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Review of an Economic Order Quantity Model with Reverse Logistics Program by S. Sunny

Suhani Agrawal, Debasmita Samal*, Manas Ranjan Mishra
Department of Mathematics, School of Science, OP Jindal University Raigarh, CG, India

Abstract
Due to the incorporation of practical issues, the economic order quantity (EOQ) model has undergone significant evolution over the past few decades. Studying the impact of reverse product flow into inventory systems has garnered attention recently. In reality, companies may implement return policies and repurpose goods and materials in an effort to win back assets and boost client loyalty. An EOQ model for reverse logistics is examined in this research. Profit maximization is a strategy used to address the inventory problem of when and how much to order when there is a reverse flow of things into the system. The model is explained by reviewing the paper.
Keywords: Inventory model, EOQ, Reverse logistics, Profit Maximization, Deterioration

INTRODUCTION

One effective approach for inventory management that helps companies meet customer demand and maintain inventories at the lowest possible cost is the Economic Order Quantity (EOQ) model. The model was first described by Harris[1] in one of the earliest papers on mathematical models of inventory management. The fundamental EOQ model is predicated on the following hypotheses: homogeneous, flawless quality, non-perishable, instantaneous replenishment, and a set demand rate. However, the EOQ model might stop offering a minimum-cost solution if these presumptions are loosened. In order to operate a reverse logistics program with an increasing rate of reverse logistic investment cost, this study focuses on creating a framework for replenishing an inventory system. The analysis aims to answer questions such as the optimal choice for the return policy, order quantity, cycle length, and unit selling price of the item given that the investment cost in the reverse logistics scheme increases with an increase in return rate. Conventional inventory system modeling often ignores the dynamic nature of reverse flow of items due to the complexity of developing a framework accommodating a return structure. This can lead to incorrect inventory decisions and inefficient use of inventory data. A reverse logistic EOQ model can be developed rapidly and cheaply, allowing for the redefinition of EOQ models and the utilization of returned items. However, this approach presents new challenges, the most significant of which is the complexity of the procedures required to adequately characterize the optimal solution. In the present analysis, we propose an efficient EOQ framework with a reverse logistic program, providing flexibility in the model to be used for a wide range of inventory problems, including systems containing items with expiry dates.

As mentioned, traditional EOQ models did not consider the impact of return products However, in many real-world scenarios, businesses have return policy to increase customer loyalty and recover assets by reusing products and materials. Recently, several attempts have been made to incorporate reverse logistics scheme into the traditional inventory models. For example, in industries such as book publishing, catalogue retailing, and greeting cards, over 20% of all products sold are eventually returned to the vendor [10]. Hawks [11] defined reverse logistics as the process of moving goods from the point of destination to the point of origin for the purpose of recapturing value or proper disposal. The reverse flow of goods impacts inventory level and total inventory cost, and it must be considered when analyzing inventory systems. According to [11], returns reduce the profitability of retailers by 4.3%, and he argued that retailers are more likely to sell returns to a broker or similar entity than to the manufacturers. Manufacturers are more inclined to remanufacture or refurbish, which seems logical given that manufacturers are better at manufacturing than retailers. Gains from reverse logistics include enhanced customer satisfaction, decreased inventory levels, and reduced cost.

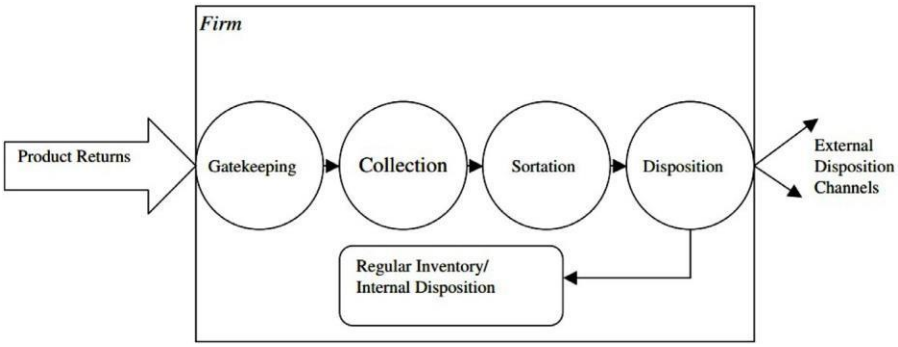

Fig. 1. A retail reverse logistics process; adapted from [12].

