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The Mathematics Behind Fractals, Their Generation, and Their Surprising Appearances in Nature

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Abstract

Fractals are intricate mathematical structures that exhibit self- similarity — a property where parts of the shape resemble the whole at different scales. Unlike traditional geometric shapes such as circles or squares, fractals can possess infinitely complex boundaries and fractional dimensions, lying between integer values. They are generated through iterative mathematical processes, often using recursive functions or algorithms, which can produce infinitely detailed patterns from simple equations. This project explores the underlying mathematics of fractals, starting with the pioneering work of Benoît Mandelbrot, who formalized the concept in the late 20th century. We examine common fractal sets, such as the Mandelbrot Set, Julia Set, and Sierpiński Triangle, detailing their generation using iteration and complex numbers. The study extends to computational methods, such as escape-time algorithms, that are used to visualize these structures in digital form. Fractals are not only theoretical constructs but also manifest widely in the natural world. Coastlines, mountain ranges, river networks, snowflakes, clouds, and even biological systems like lung bronchi and blood vessels reveal fractal- like patterns. Their efficiency in filling space and optimizing growth makes them a recurring blueprint in natural processes. The analysis highlights how fractals bridge mathematics, computer science, and natural phenomena. By understanding their properties and generation, we uncover powerful tools for modeling complex systems. This project concludes with a discussion on their practical applications in fields such as computer graphics, antenna design, and environmental modeling, illustrating that fractals are both a mathematical curiosity and a cornerstone of scientific and technological innovation.

Introduction

Fractals represent a class of mathematical shapes characterized by self- similarity and infinite complexity. Unlike Euclidean geometry, which deals with smooth and regular shapes, fractal geometry deals with irregular, fragmented patterns that remain complex regardless of magnification. The term fractal was coined by Benoît Mandelbrot in 1975, deriving from the Latin fractus, meaning "broken" or "fractured".

Fractals can be generated through iterative processes, often starting with a simple geometric figure or equation and applying a transformation repeatedly. This gives rise to intricate patterns that can be deterministic (like the Koch Snowflake) or stochastic (like natural terrain generation). Their unique property of having a non-integer, or fractional, dimension allows them to describe real-world shapes more accurately than traditional geometry.

From modeling galaxies to predicting market fluctuations, fractals provide insight into complex, self-organizing systems.

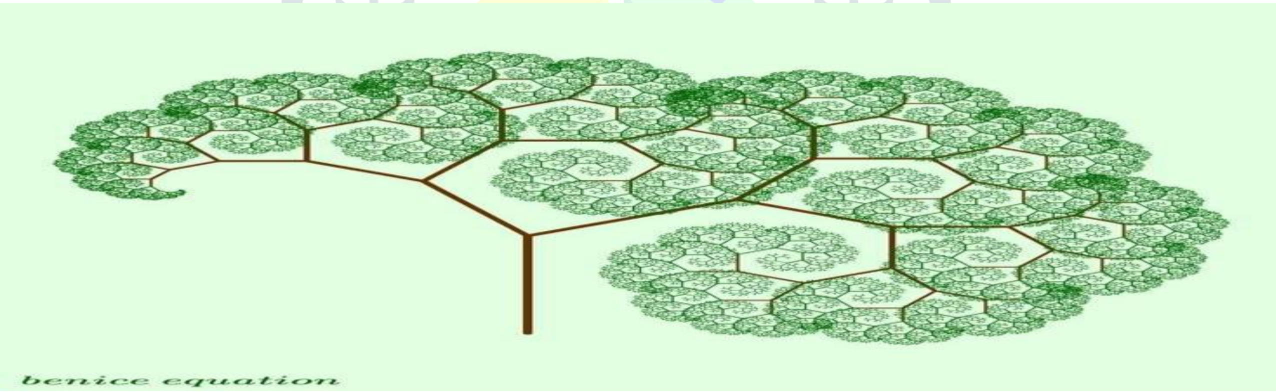
The word "fractal" often has different connotations for mathematicians and the general public, where the public is more likely to be familiar with fractal art than the mathematical concept. The mathematical concept is difficult to define formally, even for mathematicians, but key features can be understood with a little mathematical background.

