



# SOLITONS IN OPTICAL COMMUNICATION SYSTEM

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**Abstract:** Communication system has been revolutionized with the advent of the optical fiber as it enables the high-speed data transmission up to thousands of kilometers across the globe with minimum distortion. Different methods are used to improve the performance of optical communication system but optical soliton is the most attractive phenomenon which use the dispersion and non-linear effect in such a way that they compensate each other and thus allows the pulse to travel without any distortion. This review was focused on the potential of soliton in optical communication system. The review started from the fundamentals of communication system. Then, it traced the physics of pulse propagation via optical fibre. The Maxwell's equation is used to derive the nonlinear Schrödinger equation (NLSE), and its dark and bright soliton solution is investigated. The higher-order NLSE is introduced for the future perspectives.

**Index Terms -** Nonlinear Schrödinger equation, optic communication system, optical soliton.

## I. INTRODUCTION

Today, India is moving forward toward the goals of mission “Viksit Bharat@2047” by taking advantage of the potential of science, innovation and technology. A country will be developed through self sustained growth of economy, social empowerment and indigenous technological advancements. More specifically, basic science research is needed to speed up industrial growth which will lead to overall growth of the nation. The National Broadband Mission (NBM) is trying to bridge the gap between urban and rural India by connecting every Gram Panchayat with universal, affordable and high-quality broadband. As the NBM aims to extends the operational optical fiber cable connectivity to 2.70 lakh villages by 2030. So naturally it becomes important for us to understand the functioning of the optic communication system and hence to develop the indigenous technology in this field. So, in this review we will first discuss the fundamentals of the optic communication system and the we will switch to the theoretical model which governs the signal transmission and in the last section we will consider the future perspective of optic communication by considering higher order nonlinear Schrödinger equation, graded-index waveguide and tapered graded-index waveguide

## II. FUNDAMENTALS OF OPTIC COMMUNICATION SYSTEM

Optical fiber communication is a communication method in which light is used to transmit information from one place to other one. It consists of an optical transmitter and an optical receiver connected to the two ends of communication channel using optical fiber as the connecting media [1]. The three main components of the system are: Optical transmitter, Communication channel and the Optical receiver.

### 2.1 Optical transmitter

It is basically an electro-optic device which converts the electrical signal into an optical one and then launches this optical signal into the communication channel for the transmission. Further, Optical transmitter also consists of an optical source and optical modulator. Optical Source is generally a semiconductor laser diode or light emitting diode (LED) that generates coherent light. LEDs are used when the signal is to be transmitted over a short distance. As the bandwidth of the optical signal is not so higher, so it is employed for low data rate application. While the bandwidth of the semiconductor laser diode is high so this is used when we have to transmit information over long distances and with higher data rates. The Optical modulator is utilized to modulate the signal amplitude, phase or polarization in a controlled manner. The amplitude of the signal can be modulated by changing the current driving the optical source but this causes the pulse spreading and the loss of information. Usually, an external modulator is used for the purpose of modulation.

### 2.2 Optical receiver

It changes an optical signal into an electrical signal letting data be processed by electronic devices. Photodetector and demodulator are the two parts of the optical receiver. Photodetector is generally a semiconductor device that changes the signal into an electrical signal. The response time of a photodiode decides how fast data can be recovered. Photodiode offers high speed responses. For applications, we may use P-N photodiode, P-I-N photodiode or avalanche photodiode. Usually, the signal is amplified by an amplifier so that it becomes detectable. Demodulator is a circuit that reverses the work of a modulator and pulls out the original data that was encoded onto the light signal during transmission through the optical fiber.

### 2.3 Communication channel

Optical fiber cables serve as communication channels that enable the transmission of optical signals from the optical transmitter to the optical receiver. An optical fiber is a cylindrical dielectric waveguide composed of low-loss materials, most commonly high-purity fused silica glass. It is structured in four layers: core, cladding, buffer, and jacket.

