



An Inventory Model For Exponentially Increasing Demand With Shortages and Effective Marketing Strategy

Neha Soni¹, Sushma Duraphe²

Corresponding Author

NEHA SONI *

Research Scholar of Department of Mathematics ,Govt. Motilal Vigyan Mahavidyalaya,

Old Vidhan Sabha Square, Jahangirabad, Bhopal, Madhya Pradesh 462008

Address- Flat no 407A, Metro Aashiyana near sardar patel public school, Misrod

Bhopal, Madhya Pradesh 462026

SUSHMA DURAPHE

Associate Professor of Department of Mathematics, Govt. Motilal Vigyan Mahavidyalaya

Old Vidhan Sabha Square, Jahangirabad, Bhopal, Madhya Pradesh 462008

Address- 84, Deepak society, chuna bhatti, Bhopal, Madhya Pradesh 462016

ABSTRACT

In this paper, we develop an EOQ model by taking exponentially increasing demand. In the proposed model, the holding cost is a function of time. Shortages are allowed and completely backlogged. We considered constant deterioration. The model is based on those essential products their demand never declines. Also, invest some amount in advertisement and marketing. An effective marketing strategy is required to increase sales and to ensure the sale of the entire stock before deterioration. A mathematical model is developed by using a differential equation and finding the minimum total cost.

Keywords

Inventory model, exponentially increasing demand, deterioration, the total cost, holding cost, EOQ, shortages, etc.

1. INTRODUCTION

Effective marketing plays an important role in communicating with targeted customers. Higher advertising frequency increases costs but can increase your chances of acquiring new customers. Furthermore, faced with a wide range of products that meet specific needs, customers typically make purchases according to expectations regarding value and satisfaction. When customers are satisfied with the shopping experience, they are more likely to purchase again and share their experiences with others. Therefore, companies are concerned about increasing customer value and service satisfaction by developing and managing customer relationships. This maintains the company's competitive edge and can improve its market share. This paper presents an inventory model with exponentially increased demand due to effective marketing. This paper includes shortage costs. Usually, an estimated figure comes after our perception of multiple costs such as customer loss, lost sales, stock-out fines, and disputes in the contract. This way inventory shortages do not result in immediate loss of sales or profits. The seller may commit to delivering the product within a particular lead time. In this model, shortages are permitted and completely backlogged. Deterioration and holding costs are also considered in this model.

2. BACKGROUND OF RESEARCH

In classical inventory models, the demand rate is assumed to be constant but demand for physical goods may be time-dependent, stock-dependent, and also depend on the marketing of the product. Shah and Pandey (2009) designed a deteriorating inventory model in which demand depends on advertisement and stock display. Shukla et al. (2013) designed an EOQ inventory model for deteriorating items with an exponential time-dependent demand rate. Dash et al. (2013) developed an inventory model with exponential declining demand and time-varying holding costs. Sharma and Chaudhary (2013) developed an inventory model for deterioration following the Weibull distribution. Verma and Verma (2014) developed an inventory model with exponentially decreasing demand and linearly increasing deterioration. Sharmila and Uthayakumar (2015) presented the fuzzy inventory model for deteriorating items for power demands under fully backlogged conditions. Sekar and Uthayakumar (2017) reported four stages of the production inventory model for deteriorating goods in which three (Beginning, growth, and maturity) different stages of production and a decline phase are considered and take exponentially increasing demand. Mashud (2020) developed an EOQ deteriorating inventory model with different types of demand and fully backlogged shortages. Cheng et al. (2023) derived an inventory model with advertisement and customer relationship management sensitive demand for the product's life cycle.