The retail-level reverse- chain process is shown it the figure 1. Gatekeeping, collection, sorting, and disposal—collectively referred to as the central return center—are the fundamental elements of reverse logistics. In-depth discussion of each unit of this central return center is provided by [12]. Inventory decisions are directly impacted by the costs incurred in the operation of reverse logistics. Reverse logistics has garnered significant interest from supply-chain management experts in the past few years. The necessity to look into how returned goods affect inventory choices is what spurred this analysis. The decision to invest in a reverse logistics program in this instance is based on the inventory model or various model values, and the output that is produced is often determined by the order size, price, and cycle time. A novel and essential concept for inventory modeling is the integration of a reverse-logistics software. On the other hand, demand rate and deterioration have long been acknowledged as crucial components of an inventory model. The model was expanded to incorporate components with a constant rate of deterioration bySanni et al. [13]O'Neill and Sanni [14] have introduced a broad method for deriving an analysis of the EOQ problem.

A deterministic EOQ-based model with fixed lead times for external orders and recovery, as well as constant demand and return rates, was developed by [16], one of the first studies on reverse logistic EOQ control model. Every workable inventory system eventually experiences some sort of breakdown. It is necessary to incorporate losses arising from item deterioration in an inventory system analysis. The availability of goods to meet consumer demand and the average cost of inventory as a whole are both impacted by these losses. These reverse logistics models loosen the item deterioration rate and ignore the impact of unit selling on order size. These models function best when the items are non-perishable throughout the cycle; they are not very effective in representing circumstances when objects deteriorate.

The analysis conducted by Alinovi et al. [15] focuses on describing the effects of manufacturing lead time, return fraction, and demand uncertainty on inventory decisions. In these decisions, the order quantity and cycle length are fixed parameters that are presumed to be derived from the firm'sestimations or external constraints. They used the return rate as a choice variable, ignoring the effects of item deterioration.

The majority of the literature's attention has gone on approaches for figuring out the best return rate or choosing an order size in order to reduce system costs. When taking into consideration the reverse flow of objects, these models have paid little to no regard to the combined outcomes that are obtained. Therefore, there is still much to learn about the issue ofusing a reverse-logistics arrangement in inventory decisions.

Compared to the previous research, this work aims at modifying the EOQ model to give clear representation of real contexts; with this purpose in mind particular attention has been paid to

- Formulating the EOQ model to include price-dependent demand and time-varying deterioration rate under reverse logistics program with increasing investment cost;
- Determining optimal solutions for the return policy, order quantity, cycle length and unit selling price; where return rate, order size, selling price are the decision variables.

On the other hand, we do not focus on demand uncertainty and instead believe that the sellingprice is the main element determining customer demand since we presume that the corporation or business can effectively estimate demand.

All functional inventory systems eventually break down in one way or another. An inventory system study must include losses resulting from item deterioration. These losses affect both the average cost of inventories overall and the availability of items to meet consumer demand. These reverse logistics models disregard the effect of unit selling on order size and loosen the item degrading rate. These models work best in scenarios where the

things are non-perishable throughout the cycle; they are not very good at simulating situations where the objects degrade.

This paper is a review of the paper- An Economic Order Quantity Modelwith Reverse logistics program by Sanni et.al [24].

Mathematical Development

In this section, we follow the model form set out in Chen et al. [17] using similarassumptions. The notations in the EOQ model component are as follows:

a: Fixed ordering cost per replenishment
b: Purchase cost per unit item
c: Inventory holding per unit per unit time
p: Unit selling price of product
D(p): Price-dependent demand rate
θ(t): Deterioration rate as a function of time with θ denoting constant deterioration
y: Order quantity per replenishment
T: Cycle length.

The notations for modeling used product collection operation are asfollows:

b_1: The unit price of used collected product sold by the firm to its supplier
b_2: The unit cost of collecting, holding, sorting and handing a re-turned item, which includes fees paid by the firm to consumers.
v: Return rate for collecting used product.
R(v): Sum of return rate for used product in the current replenishment cycle.

The following assumptions are made in the model development

1. A single-item inventory system is considered which receives an order size of y > 0

units at the starting time = 0.

2. No shortage is allowed.
3. The demand is a deterministic price dependent demand.
4. The possibility of item deteriorating inventory is assumed.
5. A reverse logistics program is setup to absorb returned item and reused.
6. A substantial amount, s is invested in the operation of reverse flow of item.
7. The investment cost of running a reverse logistic scheme is a quadratic function of return rate coefficient, ν. This ensures that the investment cost increases at an increasing rate of returned items.
8. The inventory holding costs for collected items are ignored.
9. We assumed a positive salvage value $(b_1 - b_2)$ of collected used product.

