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An Overview on The Development of Generalized Fuzzy Information Measures

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Abstract

Mathematical techniques known as generalized fuzzy information measures are employed to quantify fuzziness, ambiguity, or uncertainty in data that is represented by fuzzy sets. These measures play a crucial role in fuzzy logic, fuzzy systems, and fuzzy set theory, as they provide a means to analyse and manipulate uncertain or imprecise information. The development of generalized fuzzy information measures has been an active area of research, aiming to extend and refine existing measures to better capture the complexities of fuzzy data. The article discusses fuzzy information measures and their generalizations, adopting both objective and descriptive methodologies. Applications for generalized fuzzy information measures can be found in several domains, including the analysis of images, data mining, machine learning, recognition of patterns, and decision-making. These measures provide a means to handle imprecise and uncertain information in real-world datasets, where traditional crisp methods may be inadequate. It covers problem-solving and problem-posing methods, along with fundamental ideas, providing a summary of various fuzzy entropy measures and their generalizations, emphasizing their importance for the advancement of fuzzy information theory.

Key Words: Fuzzy Entropy/ Uncertainty/ Fuzziness/Measures of Information.

Mathematics Subject Classification: 94 D 05

Introduction:

The essence of why fuzzy reasoning and fuzzy sets are essential in dealing with real-world decision-making scenarios where uncertainty and ambiguity are prevalent. Here breakdowns of the mentioned key points are given:

1. **Uncertainty and Fuzziness in Human Cognition**: Human cognition often deals with uncertainty and fuzziness rather than crisp, well-defined concepts. Fuzzy reasoning acknowledges this inherent fuzziness in human thought processes and provides a framework to model and deal with it effectively.
2. **Information Processing and Ambiguity Reduction**: Information processing, whether by humans or machines, aims to reduce ambiguity and uncertainty in the data being processed. The measure of information quantifies the reduction in probabilistic uncertainty achieved through an experiment or decision-making process.
3. **Shannon's Entropy**: Shannon proposed the idea of entropy to measure the uncertainty or randomness in a probability distribution. It has found extensive applications across various fields such as pattern recognition, statistical mechanics, finance, communication theory, and neural networks.

4. **Zadeh's Fuzzy Set Theory**: Lotfi A. Zadeh's fuzzy set theory provides a mathematical framework to represent and manipulate imprecise or vague concepts. It has been applied in diverse scientific and technological domains including image processing, decision making, clustering, and more.
5. **Generalization with Fuzzy Sets**: Fuzzy sets offer a means to generalize both qualitative and quantitative data, allowing for the representation of vague or imprecise information. This generalization capability is especially helpful in circumstances where crisp, binary distinctions are insufficient to capture the complexity of real-world phenomena.

In the end, fuzzy sets and fuzzy reasoning offer useful tools for handling ambiguity, vagueness, and uncertainty in information processing and decision-making tasks, allowing more flexible and resilient methods across a range of domains.

Fuzzy Set Theory

Recall that a membership characteristic $\mu_A: U \to [0,1]$ characterizes a fuzzy A in U subset (collection of phrases) and expresses the degree of $x \in U$ membership of in A as follows.

$\mu_A(x) = 0$ If x is not a part of A and there is no doubt

$= 1$ if there is no doubt and x is a member of A

$= 0.5$ If there is the most uncertainty

In fact, $\mu_A(x)$ interacts with each $x \in U$ Position within the collective A. When $\mu_A(x)$ has a value of $\{0,1\}$ It is the distinctive feature of an accurate, or nonfuzzy, set.