3. NOTATION

$I(t)$ = Inventory level with respect to time t ,

D = Demand,

θ = Deterioration rate,

C_o = Ordering cost,

C_h = Holding cost,

C_s = shortage cost,

C_d =Deterioration cost

C_A = Advertisement cost

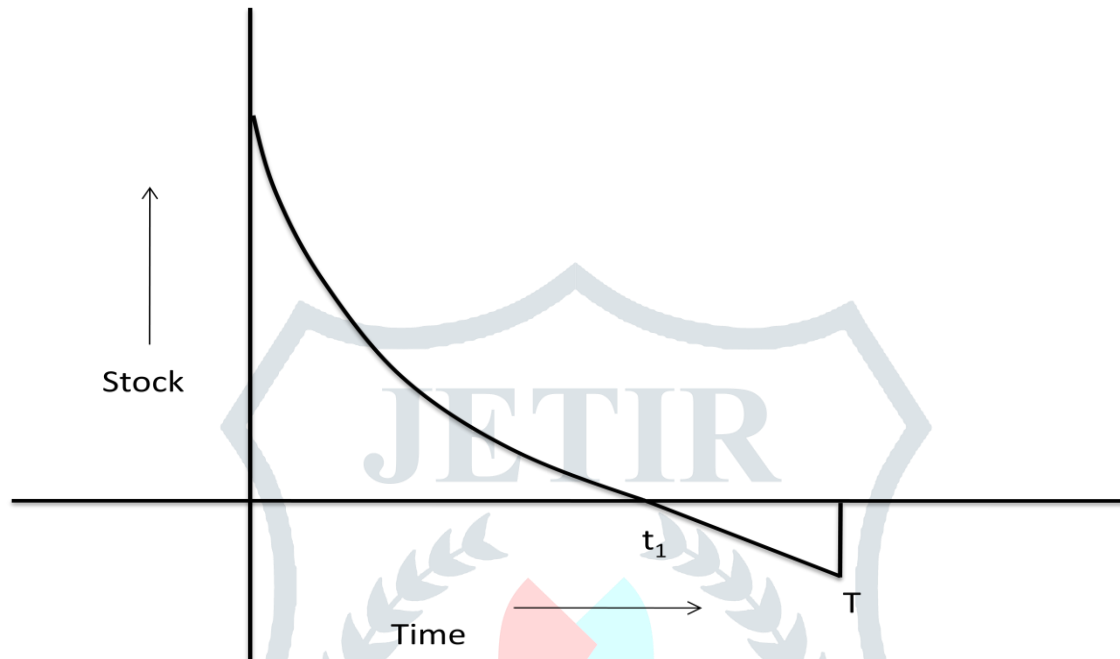


Figure 1. Graphical representation of inventory versus time.

4. ASSUMPTION

To develop the proposed model, we adopt the following assumptions

1. The demand for the product is assumed to increase exponentially as given by the function $D(t) = ae^{bt}$, where $a > 0$, $0 < b < 1$.
2. Replenish inventory arrives all at once when the inventory level drops to zero.
3. Lead time is zero.
4. Shortages are allowed and fully backlogged
5. Deterioration rate is constant.
6. We consider the constant cost of advertisement in this model.

5. MATHEMATICAL MODEL

The inventory level varies due to both demand and deterioration of material. At $t = t_1$ the inventory level achieves zero, after which shortages are allowed during the time interval (t_1, T) and all of the demand during the shortages period are fully backlogged. The inventory levels of the model are given by the following differential equation.

$$\frac{dI(t)}{dt} + \theta I(t) = -ae^{bt}, \quad 0 < t < t_1 \quad (1)$$

$$\text{And } \frac{dI(t)}{dt} = -ae^{bt}, \quad t_1 < t < T \quad (2)$$

The solution of equation (1) and (2) with the condition $I(t_1) = 0$ is given by

$$I(t) = \frac{a}{\theta+b} [e^{(\theta+b)t_1-\theta t} - e^{bt}] \quad , \quad 0 < t < t_1 \quad (3)$$

And

$$I(t) = \frac{a}{b} [e^{bt} - e^{bt_1}] \quad , \quad t_1 < t < T \quad (4)$$

$$\text{The ordering cost } OC = C_o \quad (5)$$

The holding cost during $[0, t_1]$ is given by

$$HC = C_h \int_0^{t_1} I(t) dt, \quad (6)$$

Where C_h = holding cost per unit time

$$= C_h \frac{a}{\theta+b} \left[\frac{e^{bt_1}(e^{\theta t_1}-1)}{\theta} - \frac{(e^{bt_1}-1)}{b} \right] \quad (7)$$

The shortage cost during $[t_1, T]$ is given by

$$SC = c_s \int_{t_1}^T I(t) dt, \quad (8)$$

$$= C_s \frac{a}{b^2} [e^{bT} - e^{bt_1} - e^{bt_1}b(T - t_1)] \quad (9)$$

The deterioration cost is given by

$$DC = \theta c_d, \quad (10)$$

where, θ =deterioration rate

The advertisement cost is given by

$$AC = c_a, \quad (11)$$

Total cost = Ordering cost (OC) + Holding cost (HC) + Shortage cost (SC) + Deterioration cost (DC) + Advertisement cost (AC).