The core of an optical fiber, typically composed of highly pure silica, is the central region that guides light. The core possesses a higher refractive index than the surrounding cladding, which consists of one or more layers of material with a lower refractive index. Light signals propagate through the core by total internal reflection (TIR), ensuring that the optical signal remains confined within the core due to the refractive index difference at the core-cladding interface. Optical fibers are classified into two types: single-mode and multimode fibers, based on the number of light modes they support. Single-mode fibers have a very small core diameter and support only one mode of light at a time, making them suitable for long-distance communication with minimal signal loss. In contrast, multimode fibers have a larger core diameter, allowing multiple modes of light to propagate simultaneously. Multimode fibers are further categorized as step-index or graded-index fibers, depending on the variation of refractive index within the core. In step-index fiber the refractive index changes abruptly at core-cladding interface while in the graded-index fiber refractive index varies gradually as we move away from centre of core.

### 2.4 Factors affecting the strength of signal propagating through the optical fiber

As the signal propagates through the optical channel there are numerous factors like dispersion, absorption, Rayleigh scattering and other nonlinear effects also arises when we use optical source with higher power, which affects the strength of the signal and hence leads to attenuation of the signal. Here we will be emphasized on dispersion and the nonlinear effect.

#### 2.4.1 Group velocity dispersion

When the pulse moves through the channel, the different frequency components will advance with different velocities and hence this will lead to the broadening of the pulse. And because of the pulse broadening shape of the light pulse will be distorted which will affect the integrity of signal. Hence, dispersion poses a serious challenge to transmit data without any distortion through the optical fiber.

#### 2.4.2 Nonlinear effect (Kerr effect)

Whenever a light pulse of higher intensity is transmitted through optical fiber it will modifies the refractive index of the media and hence give rise to various nonlinear effect like Kerr effect, Self-Phase Modulation (SPM), Self-Frequency Shift (SFS), Stimulated Raman Scattering (SRS) etc. But we will restrict only with Kerr Effect which is fundamental for the formation of the soliton. Due to Kerr effect the refractive index of the media becomes dependent upon the intensity of light pulse as given as:  $n = n_0 + n_{nl}(I)$ , where  $n_0$  is the linear refractive index coefficient and the  $n_{nl}(I)$  term determines the variation in refractive index with intensity.

#### 2.4.3 Soliton formation

When the group velocity dispersion is exactly trade off by the Kerr nonlinearity of the optical media then the soliton is formed. Optical soliton preserves their shape and the velocity for large propagation distance and hence they can be used for high speed and large distance communication [2]. We will explore the model equation governing the pulse propagation through the optical channel in next section.

### III. MODEL EQUATION FOR WAVE PROPAGATION THROUGH THE OPTICAL FIBER CABLE

The nonlinear equation governing the light signal through the optical waveguide is well known nonlinear Schrödinger equation (NLSE) for the complex envelope of light field whose general form is

$$i \frac{\partial \psi}{\partial z} + a_1 \frac{\partial^2 \psi}{\partial x^2} + a_2 |\psi|^2 \psi = 0; \quad (1)$$

where  $\psi(x, z)$  is the complex envelope of the electric field,  $a_1$  is coefficient of the group velocity dispersion,  $a_2$  represents cubic nonlinearity. And this is equation is called nonlinear Schrödinger equation because its form resembles with the Schrödinger equation in quantum mechanics [2].

#### 3.1 Derivation of NLSE

The propagation of light field through an optical medium can be studied by using the Maxwell's equations which describes the relation between four field vectors such as the electric field  $\mathbf{E}$ , the electric displacement  $\mathbf{D}$ , the magnetic field  $\mathbf{H}$  and the magnetic flux density  $\mathbf{B}$ . Maxwell's equations in vector forms are

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \quad (2)$$

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}, \quad (3)$$

$$\nabla \cdot \mathbf{D} = \rho, \quad (4)$$

$$\nabla \cdot \mathbf{B} = 0, \quad (5)$$

where  $\mathbf{J}$  and  $\rho$  are the free current and charge densities, respectively. Both  $\mathbf{J}$  and  $\rho$  vanishes for a magnetic medium and the flux densities are given as

$$\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P}, \quad (6)$$

$$\mathbf{B} = \mu_0 \mathbf{H}, \quad (7)$$

where  $\mathbf{P}$  is the induced electric polarizations,  $\epsilon_0$  and  $\mu_0$  are the permittivity and the permeability of the free space, respectively.