Generalization of Fuzzy Entropy Measures

In 1968, Zadeh [1] presented the notion of fuzziness being measured by fuzzy entropy. Since $\mu_A(x)$ and $1 - \mu_A(x)$ yields the same kind of ambiguity and hence correspond to the entropy due to Shannon [2], De Luca and Termini [3]proposed the fuzzy entropy measure that follows:

$$H(A) = -\left[\sum_{i=1}^{n} \mu_A(x_i) log \mu_A(x_i) + \sum_{i=1}^{n} \big(1 - \mu_A(x_i)\big) log\big(1 - \mu_A(x_i)\big)\right] \qquad (2)$$

A collection of properties presented by De Luca and Termini [3] are commonly used as a standard for establishing new fuzzy entropies. Entropy, a measure of fuzziness in fuzzy set theory, reflects the average degree of ambiguity or Have difficulties making a decision if an element is a part of a set or not. Therefore, to be considered a legitimate fuzzy entropy, A measure of a fuzzy set's average fuzziness must at least satisfy the following requirements.:

i) $H(A) = 0$ when $\mu_A(x_i) = 0$ *or* 1.
ii) $H(A)$ increases as $\mu_A(x_i)$ increases between 0 to 0.5.
iii) $H(A)$ decreases as $\mu_A(x_i)$ increases between 0.5 to 1.
iv) $H(A) = H(\bar{A})$, i.e. $\mu_A(x_i) = 1 - \mu_A(x_i)$
v) $H(A)$ represents a concave function of $\mu_A(x_i)$.

In Table 1, the generalized fuzzy entropies are briefly reviewed.

Table 1. Generalization of Fuzzy Entropies

S. No.	Definition
	Non Parametric Measure of Fuzzy Entropy
1.	De Luca and Termini [3]: $H(A) = -\left[\sum_{j=1}^{n} \mu_A(x_j) log \mu_A(x_j) + \sum_{j=1}^{n} \left(1-\mu_A(x_j)\right) log \left(1-\mu_A(x_j)\right)\right]$
2.	Bhandari and Pal [4]: $H_e(A) = \frac{1}{m(\sqrt{e}-1)} \sum_{i=1}^{n} \mu_A(x_i) e^{\left(1-\mu_A(x_i)\right)} + \left(1-\mu_A(x_i)\right) e^{\mu_A(x_i)}$
3.	Kaufmann [5]: $H(A) = -\frac{1}{logn} \sum_{i=1}^{n} \emptyset_A(x_i) log \emptyset_A(x_i)$
4.	Ebanks [6]: $H(A) = \sum_{i=1}^{n} g\,(\mu_i)$
5.	Kosko [7]: $H(q, A) = d^q(A, A^{near})/d^q(A, A^{far})$
6.	Pal et al. [8]: $H_*(A) = k \sum_{i=1}^{n} g_1\,(\mu_i), k \in R^+, \hat{g}_1(t) = f(t) + f(1-t), g_1(t) = \hat{g}_1(t) - \min_{0 \le t \le 1} \{\hat{g}_1(t)\}, f \text{ is concave}$
	Parametric Measure of Fuzzy Entropy
7.	Bhandari and Pal [4]: $H_\gamma(A) = \frac{1}{1-\gamma} \sum_{i=1}^{n} \left[\left(\mu_A(x_i)\right)^\gamma + \left(1-\mu_A(x_i)\right)^\gamma\right]; \gamma \neq 1, \gamma > 0$
8.	Kapur [9]: $H^\propto(A) = \frac{1}{1-\alpha} \sum_{i=1}^{n} \left[\left(\mu_A(x_i)\right)^\alpha + \left(1-\mu_A(x_i)\right)^\alpha - 1\right]; \alpha \neq 1, \alpha > 0$
9.	Kapur [9]: $H_{\gamma,\delta}(A) = \frac{1}{\delta-\gamma} log \sum_{i=1}^{n} \frac{\left[\left(\mu_A(x_i)\right)^\gamma + \left(1-\mu_A(x_i)\right)^\gamma\right]}{\left[\left(\mu_A(x_i)\right)^\delta + \left(1-\mu_A(x_i)\right)^\delta\right]}; \gamma \ge 1, \delta \le 1 \text{ or } \gamma \le 1, \delta \ge 1$ Kapur [9]: $H_{\gamma,\delta}(A) = \frac{1}{\gamma+\delta-2} \sum_{i=1}^{n} \left[{\mu_A}^\gamma(x_i) + \left(1-\mu_A(x_i)\right)^\gamma + {\mu_A}^\delta(x_i) + \left(1-\mu_A(x_i)\right)^\delta - 2\right]$