We analyze the issue of maximizing profits for a company that uses a reverse-logistics program. The unit selling price that a company sets for its goods influences demand, and some items degrade throughout the course of the cycle. Let us assume that the company wants to maximize its profits for a given order size (y), unit selling price (p), and return rateparameter (T > 0 and p > 0). Since y > 0, the profit is comprised of sales revenue from both new and returned items, less purchase, holding, and operating costs for the reverse logisticssystem. Consequently, the average profit rate for all T>0 is provided by

$$\pi(p,\nu,T) = [p + (b_1 - b_2)R(\nu)]D(p) - 1/T\,[a + b.y(p,T) + c.h(p,T) + c_1(\nu)]$$

where *b* is the purchase cost per unit item, *a* is the fixed cost per order and *c* is carrying cost of item per unit per unit time. The parameter b_1 represents the unit cost of collecting, holding, sorting and handling a returned item, which includes the collection fees paid by thefirm to consumers. The parameter b_2 is the unit price of a used returned product sold by the firm to its supplier; here we assume a positive salvage value $b_1 - b_2$ of collected used products. Some expenses are associated with running a reverse-logistics scheme. The investmentcost of the firm operating a reverse-logistics program is assumed to be a quadratic function of the return-rate coefficient, $c_1(\nu) = s\nu^2$. This assumption ensures that the incremental rate of the operational cost of the reverse-logistic program depends on the return rate. In linewith [17], it is assumed that products sold in past replenishment cycles

are collected in the current replenishment cycle, and that the return-rate for product sold in the past ith replenishment cycle follows a geometric series, with initial return-rate coefficient ν and common ratio μ, $0 < \mu < 1$. The return rate for the products sold in the past ith replenishment cycle is $\nu\mu^{i-1}$ and the sum of the return-rate for used products in the current replenishment cycle is represented by $R(\nu) = \sum^{\infty} \nu\mu^{i-1} = \nu/(1-\mu)$. The parameter μ can be estimated, from past data, by taking the ratio of the return rates of any two consecutive replenishment cycles— say for e.g. $\mu = \nu^{i}/\nu^{i-1}$ while ν is determined in an optimal manner. It is only reasonable to assume the total number of collected used items be non-negative, and that this number should not exceed the number of items sold in each replenishment cycle. Thus, the scope of the firm's profit maximization is limited by the constraint $0 < R(\nu) \leq 1$.

To determine the inventory level and total item-holding time, let the inventory start at the order size y, which then depletes due to the simultaneous effects of demand and deterioration until it becomes empty at the cycle time = T. At this time, it is replenishedby a new order, and the process repeats. The inventory process $I(p,T) = \{I(t \mid p,T) \mid 0 \leq t \leq T\}$ over the time T;

its variation is described by the inventory linear first-order differential equation

$$\frac{dI}{dt}(t \setminus p,T) = -D(p) - \theta(t)I(t \setminus p,T) \text{ for all } 0 \leq t \leq T, \quad (2)$$

Where $I(t)$ is the inventory level function, $D(p)$ and $\theta(t)$ are the demand and deterioration functions respectively.

To facilitate the analysis, define the cumulative deterioration function Θ is given by

$\Theta(t) = \int \theta(s)\,ds$ for all $0 \leq t \leq T$ for all $0 \leq t \leq T$

Solving Eq.(2), using the cumulative deterioration, the inventory-level function is:

$$I(t \mid p,T) = D(p)\int_0^T exp\{\Theta(s) - \Theta(t)\}ds \quad (3)$$

The order size at the beginning of inventory system is given by:

$$y(p,T) = D(p)\int_0^T exp\{\Theta(s)\}ds \quad (4)$$

The total item-holding time is given by:

$$h(p,T) = \int_0^T I(t \mid p,T)dt = D(p)\int_0^T \int_t^T \exp\{\theta(s) - \theta(t)\}ds\,dt \quad (5)$$

The above analysis is a general form of the inventory functions- specific forms are determined by the features of demand and deterioration rates. The general results can be applied to specific models to obtain the optimal policy for various kinds of inventory systemsunder a reverse-logistics program. We present the case in which a constant proportion of inventory items deteriorate per cycle, followed by the case which the deterioration rate is time varying.

Considering a constant deteriorating inventory item, the equation governing the inventory

Process $I(p,T) = \{I(t \mid p,T) \mid 0 \leq t \leq T\}$ is

$$I(0 \setminus p.T) = y(p,T), \quad \frac{dI}{dt}(t \setminus p,T) + \theta I(t \setminus p,T) = -D(p)$$

for all $0 \leq t \leq T$, where θ is the constant proportion items deteriorating per cycle time.