The feature of "self-similarity", for instance, is easily understood by analogy to zooming in with a lens or other device that zooms in on digital images to uncover finer, previously invisible, new structure. If this is done on fractals, however, no new detail appears; nothing changes and the same pattern repeats over and over, or for some fractals, nearly the same pattern reappears over and over. Self-similarity itself is not necessarily counter-intuitive (e.g., people have pondered self-similarity informally such as in the infinite regress in parallel mirrors or the homunculus, the little man inside the head of the little man inside the head ..). The difference for fractals is that the pattern reproduced must be detailed.[1]

This idea of being detailed relates to another feature that can be understood without much mathematical background: Having a fractal dimension greater than its topological dimension, for instance, refers to how a fractal scales compared to how geometric shapes are usually perceived. A straight line, for instance, is conventionally understood to be one-dimensional; if such a figure is rep-tiled into pieces each 1/3 the length of the original, then there are always three equal pieces. A solid square is understood to be two-dimensional; if such a figure is rep-tiled into pieces each scaled down by a factor of 1/3 in both dimensions, there are a total of 32 = 9 pieces.

We see that for ordinary self-similar objects, being n-dimensional means that when it is rep-tiled into pieces each scaled down by a scale-factor of 1/r, there are a total of rn pieces. Now, consider the Koch curve. It can be rep-tiled into four sub-copies, each scaled down by a scale-factor of 1/3. So, strictly by analogy, we can consider the "dimension" of the Koch curve as being the unique real number D that satisfies 3D = 4. This number is called the fractal dimension of the Koch curve; it is not the conventionally perceived dimension of a curve. In general, a key property of fractals is that the fractal dimension differs from the conventionally understood dimension (formally called the topological dimension). [3]

Fractal tree diagram



Literature Review

Early studies on fractal-like patterns date back to the work of mathematicians such as Georg Cantor (Cantor set), Helge von Koch (Koch curve), and Wacław Sierpiński (Sierpiński triangle). However, it was Mandelbrot who unified these ideas under a common theory and demonstrated their relevance to natural phenomena.

Key contributions include:

Mandelbrot (1982) – Formalized fractal geometry and introduced the Mandelbrot set.

Barnsley (1988) – Applied iterated function systems (IFS) for computer generation of fractals.

Turcotte (1997) – Illustrated fractals in geophysics, particularly in earthquakes and landscape formation.

Studies on biological fractals have shown that vascular systems, leaf venation, and neural structures exhibit fractal scaling laws.

The development of computer graphics has been instrumental in visualizing fractals, making them both a research tool and an artistic medium.

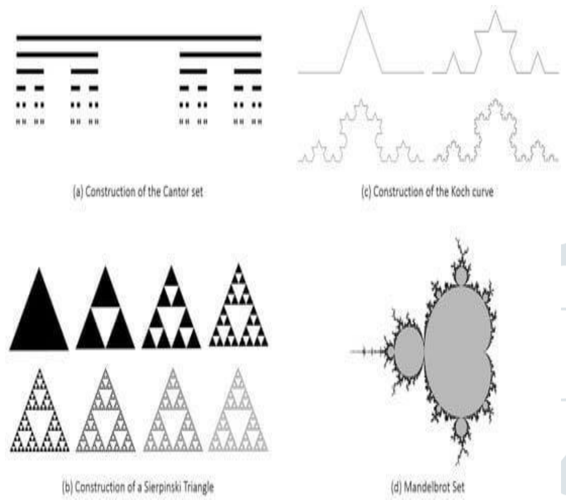
Basically, Fractals are used in many areas for its importance such as- Astronomy : for the analysis purpose of

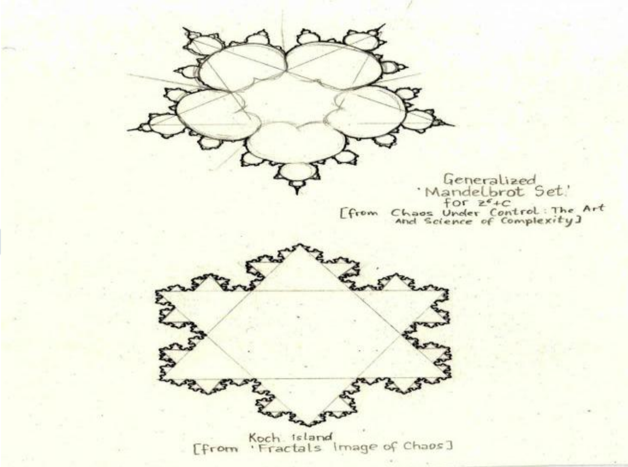
Saturn's rings , galaxies etc.

