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2-Odd Labeling in the Context of Switching of a Vertex

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ABSTRACT—**A graph G is a 2-odd graph if its vertices can be labeled with distinct integers such that for any two adjacent vertices, the absolute difference of their labels is either an odd integer or 2.In this Paper, we prove that the graphs obtained by switching of a vertex in path, cycle, wheel, helm, Jewel graph admit 2-odd labeling.**

Keywords—2-Odd labeling, 2-Odd graph, Switching of a vertex

I. INTRODUCTION

We begin with a finite, connected and undirected graph G = (V(G),E(G)) without loops and multiple edges. Throughout this paper $|V(G)|$and $|E(G)|$respectively denote the number of vertices and number of edges in G. Definitions and existing results are provided as follow.

Note : A 2-odd labeling of a graph G is not unique.

Definition 1.1. A graph labeling is an assignment of integers to the vertices or edges or both subject to certain condition(s). If the domain of the mapping is the set of vertices (or edges) then the labeling is called a vertex labeling (or an edge labeling).

Definition 1.2. A graph admits a 2-odd labeling if its vertices can be labeled with distinct integers such that for any two adjacent vertices, the absolute difference of their labels is either an odd integer or 2.

Definition 1.3. A vertex switching G_v of a graph G is the graph obtained by taking a vertex v of G, removing all the edges to v and adding edges joining v to every other vertex which are not adjacent to v in G.

Definition 1.4. The wheel graph W_n is defined to be the join $K_1 + C_n$. The vertex corresponding to K_1 is known as apex vertex and vertices corresponding to cycle are known as rim vertices while the edges corresponding to C_n are known as rim edges. We continue to recognize apex of wheel as the apex of the respective graphs obtained from wheel.

Definition 1.5. The helm H_n is the graph obtained from a wheel W_n by attaching a pendant edge to each rim vertex.

Definition 1.6. The Jewel graph J_n is the graph with the vertex set $V(J_N) = \{ f_u;\ v;\ x;\ y;\ u_i : 1 \le\ i \le n \}$ and the edge $set\ E(J_n) = \{ux;\ uy;\ xy;\ xv;\ yu;\ uu_i;\ vu_i :\ 1 \le\ i \le\ n \}$.

II. MAIN RESULTS

Theorem 2.1 : Switching of a pendant vertex in a path P_n admits a 2-Odd labeling.

Proof : Let $v_0, v_1, \dots, v_{n-1}$ be the vertices of path P_n and Gv denotes the graph obtained by switching of a pendant vertex v of G. Without loss of generality let the switched vertex be v0.
The graph has $\left|V\left(G_{V_0}\right)\right| = n\ and \left|E\left(G_{V_0}\right)\right| = 2n - 4$.
We define one-to-one function $f : V(P_n) \mapsto Z$ as follows :
Let $f(v_0) = 1$
f(vi) = 2i ; for $1 \le\ i \le\ n - 1$
An easily we can check that f is the required 2-odd labeling of switching of a pendant vertex v of path Pn . $|f(v_0) - f(v_i)|$ are all odd integers for $2 \le\ i \le\ n - 1$ and $|f(v_i) - f(v_{i+1})|\ = 2$ for $2 \le i \le n - 2$.

Illustration :

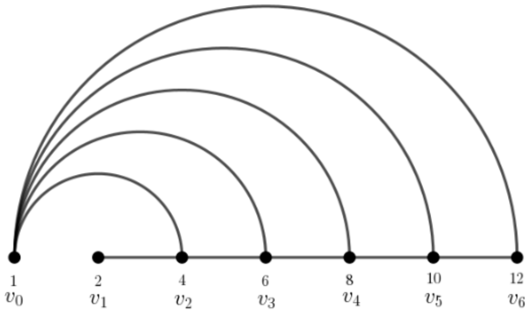

Figure 1: 2-odd labeling of switching of a pendant vertex in a path P_7

Theorem 2.2 : Switching of a vertex in a cycle C_n admits a 2-Odd labeling.