So, The total cost per unit time TC is

$$TC(t_1, T) = \frac{1}{T} \left[C_o + C_h \frac{a}{\theta+b} \left[\frac{e^{bt_1}(e^{\theta t_1}-1)}{\theta} - \frac{(e^{bt_1}-1)}{b} \right] + C_s \frac{a}{b^2} [e^{bT} - e^{bt_1} - e^{bt_1}b(T - t_1)] + \theta C_d + C_a \right] \quad (12)$$

To minimize the total cost per unit time $TC(t_1, T)$, the optimal value of T and t_1 can be obtained by solving the following equations:

$$\frac{\partial TC(t_1, T)}{\partial t_1} = 0 \text{ and} \quad (13)$$

$$\frac{\partial TC(t_1, T)}{\partial T} = 0 \quad (14)$$

Again partially differentiate eq.(13) and eq.(14) with respect to t_1 and T by using suitable software MATLAB and obtained

$$\left(\frac{\partial^2 TC(t_1, T)}{\partial t_1^2}\right) \left(\frac{\partial^2 TC(t_1, T)}{\partial T^2}\right) - \left(\frac{\partial^2 TC(t_1, T)}{\partial t_1 \partial T}\right) > 0 \quad (15)$$

$$\frac{\partial^2 TC(t_1, T)}{\partial t_1^2} > 0 \text{ and } \frac{\partial^2 TC(t_1, T)}{\partial T^2} > 0 \quad (16)$$

The value eq.(12) satisfies the condition of eq.(15) and eq.(16) so the value of total cost is minimum.

6. CONCLUSION

In this paper, we design an inventory model for exponentially increasing demand. This model allowed planned shortages and was fully backlogged. Also, we use some marketing strategy. Effective marketing plays a very important role in increasing the demand for any product. In this model, we minimize total inventory cost. This model can be extended in several ways. For example, we may extend the model by adding a pricing strategy into consideration. Also, we extend the model by taking linear demand and variable deterioration.

References

1. Shukla, H., Vivek Shukla, and S. Yadava. "EOQ model for deteriorating items with exponential demand rate and shortages." *Uncertain Supply Chain Management* 1, no. 2 (2013): 67-76.
2. Sharma, Vikas, and Rekha Rani Chaudhary. "An inventory model for deteriorating items with Weibull deterioration with time dependent demand and shortages." *Research Journal of Management Sciences* ISSN 2319 (2013): 1171.
3. Mashud, Abu Hashan Md. "An EOQ deteriorating inventory model with different types of demand and fully backlogged shortages." *International Journal of Logistics Systems and Management* 36, no. 1 (2020): 16-45.
4. Cheng, Mei-Chuan, Chun-Tao Chang, and Tsu-Pang Hsieh. "An Inventory Model with Advertisement-and Customer-Relationship-Management-Sensitive Demand for a Product's Life Cycle." *Mathematics* 11, no. 6 (2023): 1555.
5. Dash, Bhanu Priya, Trailokyanath Singh, and Hadibandhu Pattnayak. "An inventory model for deteriorating items with exponential declining demand and time-varying holding cost." *American Journal of Operations Research* 2014 (2014).
6. Sekar, T., and R. Uthayakumar. "A manufacturing inventory model for exponentially increasing demand with preservation technology and shortage." *International Journal of Operations Research* 15, no. 2 (2018): 61-70.

7. Shah, Nita H., and Poonam Pandey. "Deteriorating inventory model when demand depends on advertisement and stock display." *International Journal of Operations Research* 6, no. 2 (2009): 33-44.
8. Vijai Stanly, Sharmila, and R. Uthayakumar. "Inventory model for deteriorating items involving fuzzy with shortages and exponential demand." *International Journal of Supply and operations management* 2, no. 3 (2015): 888-904.
9. Verma, Vinod Kumar, and B. B. Verma. "An Inventory Model with Exponentially Decreasing Demand and Linearly Increasing Deterioration." *International Journal of Physical, Chemical and Mathematical Sciences* 3, no. 1 (2014).