Biology/Chemistry : For purpose of illustrating chemical reactions, human anatomy, molecules, plants and bacterial cultures.

Others : For the purpose of representing the required clouds, borders, the coast , data representation, diffusion, economy, fractal music, fractal art, landscapes, special effects and so forth.

Generation of fractals is repeating the same shape repeatedly can result in fractals. To get the appropriate shape and size , we can iterate indefinitely. Recursion is the computer language term for making such form as per requirement





Methodology

This study involved:

Mathematical Exploration – Understanding recursive equations and complex number iterations (e.g.).

Algorithmic Implementation – Using escape-time algorithms to plot the Mandelbrot and Julia sets.

Natural Pattern Analysis – Collecting high-resolution images of coastlines, plants, and clouds to compare with mathematical fractals using fractal dimension estimation (box-counting method).

Fractal analysis: This is the process of using various methods to characterize a system's fractal properties.

Box-counting method: A relatively simple method where you count how many boxes of a certain size it takes to cover the fractal. By changing the box size and repeating the count, you can estimate the fractal dimension

Hausdorff dimension: A more mathematically rigorous method for calculating fractal dimension, often expressed as

D=log(N)/log(s)cap D equals log open paren cap N close paren / log s

$$D=\log(N)/\log(s)$$

Fractal interpolation: Used in image processing to increase resolution. An image is encoded using fractal compression, then decompressed at a higher resolution, often preserving geometric details better than traditional methods.

Fractal antennas: These use fractal geometry to create antennas that are smaller and can pick up a wider range of frequencies than traditional antennas.

History

Recursion, leading to the popular meaning of the term "fractal".The history of fractals traces a path from chiefly theoretical studies to modern applications in computer graphics, with several notable people contributing canonical fractal forms along the way. A common theme in traditional African architecture is the use of fractal scaling, whereby small parts of the structure tend to look similar to larger parts, such as a circular village made of circular houses. According to Pickover, the mathematics behind fractals began to take shape in the 17th century when the mathematician and philosopher Gottfried Leibniz pondered recursive self-similarity (although he made the mistake of thinking that only the straight line was self-similar in this sense).[32]

In his writings, Leibniz used the term "fractional exponents", but lamented that "Geometry" did not yet know of them. Indeed, according to various historical accounts, after that point few mathematicians tackled the issues and the work of those who did remained obscured largely because of resistance to such unfamiliar emerging concepts, which were sometimes referred to as mathematical "monsters".Thus, it was not until two centuries had passed that on July 18, 1872 Karl Weierstrass presented the first definition of a function with a graph that would today be considered a fractal, having the non-intuitive property of being everywhere continuous but nowhere differentiable at the Royal Prussian Academy of Sciences.[8]

In addition, the quotient difference becomes arbitrarily large as the summation index increases. Not long after that, in 1883, Georg Cantor, who attended lectures by Weierstrass, published examples of subsets of the real line known as Cantor sets, which had unusual properties and are now recognized as fractals. Also in the last part of that century, Felix Klein and Henri Poincaré introduced a category of fractal that has come to be called "self-inverse" fractals.[1]

One of the next milestones came in 1904, when Helge von Koch, extending ideas of Poincaré and dissatisfied with Weierstrass's abstract and analytic definition, gave a more geometric definition including hand- drawn images of a similar function, which is now called the Koch snowflake. Another milestone came a decade later in 1915, when Wacław Sierpiński constructed his famous triangle then, one year later, his carpet. By 1918, two French mathematicians, Pierre Fatou and Gaston Julia, though working independently, arrived essentially simultaneously at results describing what is now seen as fractal behaviour associated with mapping complex numbers and iterative functions and leading to further ideas about attractors and repellors (i.e., points that attract or repel other points), which have become very important in the study of fractals.[35]

Very shortly after that work was submitted, by March 1918, Felix Hausdorff expanded the definition of "dimension", significantly for the evolution of the definition of fractals, to allow for sets to have non-integer dimensions.The idea of self-similar curves was taken further by Paul Lévy, who, in his 1938 paper Plane or Space Curves and Surfaces Consisting of Parts Similar to the Whole, described a new fractal curve, the Lévy C curve

Definition and characteristics

One often cited description that Mandelbrot published to describe geometric fractals is "a rough or fragmented geometric shape that can be split into parts, each of which is (at least approximately) a reduced-size copy of the whole"; this is generally helpful but limited. Authors disagree on the exact definition of fractal, but most usually elaborate on the basic ideas of self-similarity and the unusual relationship fractals have with the space they are embedded in.