11.	Kapur [9]: $H_{\gamma}^{\beta}(A)=\frac{1}{\beta-\gamma}\sum_{i=1}^{n}\left[\left(\mu_A(x_i)\right)^{\gamma}+\left(1-\mu_A(x_i)\right)^{\gamma}-\left(\mu_A(x_i)\right)^{\beta}-\left(1-\mu_A(x_i)\right)^{\beta}\right];\gamma\geq 1,$ $\beta\leq 1\ or\ \gamma\leq 1,\beta\geq 1\ and\ \gamma=\ \beta\ only\ if\ both\ are\ unity$
12.	Prakash [10]: $H_{\gamma}^{\delta}(A)=[(1-\gamma)\delta]^{-1}\sum_{i=1}^{n}\left[\left\{\left(\mu_A(x_i)\right)^{\gamma}+\left(1-\mu_A(x_i)\right)^{\gamma}\right\}^{\delta}-1\right];\gamma>0,\gamma\neq 1,\delta\neq 0$
13.	Hooda [11]: $H^{\beta}(A)=\frac{1}{1-\beta}\left[2^{(\beta-1)\sum_{i=1}^{n}\mu_A(x_i)log\mu_A(x_i)+\left(1-\mu_A(x_i)\right)log\left(1-\mu_A(x_i)\right)}-1\right];\beta>0,\beta\neq 1$ $H_{\alpha}^{\beta}(A)=\frac{1}{1-\beta}\sum_{i=1}^{n}\left[\left({\mu_A}^{\alpha}(x_i)+\left(1-\mu_A(x_i)\right)^{\alpha}\right)^{\frac{\beta-1}{\alpha-1}}-1\right];\alpha\neq\beta,\alpha,\beta>0,\alpha\neq 1$
14.	Hooda [11]: $H_v(A)=\frac{v}{v-1}\sum_{i=1}^{n}1-\left({\mu_A}^{v}(x_i)+\left(1-\mu_A(x_i)\right)^{v}\right)^{\frac{1}{v}};v>0,v\neq 1$
15.	Prakash and Sharma [12]: $K_a(A)=\sum_{i=1}^{n}\left[\log\left(1+a\mu_A(x_i)\right)+\log\left(1+a\left(1-\mu_A(x_i)\right)\right)-\log(1+a)\right];a>0$ $H_a(A)=-\sum_{i=1}^{n}\left[\mu_A(x_i)\log\mu_A(x_i)+\left(1-\mu_A(x_i)\right)log\left(1-\mu_A(x_i)\right)\right]$ $-\frac{1}{a}\sum_{i=1}^{n}\left[\left(1+a\mu_A(x_i)\right)\log\left(1+a\mu_A(x_i)\right)+\left(1+a\left(1-\mu_A(x_i)\right)\right)\log\left(1+a\left(1-\mu_A(x_i)\right)\right)\right.$ $\left.-(1+a)\log(1+a)\right];a>0$
16.	Hooda and Bajaj [13]: $H_R^{\beta}(A)=\frac{R}{R+\beta-2}\sum_{i=1}^{n}\left[1-\left({\mu_A}^{\frac{R}{2-\beta}}(x_i)+\left(1-\mu_A(x_i)\right)^{\frac{R}{2-\beta}}\right)^{\frac{2-\beta}{R}}\right];0<\beta\leq 1,R>0\ and\ R+\beta$ $\neq 2$
17.	Hooda and Jain [14]: $H_a(A)=-2^{\alpha-1}\sum_{i=1}^{n}\left[{\mu_A}^{\alpha}(x_i)\log\left(\mu_A(x_i)\right)+\left(1-\mu_A(x_i)\right)^{\alpha}log\left(1-\mu_A(x_i)\right)\right];where\ 0.5<\alpha$ $<2\ and\ 0log0=0.$ $H_{\alpha}^{\beta}(A)=\frac{1}{\beta-\alpha}\sum_{i=1}^{n}\left[{\mu_A}^{\alpha}(x_i)+\left(1-\mu_A(x_i)\right)^{\alpha}-{\mu_A}^{\beta}(x_i)-\left(1-\mu_A(x_i)\right)^{\beta}\right];0<\alpha<1,\beta\geq 1\ or\ 0$ $<\beta<1,\alpha\geq 1$
18.	Prakash and Gandhi [15]: $H_1(A)=-\sum_{i=1}^{n}\left[sin\frac{\pi}{2n\mu_A(x_i)}+sin\frac{\pi}{2n\left(1-\mu_A(x_i)\right)}-n^2sin\frac{\pi}{2n}\right];n>3$ $H_2(A)=-\sum_{i=1}^{n}\left[tan\frac{\pi}{2n\mu_A(x_i)}+tan\frac{\pi}{2n\left(1-\mu_A(x_i)\right)}-n^2tan\frac{\pi}{2n}\right];n>3$

