



Dyonic Condensation and Superconductivity in Restricted Chromodynamics

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ABSTRACT

Within the framework of restricted chromodynamics (RCD) in the $SU(3)$ gauge concept, an investigation into the monopole condensation and the subsequent state of the chromo-magnetic superconductivity was carried out. In the existence of two vectors and two scalar modes, it has been demonstrated that the obtained Lagrangian is a result of dual superconductivity, color confinement, and dyonic condensation.

I. INTRODUCTION

The concept of a chromo-magnetic superconductor [1] and the magnetic superconductivity state [2] are both incorporated into the condensation of monopoles. In this state, the Meissner effect, which confines the magnetic field in simple superconductivity, will be supplanted by the dual Meissner effect, which will restrict the color electric field. It leads to a relationship among the chromo-magnetic superconductor as well as the quantum chromo-dynamic situation, in which the solenoidal monopole current is squeezing the Abelian electric field [3,4], as well as the color restriction occurs as a consequence of the dual Meissner effect, which is induced by monopole condensation. A dual gauge concept known as restricted chromodynamics (RCD) was developed from quantum chromodynamics (QCD) in $SU(2)$ concept [5-8]. This concept was constructed by utilizing the restriction of electric flux as a consequence of magnetic monopole condensation. The concept was made by utilizing the notion of restricting electric flux as a consequence of magnetic monopole condensation. This approach allows for a deeper understanding of the interplay between electric and magnetic fields, highlighting how dynamics change under certain dual gauge conditions. This dual dynamic ensures the encapsulation of dyonic quarks by the commonly occurring Meissner effect. The total amount of dynamical degrees of freedom has been decreased due to the imposition of an additional internal symmetry known as magnetic symmetry [5-11], which has been used to extract this RCD from QCD. Efforts have been undertaken [12,13] to determine the connection between the phenomenon of the dynamical violation of magnetic symmetry and superconductivity, which includes the restriction aspect in RCD vacuum.

A series of studies by B.S. Rajput and collaborators [14-21] introduced the framework of Radial Compactification Dynamics (RCD) as a mechanism to describe confinement. This model suggests that confinement arises due to dyonic condensation in the vacuum, which induces screening currents capable of simultaneously trapping chromo-electric and chromo-magnetic fluxes. These screening currents give rise to flux tubes, providing a natural explanation for the formation of color strings between quarks and antiquarks. The mathematical models presented

in these works [13-21] extend the concept of the generalized Meissner effect, showing how both electric and magnetic components of the gauge fields become restricted within confined regions. Additionally, their formulations incorporate the roles of vacuum polarization, monopole-like configurations, and topological excitations, thus linking confinement to intrinsic properties of the QCD vacuum rather than relying solely on strong coupling dynamics.

Complementing these analytical models, Masayasu Hasegawa [22] examined the division between strong and weak coupling regimes and their impact on spectral characteristics of confined states. M.N. Chernodub and collaborators [23-25] employed lattice gauge simulations to identify monopole condensation and vortex excitations as critical factors driving the confinement–deconfinement transition. Their computational studies lend strong support to the dual superconductor picture by illustrating how topological defects facilitate the persistence of flux tubes even in extreme conditions.

Diamantini, Tmgenberger, and Vonokur [26] further expanded this view by exploring topological superconductivity in related physical systems, demonstrating analogies between vortex dynamics in condensed matter and flux confinement in QCD. Chester [27] analyzed quantum anomalies and strong correlations, suggesting that non-trivial topological phases can enhance the stability of the confined state.

In this paper, it has been demonstrated that the notion of chromo-dyonic superconductor strengthens the RCD formulation. Furthermore, it was shown in the restriction stage, dyonic condensation of the vacuum leads to the intricate current of screening. The current restricts each of the chromo-magnetic and chromo-electric fluxes due to the process of generalised Meissner effect (both the conventional kind as well as its dual). Extending the RCD in the feasible color gauge group SU(3) by utilizing dual internal Killing vectors as λ_3 -like octet, as well as λ_8 -like octet, RCD Lagrangian of SU(3) concept, as achieved in the magnetic gauge, and it was represented for leading to color confinement, dyonic condensation, and the subsequent superconductivity in SU(3) concept, with the place of two vector modes as well as two scalar modes as the result of the place of two magnetic octets (λ_3 -like and λ_8 -like). As a result of the dynamic breakdown of magnetic symmetry, it was demonstrated that the vacuum gains characteristics that are comparable to those of the relativistic superconductor. In this case, the quantum fields produce expectation levels that are not zero and generate screening currents.