One point agreed on is that fractal patterns are characterized by fractal dimensions, but whereas these numbers quantify features; complexity (i.e., changing detail with changing scale), they neither uniquely describe nor specify details of how to construct particular fractal patterns. In 1975 when Mandelbrot coined the word "fractal", he did so to

denote an object whose Hausdorff–Besicovitch dimension is greater than its topological dimension. However, this requirement is not met by space- filling curves such as the Hilbert curve. [28]

Because of the trouble involved in finding one definition for fractals, some argue that fractals should not be strictly defined at all. According to Falconer, fractals should be only generally characterized by a gestalt of the following Self-similarity, which may include:

Exact self-similarity: identical at all scales, such as the Koch snowflake

Quasi self-similarity: approximates the same pattern at different scales; may contain small copies of the entire fractal in distorted and degenerate forms; e.g., the Mandelbrot set's satellites are approximations of the entire set, but not exact copies.

Statistical self-similarity: repeats a pattern stochastically so numerical or statistical measures are preserved across scales; e.g., randomly generated fractals like the well-known example of the coastline of Britain for which one would not expect to find a segment scaled and repeated as neatly as the repeated unit that defines fractals like the Koch snowflake.

Qualitative self-similarity: as in a time series Multifractal scaling: characterized by more than one fractal dimension or scaling rule.

Fine or detailed structure at arbitrarily small scales. A consequence of this structure is fractals may have emergent properties

Irregularity locally and globally that cannot easily be described in the language of traditional Euclidean geometry other than as the limit of a recursively defined sequence of stages. For images of fractal patterns, this has been expressed by phrases such as "smoothly piling up surfaces" and "swirls upon swirls";

As a group, these criteria form guidelines for excluding certain cases, such as those that may be self-similar without having other typically fractal features. A straight line, for instance, is self-similar but not fractal because it lacks detail, and is easily described in Euclidean language without a need for recursion.[1]

When Mandelbrot introduced the term fractal, he excluded magnification range as a defining characteristic in order to accommodate physical fractals with more limited ranges than their mathematical counterparts.[34]

Common techniques for generating fractals

Images of fractals can be created by fractal generating programs. Because of the butterfly effect, a small change in a single variable can have an unpredictable outcome.

Iterated function systems (IFS) – use fixed geometric replacement rules; may be stochastic or deterministic; e.g., Koch snowflake, Cantor set, Haferman carpet, Sierpinski carpet, Sierpinski gasket, Peano curve, Harter-Heighway dragon curve, T-square, Menger sponge

Strange attractors – use iterations of a map or solutions of a system of initial-value differential or difference equations that exhibit chaos (e.g., see multifractal image, or the logistic map)

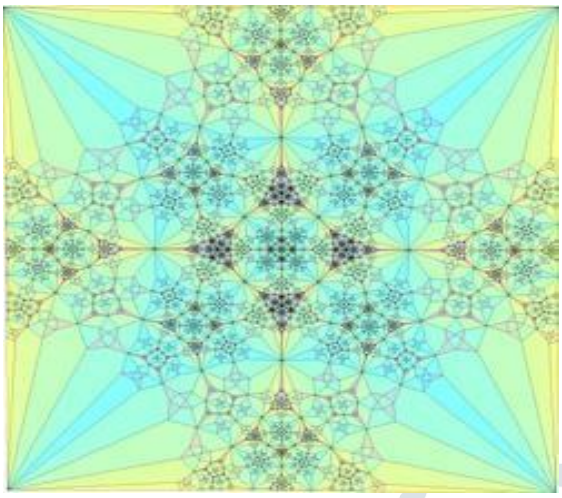
L-systems – use string rewriting; may resemble branching patterns, such as in plants, biological cells (e.g., neurons and immune system cells), blood vessels, pulmonary structure etc. or turtle graphics patterns such as space-filling curves and tiling’s