19.	Verma and Sharma [16]: $E_a(A)=\frac{1}{n(e^{(1-0.5^{\alpha})}-1)}\sum_{i=1}^{n}\left[\mu_A(x_i)e^{\left(1-\mu_A{}^{\alpha}(x_i)\right)}+\left(1-\mu_A(x_i)\right)e^{\left(1-\left(1-\mu_A(x_i)\right)^{\alpha}\right)}-1\right];\alpha>0$
20.	Hooda and Jain [17]: $H_R^{(\alpha,\beta)}(A)=\frac{R}{R+\beta-2\alpha}\sum_{i=1}^{n}\left[1-\left(\mu_A{}^{\frac{R}{2\alpha-\beta}}(x_i)+\left(1-\mu_A(x_i)\right)^{\frac{R}{2\alpha-\beta}}\right)^{\frac{2\alpha-\beta}{R}}\right];\alpha\geq 1,0<\beta\leq 1,R>0,R\neq 1\ and\ R+\beta\neq 2\alpha$
21.	Kumar et al. [18]: $H_{R,\rho,\eta}^{(\alpha,\beta)}(A)=\rho\frac{R}{R+\alpha-2}\sum_{i=1}^{n}\left[1-\left(\mu_A{}^{\frac{R}{2-\alpha}}(x_i)+\left(1-\mu_A(x_i)\right)^{\frac{R}{2-\alpha}}\right)^{\frac{2-\alpha}{R}}\right]+\eta\frac{R}{R+\beta-2}\sum_{i=1}^{n}\left[1-\left(\mu_A{}^{\frac{R}{2-\beta}}(x_i)+\left(1-\mu_A(x_i)\right)^{\frac{R}{2-\beta}}\right)^{\frac{2-\beta}{R}}\right];\rho,\eta>0,\alpha>0,0<\beta\leq 1,R>0,R\neq 1,R+\alpha\neq 2,R+\beta\neq 2$
22.	Kumar et al. [19]: $H_1(A)=-\frac{1}{a}\sum_{i=1}^{n}\left[\mu_A{}^{\alpha\mu_A(x_i)}(x_i)+\left(1-\mu_A(x_i)\right)^{\alpha\left(1-\mu_A(x_i)\right)}-3-\alpha+\alpha^{\mu_A(x_i)}+\alpha^{\left(1-\mu_A(x_i)\right)}\right]$ $H^{\alpha}(A)=-\frac{1}{a}\sum_{i=1}^{n}\left[\mu_A{}^{\alpha\mu_A(x_i)}(x_i)+\left(1-\mu_A(x_i)\right)^{\alpha\left(1-\mu_A(x_i)\right)}-2\right]-\frac{1}{\alpha-1}\sum_{i=1}^{n}[\mu_A{}^{\alpha}(x_i)\left(1-\mu_A(x_i)\right)^{\alpha}-1]$ $H_{\alpha}(A)=-\frac{1}{a}\sum_{i=1}^{n}\left[\mu_A{}^{\alpha\mu_A(x_i)}(x_i)+\left(1-\mu_A(x_i)\right)^{\alpha\left(1-\mu_A(x_i)\right)}-2+\mu_A(x_i)log\mu_A(x_i)+\left(1-\mu_A(x_i)\right)log\left(1-\mu_A(x_i)\right)\right];under\ the\ assumption\ 0^{0.\alpha}=1$
23.	Tomar and Ohlan [20]: $H_R^{\alpha}(A)=\frac{R}{R-\alpha}\sum_{i=1}^{n}1-\left(\mu_A{}^{\frac{R}{\alpha}}(x_i)+\left(1-\mu_A(x_i)\right)^{\frac{R}{\alpha}}\right)^{\frac{\alpha}{R}};0<\alpha\leq 1,R>0,R\neq 1$ $H_R^{m}(A)=\frac{R-m+1}{R-m}\sum_{i=1}^{n}1-\left(\mu_A{}^{R-m+1}(x_i)+\left(1-\mu_A(x_i)\right)^{R-m+1}\right)^{\frac{1}{R-m+1}};R-m+1>0,R\neq m,R,m>0(\neq 1)$
24.	Singh, Hooda and Malik [21] : $H_1(A)=\sum_{i=1}^{n}\left[sin\frac{\pi\mu_A(x_i)}{2}+sin\frac{\pi\left(1-\mu_A(x_i)\right)}{2}-1\right]$ $H_2(A)=\sum_{i=1}^{n}\left[cos\frac{\pi\mu_A(x_i)}{2}+cos\frac{\pi\left(1-\mu_A(x_i)\right)}{2}-1\right]$ $H_3(A)=\sum_{i=1}^{n}\left[sin\pi\mu_A(x_i)+sin\pi\left(1-\mu_A(x_i)\right)-1\right]$ $H_4(A)=\sum_{i=1}^{n}\left[sin\beta\mu_A(x_i)+sin\beta\left(1-\mu_A(x_i)\right)-1\right]-sin\beta$