Considering the curl of Eq. (2) and using other relations we have

$$\nabla^2 \mathbf{E} - \frac{1}{c^2} \frac{\partial^2 \mathbf{E}}{\partial t^2} = \frac{1}{\epsilon_0 c^2} \frac{\partial^2 \mathbf{P}}{\partial t^2}, \quad (8)$$

where  $c$  is the speed of the light in free space. And the induced polarization in a nonlinear dielectric medium is expressed as

$$\mathbf{P} = \epsilon_0 [\chi^{(1)} \cdot \mathbf{E} + \chi^{(2)} \cdot \mathbf{E}\mathbf{E} + \chi^{(3)} \cdot \mathbf{E}\mathbf{E}\mathbf{E} + \dots \dots \dots] \quad (9)$$

where  $\chi^{(i)}$  is the  $i$ -th order susceptibility tensor. In case of dielectric media, only  $\chi^{(1)}$  contributes significantly. The third order susceptibility tensor  $\chi^{(3)}$  is the lowest order nonlinear effect which gives rise to nonlinear effect such as intensity dependent refractive index, third-order harmonic generation and four wave mixing.

Considering that the electric field is polarized in the  $y$ -axis, confined along the  $x$ -axis and propagating in  $z$ -axis, the general solution of Eq. (8) is given as

$$\mathbf{E} = \frac{1}{2} \{ u(z, x) \exp [i(\beta_0 z - \omega_0 t)] + c. c. \} \hat{x} \quad (10)$$

and the  $\beta_0$  is propagation constant is given by  $\beta_0 = k_0 n_0 = \frac{2\pi n_0}{\lambda}$ , while the  $\omega_0$  is the frequency.

Using the equation (10) into the eq. (9), the induced polarization becomes

$$\mathbf{P} = \epsilon_0 (\chi^{(1)} + \frac{3}{4} \chi^{(3)} |\mathbf{E}|^2) \mathbf{E} \quad (11)$$

Here, susceptibility can be considered as combination of linear and nonlinear parts to the polarization as  $\chi = \chi_l + \chi_{nl}$  where  $\chi_l = \chi^{(1)}$  and  $\chi_{nl} = \frac{3}{4} \chi^{(3)} |\mathbf{E}|^2$ . Consequently, the refractive index of the media becomes dependent on the intensity of wave and is given by

$$n = n_0 + n_2 |\mathbf{E}|^2 \quad (12)$$

where  $n_0$  and  $n_2$  are the linear and the nonlinear coefficient of the refractive index, respectively. Generally, the eq. (12) is written as

$$n = n_0 + n_{nl}(I) \quad (13)$$

where  $n_{nl}(I)$  specify the variation of refractive index with the intensity of light  $I = |\mathbf{E}|^2$ .

Considering the slowly varying envelope approximation (SVEA) and substituting the eq. (10) into the eq. (8), then the eq. (8) reduces to nonlinear Schrödinger equation (NLSE)

$$i \frac{\partial u}{\partial z} + \beta_0 \frac{\partial^2 u}{\partial x^2} + k_0 |u|^2 u = 0 \quad (14)$$

Using the dimensional variables

$$X = \frac{x}{w_0}, \quad Z = \frac{z}{L_D}, \quad U = (k_0 |n_2| L_D)^{1/2} u,$$

where  $L_D = \beta_0 w_0^2$  is the diffraction length, eq. (14) reduces to the following dimensionless form

$$i \frac{\partial U}{\partial Z} + \frac{1}{2} \frac{\partial^2 U}{\partial X^2} \pm |U|^2 U = 0 \quad (15)$$

where the sign ( $\pm$ ) stands for self-focusing (self-defocusing) nonlinearity.