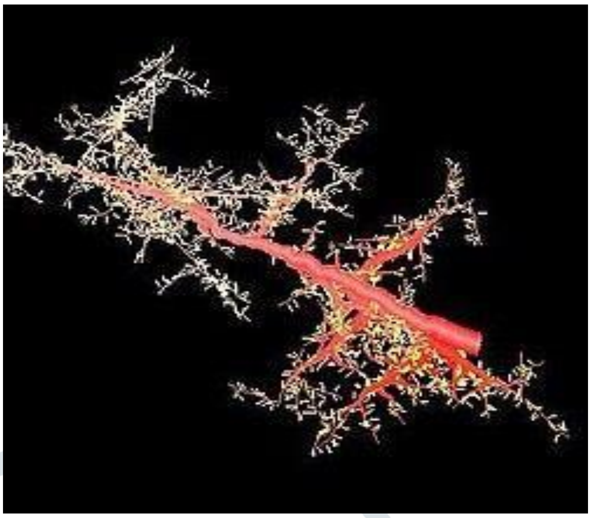
Escape-time fractals – use a formula or recurrence relation at each point in a space (such as the complex plane); usually quasi-self-similar; also known as "orbit" fractals; e.g., the Mandelbrot set, Julia set, Burning Ship fractal, Nova fractal and Lyapunov fractal. The 2d vector fields that are generated by one or two iterations of escape-time formulae also give rise to a fractal form when points (or pixel data) are passed through this field repeatedly.

Random fractals – use stochastic rules; e.g., Lévy flight, percolation clusters, self avoiding walks, fractal landscapes, trajectories of Brownian motion and the Brownian tree (i.e., dendritic fractals generated by modeling diffusion-limited aggregation or reaction-limited aggregation clusters).[4]

Finite subdivision rules – use a recursive topological algorithm for refining tilings and they are similar to the process of cell division. The iterative processes used in creating the Cantor set and the Sierpinski carpet are examples of finite subdivision rules, as is barycentric subdivision. [33]

A fractal generated by a finite subdivision rule





Applications

Simulated fractals

Fractal patterns have been modeled extensively, albeit within a range of scales rather than infinitely, owing to the practical limits of physical time and space. Models may simulate theoretical fractals or natural phenomena with fractal features. The outputs of the modelling process may be highly artistic renderings, outputs for investigation, or benchmarks for fractal analysis. Some specific applications of fractals to technology are listed elsewhere. Images and other outputs of modelling are normally referred to as being "fractals" even if they do not have strictly fractal characteristics, such as when it is possible to zoom into a region of the fractal image that does not exhibit any fractal properties. Also, these may include calculation or display artifacts which are not characteristics of true fractals.

Modeled fractals may be sounds,[14] digital images, electrochemical patterns, circadian rhythms, etc. Fractal patterns have been reconstructed in physical 3-dimensional space and virtually, often called "in silico" modeling. Models of fractals are generally created using fractal- generating software that implements techniques such as those outlined above.[4][13][21] As one illustration, trees, ferns, cells of the nervous system,[18] blood and lung vasculature,[44] and other branching patterns in nature can be modeled on a computer by using recursive algorithms and L-systems techniques. The recursive nature of some patterns is obvious in certain examples—a branch from a tree or a frond from a fern is a miniature replica of the whole: not identical, but similar in nature. Similarly, random fractals have been used to describe/create many highly irregular real-world objects, such as coastlines and mountains. A limitation of modeling fractals is that resemblance of a fractal model to a natural phenomenon does not prove that the phenomenon being modeled is formed by a process similar to the modeling algorithms.[21]

Natural phenomena with fractal features

Approximate fractals found in nature display self-similarity over extended, but finite, scale ranges. The connection between fractals and leaves, for instance, is currently being used to determine how much carbon is contained in trees. Phenomena known to have fractal features include:

Crystals Dust grain, Geometrical optics Lightning bolts, Polymers Pineapple Snowflakes Trees, Protein complexes. Etc

Fractals in cell biology

Fractals often appear in the realm of living organisms where they arise through branching processes and other complex pattern formation. Richard Taylor and co-workers have shown that the dendritic branches of neurons form fractal patterns. Ian Wong and co-workers have shown that migrating cells can form fractals by clustering and branching.[74] Nerve cells function through processes at the cell surface, with phenomena that are enhanced by largely increasing the surface to volume ratio. As a consequence nerve cells often are found to form into fractal patterns. These processes are crucial in cell physiology and different pathologies.