II. RESTRICTED CHROMODYNAMICS (RCD) AND MAGNETIC SYMMETRY IN SU(3) GAUGE THEORY

The computational underpinning of restricted chromodynamics (RCD) is based on the assertion that a non-Abelian gauge concept allows certain supplementary internal symmetry, which is known as magnetic symmetry [5-8]. In the aspect of the non-Abelian gauge concept, the united space P can be conceptualized as

$$P = M \otimes G \quad (1)$$

that is $(4+n)$ dimension manifold wherein G, in general, is the n-dimension internal space as well as M is the 4-dimension external space, produced by n Killing vectors ξ_i , which is satisfied by the conditions

$$[\xi_i, \xi_j] = f_{ij}^k \xi_k, \quad (2)$$

$$\text{and } L_{\xi_i} g_{AB} = 0. \quad (3)$$

wherein g_{AB} ($A, B=0, \dots, n+3$) is the manifold metric P having gauge symmetry as n -dimensional isometry [28,29] and $\mathcal{L}_{\xi_i}$ is the Lie derivative with ξ_i . In equation (2), the four-dimensional quotient space $M = P/G$ is the actual base manifold, where P is put as the principal fiber bundle, and f_{ji} is an internal structure parameter. There is a hypothesis that suggests that the dynamics of a magnetic monopole can be efficiently characterized a gauge concept that is founded on the magnetic symmetry and has topographical significance. This particular magnetic symmetry is a supplementary internal isometry H that possesses certain supplementary Killing vector fields that are associated with the generalized gauge concept. The subsequent Killing vectors are intentionally internal aspects and is therefore commuting with previously present fields ξ_i of G . The Cartan's subgroup of G is defined by internal isometry H and commutes with it. Consider the subsequent Killing vector fields be m_a ($a=1,2,\dots,k = \dim H$). Therefore

$$\begin{aligned} m_a &= m_a^i \xi_i, (i=1,2,3) \\ (\xi_i, m_a) &= 0, \\ (m_a, m_b) &= -f_{ab}^{(H)c} m_c \\ \mathcal{L}_{m_a} g_{AB} &= 0 \end{aligned} \quad , \quad (4)$$

where $\mathcal{L}_{m_a}$ is the Lie derivative over the magnetic symmetry direction. Because the isometry H commutes with the right isometry G , it is known as the left isometry. The topological magnetic charge in association with monopoles corresponds to the second homotopic group $\pi_2(G/H)$ elements.

The establishment in the $SU(3)$ limit of restricted chromodynamics is where we should look to begin. The magnetic structure of the framework as mentioned above can be characterized by dual internalized Killing vectors that interact with one another as well as having the gauge symmetry directly. These vectors are adjusted to unit place by the mathematical formulas that are presented below:

$$\hat{m}^2 = 1 \quad \text{and} \quad \widehat{m'}^2 = 1. \quad (5)$$

These Killing vectors are a λ_3 -like octet $\hat{m}$ and its symmetric product

$$\hat{m}' = \sqrt{3}(\hat{m} \times \hat{m}), \quad (6)$$

which is λ_8 -like.

By applying the additional internal symmetries, it is possible to derive the restricted theory (RCD) from the overall quantum conservation of energy (QCD). Let us constrain the dynamic levels of freedom of the model by enforcing the extra magnetic symmetry, which constrains the generalized non-Abelian gauge potential $\vec{V}_\mu$ for satisfying the constraint represented by

$$D_\mu \hat{m} = \partial_\mu \hat{m} + i|q|\vec{V}_\mu \times \hat{m} = 0, \quad (7)$$

and

$$D_\mu \hat{m}' = \partial_\mu \hat{m}' + i|q|\vec{V}_\mu \times \hat{m}' = 0,$$