#### IV. SOLITARY WAVES AND THE SOLITON SOLUTIONS OF THE NLSE

##### 4.1 Solitary waves and historical background

A solitary wave is a localized waves which propagates without any variation in its shape and velocity. It arises due to interplay between dispersion term and the nonlinearity of the media. Solitary wave was first observed accidentally by John Scoot Russell in 1834, while he was conducting experiments in order to find out the most efficient design for the canal boats. And he termed this wave as “wave of translation” which travels for miles before fades out.

The speed of propagation  $v$ , of a solitary wave is given by the equation

$$v = \sqrt{g(h + \rho)},$$

where  $h$  is the depth of channel,  $g$  represents the force due to gravity and  $\rho$  is the amplitude of wave. And the theoretical formulation water wave was proposed by two Dutch physicist, Kortweg and de Vries in 1895, known as KdV equation [3]. KdV equation was solved numerically in 1965 by Zabusky and Kruskal [4] for non-linear lattice. They also demonstrated the particle like behaviour of these solitary waves, as solitary waves interact elastically with each other and then because of this particle like character solitary wave is termed as soliton.

##### 4.2 Soliton solution of the NLSE

Soliton are solitary wave whose property remains intact even after the interaction between them. And because of this very particle like behaviour soliton differs from the solitary waves but nowadays, the term soliton and solitary waves are used interchangeably. And in optics soliton stands for a phenomenon where light beam or pulse propagates through a nonlinear medium without any alteration in its shape and velocity. Generally, a soliton is formed when the effect due to dispersion is exactly balanced by the nonlinearity of the medium.

Nonlinear Schrödinger equation in standard form is written as

$$i \frac{\partial \psi}{\partial z} + \frac{1}{2} \frac{\partial^2 \psi}{\partial x^2} \pm |\psi|^2 \psi = 0, \quad (16)$$

where  $\psi(z, x)$  is the complex envelope of the electric field, second term of the equation denotes group velocity dispersion (GVD) and the third term represents the cubic nonlinearity. And as the eq. (16) resembles with the form of the Schrödinger equation in quantum mechanics, that's why in the nonlinear dynamics this is known as nonlinear Schrödinger equation (NLSE). (+) sign is corresponds to self-focusing non-linearity and the (−) sign is for self-defocusing non-linearity. NLSE is completely integrable, this was demonstrated by Zakharov and Shabat by using inverse scattering transformation (IST) [5]. This equation has a class of exact solution out of them we will be emphasized on the bright and the dark soliton.

##### 4.2.1 Bright soliton

When the nonlinearity is with (+) sign i.e., self-focusing then the solution of NLSE is bright soliton. In 1973, Hasegawa and Tappert [6] theoretically predicted the existence of bright soliton. In 1980, Mollenauer et. al observed the bright soliton [7] in optical fiber for first time by using the mode-locked laser which emits short pulse with the width of picometer scale (7 ps) around 1.55  $\mu\text{m}$ . The general form of bright soliton solution is given as

$$\psi(z, x) = a \text{Sech}[a(x - vx)] e^{i(vx + (a^2 - v^2)z/2)}, \quad (17)$$

where  $a$  amplitude of soliton and  $v$  is the transverse velocity of propagating soliton. Bright soliton has a bell-shaped form (fig. 3) and the intensity of bright soliton is expressed as