Multiple subcellular structures also are found to assemble into fractals. Diego Krapf has shown that through branching processes the actin filaments in human cells assemble into fractal patterns. Similarly Matthias Weiss showed that the endoplasmic reticulum displays fractal features. The current understanding is that fractals are ubiquitous in cell biology, from proteins, to organelles, to whole cells.

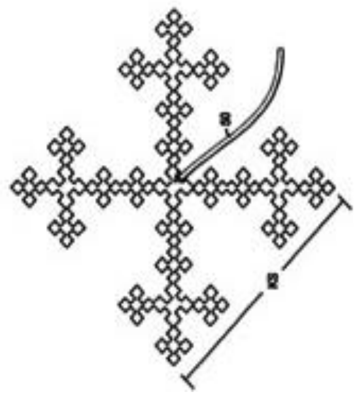
In creative works

Fractal expressionism is used to distinguish fractal art generated directly by artists from fractal art generated using mathematics and/or computers.[30] Since 1999 numerous scientific groups have performed fractal analysis on over 50 paintings created by Jackson Pollock by pouring paint directly onto horizontal canvasses, see for example. In 2015, fractal analysis was used to achieve a 93% success rate in distinguishing real from imitation Pollocks. A 2024 study used an artificial intelligence technique based on fractals to achieve a 99% success rate. Decalcomania, a technique used by artists such as Max Ernst, can produce fractal-like patterns. It involves pressing paint between two surfaces and pulling them apart. Cyberneticist Ron Eglash has suggested that fractal geometry and mathematics are prevalent in African art, games, divination, trade, and architecture. Circular houses appear in circles of circles, rectangular houses in rectangles of rectangles, and so on. Such scaling patterns can also be found in African textiles, sculpture, and even cornrow hairstyles. Hokky Situngkir also suggested the similar properties in Indonesian traditional art, batik, and ornaments found in traditional houses. [24] Ethnomathematical Ron Eglash has discussed the planned layout of Benin city using fractals as the basis, not only in the city itself and the villages but even in the rooms of houses. He commented that "When Europeans first came to Africa, they considered the architecture very disorganized and thus primitive. It never occurred to them that the Africans might have been using a form of mathematics that they hadn't even discovered yet. "In a 1996 interview with Michael Silverblatt, David Foster Wallace explained that the structure of the first draft of Infinite Jest he gave to his editor Michael Pietsch was inspired by fractals, specifically the Sierpinski triangle (a.k.a. Sierpinski gasket), but that the edited novel is "more like a lopsided Sierpinsky Gasket".[23]Some works by the Dutch artist M. C. Escher, such as Circle Limit III, contain shapes repeated to infinity that become smaller and smaller as they get near to the edges, in a pattern that would always look the same if zoomed in. Biophilic fractals are patterns designed to induce the health and well-being benefits associated with exposure to nature's scenery. These include stress-reduction and enhanced cognitive capacity. Designers and architects incorporate biophilic fractals into the built environment to counter the fact that people spend 92% of their time indoors and away from nature's scenery. The Fractal Chapel designed by INNOCAD architecture in the state hospital in Graz, Austria, is a prominent example and recipient of the 2025 IIDA (International Interior Design Association) Best of Competition Award.

Physiological responses: Fractal Fluency

Fractal fluency is a neuroscience model that proposes that, through exposure to nature's fractal scenery, people's visual systems have adapted to efficiently process fractals with ease. This adaptation occurs at many stages of the visual system, from the way people's eyes move to which regions of the brain get activated. Fluency puts the viewer in a 'comfort zone' so inducing an aesthetic experience. Neuroscience experiments have shown that Jackson Pollock's fractal paintings induce the same positive physiological responses in the observer as nature's fractals and mathematical fractals. This shows that fractal expressionism is related to fractal fluency by providing motivation for artists, such as Pollock, to use Fractal Expressionism in their art to appeal to people.

Humans appear to be especially well-adapted to processing fractal patterns with fractal dimension between 1.3 and 1.5. When humans view fractal patterns with fractal dimensions in this range, these fractals reduce physiological stress and boost cognitive abilities.