The solution to the inventory equation is $\frac{dI}{dt}(t \setminus p,T) + \theta I(t \setminus p,T) = -D(p)$

$$I.F = e^{\theta t}$$

$$I(t \mid p,T) = \frac{1}{e^{\theta t}}\int_t^T e^{\theta t}D(p)dt$$

$$= \frac{D(p)}{\theta}[e^{\theta T}.e^{-\theta t}.e^{-\theta t}]$$

$$= \frac{D(p)}{\theta}[e^{\theta(T-t)} - e^{0}] = \frac{D(p)}{\theta}[e^{\theta(T-t)} - 1] \qquad \text{for all } 0 \le t \le T$$

The order size is obtained using Eq. (4)

$$y(p,T) = D(p)\int_0^T \exp\{\theta(s)\}\,ds$$

$$= D(p)\left[\frac{e^{\theta(s)}}{\theta}\right]_0^T$$

$$= D(p)\left[\frac{e^{\theta T} - 1}{\theta}\right]_0^T$$

The total item holding time is obtained using Eq. (5)

$$h(p,T) = \int_0^T \mathrm{I(t/p,T)}\,dt$$

$$= D(p)\int_0^T \int_t^T \exp\{\theta(s) - \theta(t)\}\,ds\,dt$$

$$= \frac{D(p)}{\theta^2}\left[e^{\theta T} - \theta T - 1 - 0\right]$$

$$= \frac{D(p)}{\theta^2}\left[e^{\theta T} - \theta T - 1\right]$$

Substituting these expressions in Eq. (1), the firm's profit function can be written as

$$\pi(p,v,T) = [p + (b_1 - b_2)R(v)]D(p) - \frac{1}{T}[a + b.y(p,t) + c.h(p,T) + c_1(v)]$$

$$= \left[\left(p + \frac{c}{\theta}\right) + (b_1 - b_2)\frac{v}{1-\mu} - \left(b + \frac{c}{\theta}\right)\left(\frac{e^{\theta T} - 1}{\theta T}\right)\right]D(p) - \left[\frac{a + sv^2}{T}\right] \qquad (6)$$

There is no exact solution for the decision variables using the profit function in Eq. (6) because of the presence of 1/(θT) and exp{θT}, which does not augur well for solving explicitly for T. The exact solution for the cycle length T can be described in terms of the Lambert W- function, also called the Product Log function. Although it is possible to obtain the exact solution using iterative schemes, we adopt a quadratic approximation of the exponential term for the sake of analytic and computational convenience. Some authors have favored this approach. To obtain the specific structure of the rate of withdrawal of items fromthe inventory system, we assume a linear demand rate D (p) = m − kp where m is the fiXed demand and k is the elasticity coefficient. The average profit-rate for this model is the

$$\pi(p,v,T) = \left[\left(p + \frac{c}{\theta}\right) + (b_1 - b_2)\frac{v}{1-\mu} - \left(b + \frac{c}{\theta}\right)\left(\frac{e^{\theta T} - 1}{\theta T}\right)\right](m - kp) - \left[\frac{a + sv^2}{T}\right] \qquad (7)$$

The firm seeks to maximize the average profit rate by appropriate choice of the price p, return rate ν and cycle length T, which from the expression of y(p,T) determines the order size. We solve the optimization problem by considering four possible scenarios. Firstly, we consider the price level and the return rate to be fixed exogenous values, and determine the optimal cycle length for a given price and return rate. Secondly, we determine the optimal price level for a fixed cycle length and return rate. Thirdly, we consider the price level to be a fixed exogenous value, and determine the optimal cycle length and optimal return-rate. Finally, we consider the case in which the price level, the return-rate and the cycle length are all decision variables limited only by the return-rate constraint.

CONCLUSION

For decaying products, we presented an EOQ model with a reverse logistics approach. We looked at how the reverse logistics program and item deterioration affected inventory management choices. Businesses can reap long-term benefits and lower money spent on returns by streamlining the reverse-logistics operations. Our research adds to the body of knowledge on EOQ by demonstrating how the firm's best decisions are impacted by both item deterioration and investment in a reverse-logistics program. The potential of EOQ to evolve with the complexity of modern inventory management is what will determine its future reach. Reverse logistics considerations can make EOQ a more comprehensive and long-lasting tool, improving inventory levels for incoming products as well as the increasingly important role that returns and product lifecycles play. Future inventory management strategies will be more responsible and effective as a result of this integration. There are various methods to expand on this work. A reverse logistics program and a reorder-point inventory are now being developed by the writers. A useful addition to the model is to take into account an inventory system that consists of several things.

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