$$I_B = |\psi(x, z)|^2 \\ = a^2 \operatorname{sech}^2[a(x-vz)].$$

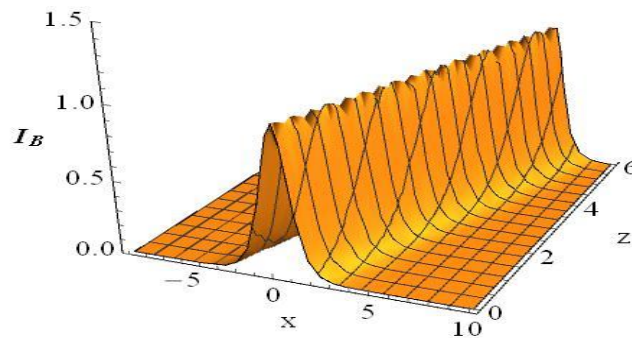


Fig.1 Intensity plot of bright soliton for  $a=1, v=1$ .

#### 4.2.2 Dark soliton

For the case of self-defocusing nonlinearity, NLSE admits dark soliton which exhibits a dip on the continuous background [8]. General form of the dark soliton for the NLSE is given as

$$\psi(z, x) = u_o [B \tanh(u_o B(x - Au_o z)) + iA] e^{-izu_o^2},$$

where  $A$  and  $B$  bears the relation  $A^2 + B^2 = 1$ , and  $u_o$  represents the continuous background. And exploiting  $A = \sin\phi$  and  $B = \cos\phi$ , intensity expression for the dark soliton is of the following form

$$I_D = u_o^2 [\cos^2\phi \tanh^2(u_o \cos\phi(x - u_o \sin\phi z)) + \sin^2\phi]$$

Here  $u_o \sin\phi$  is corresponding to velocity of the dark soliton and the  $\cos^2\phi$  is represents the magnitude of dip at the centre. And the plot for the dark soliton is as shown in fig. (4).

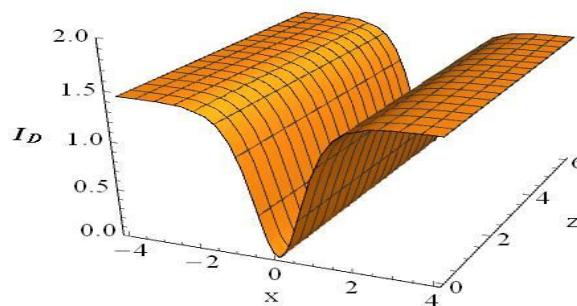


Fig. 2 Intensity plot of dark soliton for  $\phi = 0, u_o = 1.2$ .

## V. CURRENT RESEARCH

While the nonlinear Schrodinger equation gives an accurate account of the propagation of pulses with picosecond width but it turns out to be inadequate to governs the dynamics of the ultrashort pulse (femto-second pulse). Propagation of ultrashort pulses via optical fibre are of special interest because of their applications to communication system and ultrafast signal-routing system. Apart from the higher-order effect, the media may be inhomogeneous to include all such effects several research groups across the globe are investigating higher-order NLSE, variable-coefficient NLSE and NLSE with source in order to gain better optimization of ultrashort pulse propagation.

### 5.1 Higher-order nonlinear Schrödinger equation

When higher order terms third-order dispersion (TOD), self-steepening and self-frequency shift terms are incorporated into the NLSE then the equation is called higher order nonlinear Schrodinger equation (HNLSE) [9] which governs the dynamics of ultrashort pulse through the optical fibre. While the contribution of the third-order dispersion is negligibly smaller than the GVD, it gains significance with regard to ultrashort pulses when the GVD approaches zero. TOD results in the asymmetric broadening of the pulses. Self-steepening effect leads to the asymmetric broadening of the pulses in the spectrum. The self-frequency shift phenomenon arises on account of the stimulated Raman scattering, whereby the red shift of the spectrum of the pulse arises as the result of Raman gain of the long wavelength parts of the spectrum at the cost of the short wavelength parts. Such phenomena become noticeable when the pulse has a high intensity or nonlinearity of the media is strong. Considering these higher order effects, HNLSE provides a more advanced approach to nonlinear pulse propagation in various optical media.