Applications In technology

Fractal Bionics, Fractal antennas, Fractal transistor, Fractal heat exchangers, Digital imaging, Architecture[25], Urban growth, Classification of histopathology slides, Fractal landscape or Coastline complexity, Detecting 'life as we don't know it' by fractal analysis, Enzymes (Michaelis–Menten kinetics), Generation of new music, Signal and image compression, Creation of digital photographic enlargements, Fractal in soil mechanics, Computer and video game design, Computer graphics, Organic environments, Procedural generation, Fractography and fracture mechanics, Small angle scattering theory of fractally rough systems, T-shirts and other fashion, Generation of patterns for camouflage, such as MARPAT, Digital sundial, Technical analysis of price series, Fractals in networks, Medicine Neuroscience,Diagnostic imaging
Pathology,Geology,Geography,Archaeology,Soil mechanics,Seismology,Search and rescue,Morton order space filling curves for GPU cache coherency in texture mapping, rasterisation and indexing of turbulence data.

Results, Observations, and Analysis

Mathematical Fractals:

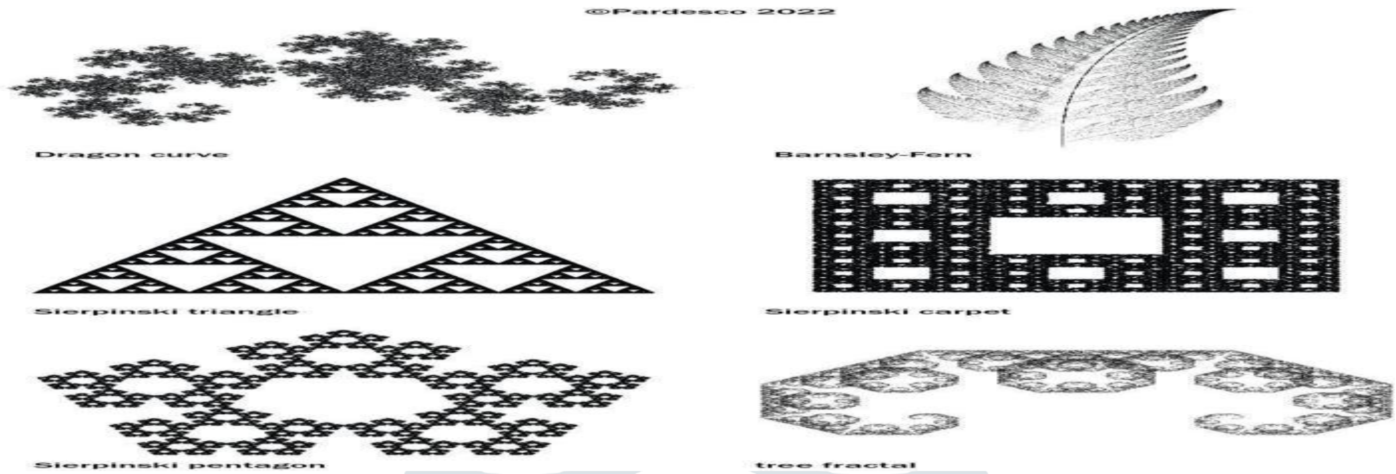
Generated Mandelbrot and Julia sets displayed infinite complexity, with intricate boundaries at every zoom level. Fractal dimension calculations revealed values between 1 and 2 for curve-based fractals and between 2 and 3 for surface-like fractals.

Natural Fractals:

Coastlines exhibited fractal dimensions around 1.25–1.35, aligning with Mandelbrot’s original coastline paradox findings. Fern leaf patterns matched those generated using iterated function systems with transformation scaling factors similar to real plants.

Observations:

Mathematical and natural fractals share scaling properties, suggesting that fractal geometry is a universal principle of pattern formation in nature.



Discussion, Conclusion, and Recommendations

This project confirms that fractals are more than abstract mathematical creations; they are intrinsic to the fabric of nature. Their self-similarity, scale invariance, and fractional dimension make them ideal for modeling phenomena that are otherwise too complex for classical geometry.

Conclusions:

Fractal mathematics bridges theoretical and real-world structures.

Iterative algorithms provide efficient ways to generate and study fractals.

Natural patterns often conform to fractal scaling laws, enhancing our understanding of growth, efficiency, and resource distribution in biological and geological systems.

Recommendations:

Further research on fractal dynamics in climate modeling and biological growth.

Integration of fractal-based algorithms in AI for image recognition of natural structures.

Development of educational tools for visualizing fractal geometry interactively.

References

1. Benoit Mandelbrot, B. B. (1982). The Fractal Geometry of Nature. W. H. Freeman & Co.
2. Michael Fielding Barnsley, M. F. (1988). Fractals Everywhere. Academic Press.