Proof : Let $v_0, v_1, \ldots, v_{n-1}$ be the vertices of cycle $C_n$and Gv denotes the graph obtained by switching of a vertex v of G.
Without loss of generality let the switched vertex be v_0.
The graph has $\left|V\left(G_{v_0}\right)\right| = n \; and \; \left|E\left(G_{v_0}\right)\right| = 2n - 5.$
We define one-to-one function $f : V(Cn) \mapsto Z$ as follows :
Let $f(v_0) = 1$
f(vi) = 2i ; for $1 \leq i \leq n - 1$
An easily we can check that f is the required 2-odd labeling of switching of a vertex v of cycle Cn . $|f(v_0) - f(v_i)|$ are all odd integers for $2 \leq i \leq n-2$ and $|f(v_i) - f(v_{i+1})| = 2$ for $2 \leq i \leq n-2$.

Illustration :

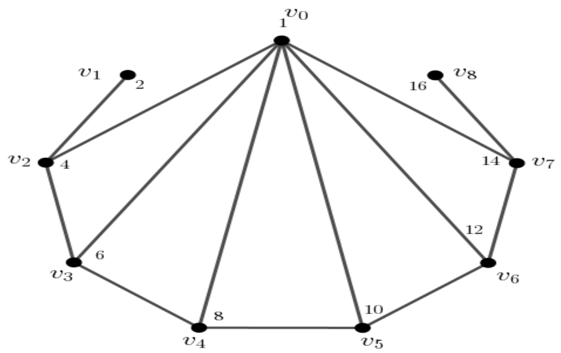

Figure 2: 2-odd labeling of switching of a vertex in a cycle C_9

Theorem 2.3 : Switching of a rim vertex in a wheel W_n admits a 2-Odd labeling.

Proof : Let $v, v_0, v_1, \ldots, v_{n-1}$ be the vertices of wheel W_n where v as the apex vertex and $v_0, v_1, \ldots, v_{n-1}$ are rim vertices. Let Gv denotes the graph obtained by switching of a rim vertex v of G.
Without loss of generality let the switched rim vertex bev_0.
The graph has $\left|V\left(G_{V_0}\right)\right| = n + 1 \; and \; \left|E\left(G_{V_0}\right)\right| = 3n - 6.$
We define one-to-one function $f : V(Wn) \mapsto Z$ as follows :
Let f(v0) = 1
f(v) = 3
f(vi) = 2i ; for $1 \leq i \leq n - 1$
An easily we can check that f is the required 2-odd labeling of switching of a rim vertex v of wheel W_n . $|f(v_0) - f(v_i)|$ are all odd integers for $2 \leq i \leq n-2$ and $|f(v_i) - f(v_{i+1})| = 2$ for $1 \leq i \leq n-2$ and $|f(v_n) - f(v_i)|$ are all odd integers for $1 \leq i \leq n-1$.

Illustration :

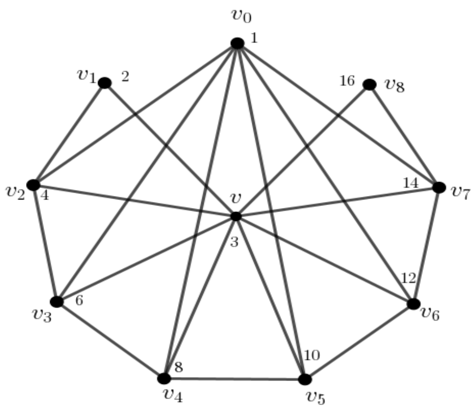

Figure 3: 2-odd labeling of switching of a rim vertex in a wheel W9

Theorem 2.4 : Switching of a apex vertex in a helm H_n admits a 2-Odd labeling.