Where gauge group, D_μ is the covariant derivative. Then the dyonic generalized four-potential

$$\vec{V}_\mu = \vec{A}_\mu - \vec{B}_\mu,$$

of QCD in SU(3) gauge concept which could be formulated as given in RCD SU(3) gauge concept:

$$\vec{V}_\mu = -iV_\mu^* \hat{m} - iV_\mu'^* \hat{m}' + \left(\frac{i}{|q|}\right) \hat{m} \times \partial_\mu \hat{m} + \left(\frac{i}{|q|}\right) \hat{m}' \times \partial_\mu \hat{m}', \quad (8)$$

where

$$\hat{m} \cdot \vec{V}_\mu = -iV_\mu^*,$$

and

$$\hat{m}' \cdot \vec{V}_\mu = -iV_\mu'^* \quad (9)$$

are, respectively, λ_3 -like and λ_8 -like uncontrolled Abelian components having controlled potential. The space-time independent $\hat{\xi}_3$ and $\hat{\xi}_8$ is coming from the magnetic gauge $\hat{m}$ and $\hat{m}'$ correspondingly, where

$$\hat{\xi}_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad \text{and} \quad \hat{\xi}_8 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \quad (10)$$

Then the generalized potential of equation (8) could be noted as

$$\vec{V}_\mu = (-iV_\mu^* + W_\mu) \hat{\xi}_3 + (-iV_\mu'^* + W_\mu') \hat{\xi}_8, \quad (11)$$

where W_μ and W_μ' could be considered in the magnetic symmetry of SU(3) gauge concept as the topological dyon potentials. The given concepts are given fixation by $\hat{m}$ and $\hat{m}'$, respectively, up to Abelian gauge degree of freedom. The general field strength could, later, be considered as

$$\begin{aligned} \vec{G}_{\mu\nu} &= \vec{G}_{\mu\nu} + \frac{i}{|q|} [\vec{V}_\mu \times \vec{V}_\nu] \\ &= (-iF_{\mu\nu} + H_{\mu\nu}) \hat{\xi}_3 + (-iF'_{\mu\nu} + H'_{\mu\nu}) \hat{\xi}_8, \end{aligned} \quad (12)$$

where

$$\begin{aligned} \vec{G}_{\mu\nu} &= \vec{V}_{\nu,\mu} - \vec{V}_{\mu,\nu}, \\ F_{\mu\nu} &= V_{\nu,\mu}^* - V_{\mu,\nu}^*, \end{aligned} \quad (13)$$

$$H_{\mu\nu} = \left(\frac{i}{|q|}\right) \hat{m} \cdot [\partial_\mu \hat{m} \times \partial_\nu \hat{m}], \quad (14)$$

$$\vec{G}_{\mu\nu} = \partial_\nu \vec{V}_\mu - \partial_\mu \vec{V}_\nu; \quad (15)$$

$$F'_{\mu\nu} = \partial_\mu V_\nu' - \partial_\nu V_\mu';$$

and

$$H'_{\mu\nu} = \partial_\mu W'_\nu - \partial_\nu W'_\mu. \quad (16)$$

In the given concept, the gauge fields are expressed for a unique time-like non-singular potential V_μ^* and V_μ^* , W_μ and W'_μ . When colored objects or quarks are lacking, in the magnetic gauge the RCD Lagrangian of SU(3) concept could be noted as

$$L = \frac{1}{4} H_{\mu\nu} H^{\mu\nu} + \frac{1}{4} H'_{\mu\nu} H'^{\mu\nu} + \frac{1}{4} [H_{\mu\nu} H'^{* \mu\nu} + H'_{\mu\nu} H^{* \mu\nu}] + \frac{1}{2} |D_\mu \phi|^2 + \frac{1}{2} |D'_\mu \phi'|^2 - V(\phi^* \phi, \phi'^* \phi') \quad (17)$$

where $D_\mu \phi = (\partial_\mu + i |q| W_\mu) \phi$;

$$D'_\mu \phi' = (\partial_\mu + i |q| W'_\mu) \phi'; \quad (18)$$

and the dyonic field operators ϕ and ϕ' correspond to m and m' respectively. Here $V(\phi^* \phi, \phi'^* \phi')$ is the subsequent potential represented for inducing the dynamic breaking of the magnetic symmetries.