Falconer, K. (2003). Fractal Geometry: Mathematical Foundations and Applications. Wiley.

3. Turcotte, D. L. (1997). Fractals and Chaos in Geology and Geophysics. Cambridge University Press.
4. Peitgen, H.-O., Jürgens, H., & Saupe, D. (2004). Chaos and Fractals: New Frontiers of Science. Springer.
5. *Mandelbrot, Benoît B. (1983). The fractal geometry of nature* .
6. Falconer, Kenneth (2003).Fractal Geometry: Mathematical Foundations and Applications. John Wiley & Sons.
7. Briggs, John (1992).Fractals: The Patterns of Chaos. London: Thames and Hudson. p.

Tamás (1992).Fractal growth phenomena. Singapore/New Jersey: World Scientific.

Gouyet, Jean-François (1996).Physics and fractal structures.

8. Mandelbrot, Benoît B. (2004).Fractals and Chaos . “A fractal set is one for which the fractal (Hausdorff-Besicovitch) dimension strictly exceeds the topological dimension”
9. Segal, S.L. (June 1978). "Riemann's example of a continuous 'nondifferentiable' function continued". The Mathematical Intelligencer Edgar, Gerald (2004).Classics on Fractals.
10. Trochet, Holly (2009)."A History of Fractal Geometry".MacTutor History of Mathematics. Archived fromthe originalon March 12, 2012.
11. Mandelbrot, Benoit (July 8, 2013). "24/7 Lecture on Fractals" . 2006 Ig Nobel Awards

Mandelbrot, BB: The Fractal Geometry of Nature. W. H. Freeman and Company, New York (1982) Edgar, Gerald (2007).

12. Measurement,Topology, and Fractal Geometry

13. Peters, Edgar (1996).Chaos and order in the capital markets: a new view of cycles, prices, and market volatility.

14. Brothers, Harlan J. (2007). "Structural Scaling in Bach's Cello Suite No. *3".Fractals.*

Tan, Can Ozan; Cohen, Michael A.; Eckberg, Dwain L.; Taylor, J. Andrew (2009)."Fractal properties of human heart period variability: Physiological and methodological implications".The Journal of Physiology Liu, Jing Z.; Zhang, Lu D.; Yue, Guang H. (2003).

15. "Fractal Dimension in Human Cerebellum Measured by Magnetic Resonance Imaging".Biophysical Journal.
16. Karperien, Audrey L.; Jelinek, Herbert F.; Buchan, Alastair M. (2008). "Box-Counting Analysis of Microglia Form in Schizophrenia, Alzheimer's Disease and Affective Disorder".Fractals.
17. Losa, Gabriele A.; Nonnenmacher, Theo F. (2005). Fractals in biology and medicine
18. Wallace, David Foster (August 4, 2006). "Bookworm on KCRW"
19. Eglash, Ron (1999). "African Fractals: Modern Computing and Indigenous Design"
20. Ostwald, Michael J., and Vaughan, Josephine (2016) The Fractal Dimension of Architecture Birhauser, Basel.
21. Baranger, Michael. "Chaos, Complexity, and Entropy: A physics talk for non-physicists"
22. Benoît Mandelbrot, Objets fractals, 1975
23. Albers, Donald J.; Alexanderson, Gerald L. (2008). "Benoît Mandelbrot: In his own words". Mathematical people: profiles and interviews
24. "fractal". Oxford English Dictionary (Online ed.). Oxford University Press.
25. Gordon, Nigel (2000). Introducing fractal geometry Eglash, Ron (1999). African Fractals Modern Computing and Indigenous Design. Pickover, Clifford A. (2009). The Math Book: From Pythagoras to the 57th Dimension, 250 Milestones in the History of Mathematics "Fractal Geometry"
26. Mandelbrot, B. (1967).
27. Batty, Michael (April 4, 1985). "Fractals – Geometry Between Dimensions".
28. Russ, John C. (1994). Fractal surfaces
29. Edgar, Gerald (2008). Measure, topology, and fractal geometry.

Karperien, Audrey (2004). Defining microglial morphology: Form, Function, and Fractal Dimension.Spencer, John; Thomas, Michael S. C.; McClelland, James L. (2009). Toward a unified theory of development: connectionism and dynamic systems theory re-considered