making payments to firms providing services. To model this, each firm's debt and receivable towards the Ministry are represented in a graph, with the Ministry represented as a vertex.

VI.APPLICATION OF GRAPH THEORY IN GEOSCIENCE

1. Alternate route planning (Vanhove et al., 2010)

The report uses the shortest path algorithms in a realistic situation where forbidden turns (Turn Prohibitions) and turn costs are taken into account. A solution to prohibited turns has been modelled by the Direct method, which is an adaptation of the Dijkstra algorithm. This method did not take into consideration Turn costs, so another method, Node Splitting, was proposed. Turn cost as well as turn prohibition restrictions were both given importance. A third method, the Line Graph method, was also proposed for the restrictions. This method is a graph transformation algorithm that has the freedom to use any shortest path algorithm. The second part of the report focused on alternative route planning for which the k shortest paths algorithm and a heuristic approach known as the deviation path algorithms (which follows Yen's approach) were used as a solution model. It has been observed that the heuristic method is always quicker than the exact approach and frequently misses none or very few pathways. The future scope suggests optimising the k shortest paths problem and the heuristic approach.

2. Minimum Spanning Tool (MST) tool development (Dutta et al., 2014)

The main topic of the report is the development of a GIS tool that employs Prim's Algorithm to generate the Minimum Spanning Tool (MST) of a road network while taking significant junction sites into account. Database creation and tool development are two parts that summarise the technique used for this 3 project. The phase of tool development comes after database building, which comprises choosing the region that needs high-quality satellite imagery and digitising the photos. The outcomes from utilising this GIS tool demonstrate that there is better satisfaction when attempting to reduce the overall road length, which is a necessary condition for connecting important nodes (junctions) with one another.

3. Forest patch connectivity diagnostics and prioritisation (Devi et al., 2013)

Using graph theory, the report proposes a method for determining the ideal forest patch locations and threshold distances. Graph theory-based connectivity indices were used to establish the ideal threshold distance and its components. The connectivity investigated in this study is based on landscape structure, i.e., structural connectedness. It is used to evaluate graph theory applications and determine the best connectivity between forest landscapes. The status of fragmentation is determined by supplying patch counts and area range categories. The Arc GIS extension is used to obtain nodes and lengths concerning forest type polygons. The study emphasises how the use of graph theory made it easier to determine the best patches for connection and find suitable habitat patches for the preservation of biodiversity. The study demonstrated that hierarchical analysis of patch size, number, inter-distance, and relative importance was required to establish optimal connectivity.

VII. CONCLUSION:

The application of Graph Theory in image segmentation is a powerful and flexible technique.The application of graph theory in mutual debt compensation facilitates efficient debt settlement among multiple entities. Through modeling debt structures and employing optimization algorithms like Klein's cycle-cancelling method, it enhances decision-making for subsidy centers or ministries seeking to minimize receivables or maximize subsidy distribution, ensuring financial stability and fairness. The above mentioned applications vary over a range of domains like smart city planning, environment conservation, development of scientific tools.

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