	$$H_5(A) = \sum_{i=1}^{n} \left[sin(\beta\mu_A(x_i) + \alpha) + sin\left(\beta\left(1 - \mu_A(x_i)\right) + \alpha\right) - 1 \right] - sin(\alpha + \beta)$$ $$H_6(A) = \sum_{i=1}^{n} \left[cos\beta\mu_A(x_i) + cos\beta\left(1 - \mu_A(x_i)\right) - 1 \right] - (1 + cos\beta)$$
25.	Singh, Hooda and Malik [21]: Fuzzy Rough Information Measures $$E_{log}(A) = \frac{1}{n}\sum_{i=1}^{n} log_2\left(2 - \frac{1}{2}\left(\left|2\underline{x_i} - 1\right| + \left|2\overline{x_i} + 1\right|\right)\right), \forall x_i \in A \; and \; x_i \in \left(\underline{x_i}, \overline{x_i}\right)$$

Conclusion

The manuscript reviews fuzzy measures and their generalizations, highlighting their significance in advancing the field of fuzzy information theory. By exploring these generalizations, researchers can pave the way for further developments in handling uncertainty and vagueness in various applications. The reviewed measures of fuzzy entropies, in particular, offer practical solutions for addressing real-world problems across diverse domains.

Fuzzy measures, stemming from fuzzy set theory, provide a means to represent and quantify uncertainty and imprecision inherent in real-world data. These measures have been subject to various generalizations, which extend their applicability and utility in fuzzy information theory. By considering the nuances of fuzzy data, such as degrees of membership and uncertainty, these generalizations offer more comprehensive frameworks for analysing and processing fuzzy information.

The manuscript underscores the practical relevance of these generalized fuzzy measures, particularly fuzzy entropies, in solving real-world problems. Entropy measures quantify uncertainty or randomness in probability distributions and play a pivotal role in various forms of applications, including, pattern recognition, decision making data mining and machine learning. In fuzzy information theory, the reviewed measures of fuzzy entropies provide valuable tools for handling uncertain or imprecise data effectively.

Overall, the manuscript emphasizes the importance of exploring and understanding fuzzy measures and their generalizations in advancing fuzzy information theory. By harnessing the insights from these measures, researchers can develop innovative solutions to tackle the challenges posed by uncertainty and vagueness in real-world scenarios, ultimately driving progress and innovation across diverse application areas.

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