Proof : Let $H_n$be a helm with v as the apex vertex $v_1, v_2, \ldots, v_n$ be the vertices of cycle and $u_1, u_2, \ldots, u_n$ be the pendant vertices. Let Gv denotes the graph obtained by switching of a apex vertex v of G.
Without loss of generality let the switched apex vertex be v.
The graph has $|V(G_V)| = 2n + 1$ and $|E(G_V)| = 3n.$
We define one-to-one function $f : V(H_n) \mapsto Z$ as follows :
Let $f(v_0) = 1$
$f(u_i) = 2i$; for $1 \leq i \leq n - 1$
$f(v_i) = 2i + 1$; for $1 \leq i \leq n - 1$
An easily we can check that f is the required 2-odd labeling of switching of a apex vertex v of helm H_n . $|f(v) - f(u_i)|$ are all odd integers for $1 \leq i \leq n$ and $|f(v_i) - f(v_{i+1})| = 2$ for $1 \leq i \leq n$ where $(V_{n+1} = v_1)$and $|f(u_i) - f(v_i)|$ are all odd integers for $1 \leq i \leq n.$

Illustration :

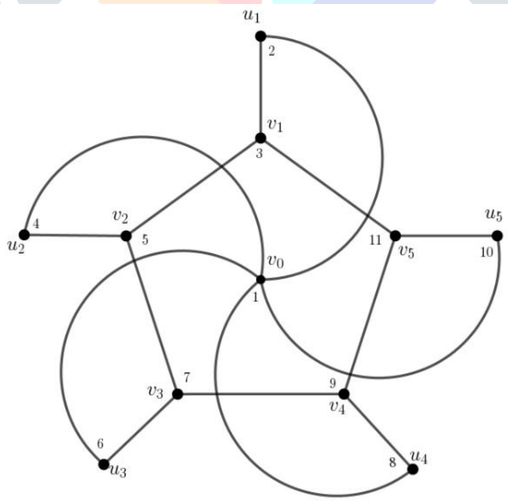

Figure 4: 2-odd labeling of switching of a apex vertex in a helm H5

Theorem 2.5 : Switching of a vertex in a jewel J_n admits a 2-Odd labeling.

Proof : Let $u_1, u_2, u_3, u_4, v_1, v_2, \ldots, v_n$ be the vertices of jewel graph J_n and Gv denotes the graph obtained by switching of a vertex v of G.
Without loss of generality let the switched vertex be u_1.
The graph has $|V(G_V)| = n + 4$ and $|E(G_V)| = 3n + 2.$
We define one-to-one function $f : V(J_n) \mapsto Z$ as follows :
Let $f(u_1) = 1$
$f(u_2) = 3$
$f(u_3) = 5$
$f(u_4) = 7$
$f(v_i) = 2i$; for $1 \leq i \leq n$
An easily we can check that f is the required 2-odd labeling of switching of a vertex v of jewel J_n. $|f(u_1) - f(v_i)|, |f(u_2) - f(v_i)|, |f(u_4) - f(v_i)|$, are all odd integers for $1 \leq i \leq n$ and $|f(u_3) - f(u_i)| = 2$ for $i = 2,4.$

Illustration :

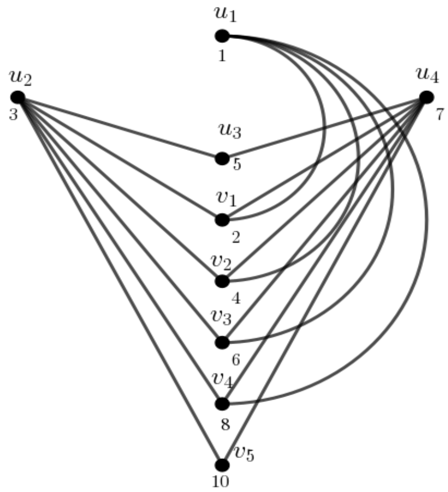

Figure 5: 2-odd labeling of switching of a vertex in a jewel graph J5

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