### 5.2 Variable-coefficient NLSE

As the NLSE is highly idealized but many physical phenomena consist non-uniform boundaries and inhomogeneous media, such as plasma, optical fibre cable, BEC etc. Naturally, the vc-NLSE turns out to be more realistic for modelling a wide spectrum of nonlinear system. In the vc-NLSE, the spatial variation of the coefficients produces additional complexity into the dynamics of optical pulse propagation. And consequently, vc-NLSE provides a more detailed description of pulse propagation in media with spatially varying properties [10].

### 5.3 NLSE with source

In order to govern the dynamics of nonlinear system efficiently, it is necessary to study the behaviour of system under the influence of external force, dissipation and noise. Dissipation leads to loss, while the external force aids in stabilizing the system by supplying energy. Recently ac-driven NLSE [11] is attracting focus of scientific community and researcher because it arises in many physical systems such as plasma driven by radio frequency fields, chaotic dispersion and as pulse propagation in twin-core fibers.

## VI. CONCLUSION

Optical solitons provide a crucial solution for distortion of signal because of dispersion in fibre-optic communication systems by trade-off between group velocity dispersion and nonlinearity described by the nonlinear Schrödinger equation. These self-preserving pulses retain their shape and speed over thousands of kilometres, and hence enabling ultra-long-haul transmission without conventional regeneration. While practical challenges including Gordon-Haus timing jitter and interchange interactions exist, recent advances in dispersion management and Raman amplification have substantially enhanced system performance. The inherent robustness and high-bit-rate compatibility of solitons position them as a compelling technology for next-generation optical networks and emerging applications in quantum communication and all-optical processing. In conclusion, optical solitons constitute a mature yet evolving technology that bridges fundamental nonlinear physics with critical engineering challenges, ensuring continued relevance in addressing the escalating bandwidth demands of modern communication infrastructure.

## REFERENCES

- [1] G.P. Agrawal. Fiber-optic communication system. John Wiley & Sons, New York, 2002.
- [2] K.S. Kivshar and G.P. Agrawal. Optical solitons: from fibers to photonic crystals. Academic Press, London, 2003.
- [3] D.J. Korteweg and G.D. Vries. On the change of form of long waves advancing in a rectangular canal, on a new type of long stationary waves. The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science, 39(240):422-443,1895.
- [4] N.J. Zabusky and M.D. Kruskal. Interaction of "solitons" in a collisionless plasma and the recurrence of initial states. Physical Review Letters, 15(6):240,1965.
- [5] A.B. Shabat and V.E. Zakharov. Exact theory of two-dimensional self-focusing and one-dimensional self-modulation of waves in nonlinear media. Soviet Physics JETP, 34:62-69,1972.
- [6] A. Hasegawa and F. Tappert. Transmission of stationary nonlinear optical pulses in dispersive dielectric fibers. I. Anomalous dispersion. Applied Physics Letters, 23:142,1973.
- [7] L.F. Mollenauer, R.H. Stolen, and J.P. Gordon. Experimental observation of picosecond pulse narrowing and solitons in optical fibers. Physical Review Letters, 45:1095-1098,1980.
- [8] Y.S. Kivshar and B.L. Davies. Dark optical solitons: physics and application. Physics Reports, 298(2):81-197,1998.
- [9] L. Zhonghao, L. Li, H. Tian and G. Zhou. New types of solitary waves solutions for higher order nonlinear Schrödinger equation. Physical Review Letters 84:4096-4099, 2000.
- [10] S.A. Ponomarenko and G.P. Agrawal. Optical similaritons in nonlinear waveguides. Optics Letters, 32(12):1659-1661, 2007.
- [11] N. Mann, S. Bhatia, A. Goyal and C.N. Kumar. Propagating chirped lambert W-kink solitons for ac-driven higher order nonlinear Schrödinger equation with quadratic-cubic nonlinearity. The European Physical Journal Plus, 137:519, 2022.