III. DYONIC CONDENSATION AND SUPERCONDUCTIVITY IN RCD IN SU(3) GAUGE THEORY

Consider the generalised potential W_μ as the electrical potential and the dyon field for the parameter, the Lagrangian given by equation (17), which is the Lagrangian in magnetic gauge of the RCD, appears to be similar to the concept of superconductivity to the Ginsburg–Landau Lagrangian. This is the case when there is no quark or any colored particle present. The dyonic condensation of the vacuum is brought about as a consequence of the magnetic symmetry of its dynamical violation, which is brought about by the effective potential. The result is the formation of the dyonic supercurrent, which has two components: the real part, which is the electric aspect, screening the electric flux that is confining the magnetic color charge (as a consequence of the typical Meissner effect), and the imaginary constituent, which is the magnetic aspect, screening the magnetic flux which is confining the electric color iso-charges (as a result of the dual Meissner effect). To put it another way, the dual Meissner effect is the cause for the movement of the electric field into a narrow flux tube among static-colored charges. This results in the formation of a potential that increases linearly and also leads to confinement.

This Lagrangian leads to color confinement, dyonic condensation, and the subsequent dual superconductivity in SU(3) concept. The conventional SU(3) Lagrangian has been utilised to get the Lagrangian given by equation (17), and as a consequence, the dynamic breaking of magnetic symmetry that is needed can be achieved by establishing the optimal potential in the given form:

$$V(\phi^* \phi, \phi'^* \phi') = -\eta (|\phi|^2 + |\phi'|^2 - v^2 - v'^2), \quad (19)$$

where v and v' are the expectation values of Higgs fields ϕ and ϕ' , η is a constant:

$$v = \langle \phi \rangle_0 \quad (20)$$

$$\text{and} \quad v' = \langle \phi' \rangle_0. \quad (21)$$

In the Prasad–Sommerfeld limit [30]

$$V(\phi^* \phi, \phi'^* \phi') = 0. \quad (22)$$

But $\nu \neq 0$ and $\nu' \neq 0$.(23)

The dyons have the least feasible energy for the given magnetic and electric charges, which are e and g respectively, when they are functioning within this limit. Because of the existence of dual magnetic vectors $\hat{m}$ and $\hat{m}'$ in SU(3) concept, two scalar modes with masses are present

$$M_\phi = \sqrt{(8\eta)}\nu \text{ and } M_{\phi'} = \sqrt{(8\eta)}\nu', (24)$$

and two vector modes with masses given by

$$M_D = |q|\nu \text{ and } M_{D'} = |q|\nu', (25)$$

respectively.

Using the given two mass scales the lengths of coherence ε and ε' as well as length of the penetration λ and λ' , in correspondence to the two magnetic vectors $\hat{m}$ and $\hat{m}'$, are as follows

$$\varepsilon = 1/M_\phi = 1/[\sqrt{(8\eta)}\nu] ; \varepsilon' = 1/M_{\phi'} = 1/[\sqrt{(8\eta)}\nu'] , (26)$$

and

$$\lambda = 1/M_D = 1/(|q|\nu) ; \lambda' = 1/M_{D'} = 1/(|q|\nu') \quad (27)$$

The phase diagram space region, wherein $\varepsilon = \lambda$ and $\varepsilon' = \lambda'$ is constituting the border among type-I as well as type-II superconductors. It may be achieved for the following values of constant η of the effective potential given by eqn. (3.1):

$$\eta = \frac{|q|^2}{8} = \frac{e^2 + g^2}{8}. (28)$$

where e and g are the electric and magnetic charges of dyon.

An important representation for the real confinement technique is provided by superconductivity, and the color confinement as a consequence of the generalised Meissner effect, which is brought about by dyonic condensation. According to the dual superconductivity concept that was developed by Alessandro et al. [31], the Yang-Mills vacuum is located near the boundary among type I and type II superconductors, and it is also located on the type II side to a marginal degree.

IV. RESULTS AND DISCUSSION

Equation (7) represents the magnetic representation of restricted chromo-dynamics in SU(3) theory wherein dual internalized Killing vectors λ_3 -like octet and λ_8 -octet represented by formula (6) were given introduction basing it on the aspect that any system having possession of a SU(3) symmetry is suffering with a non-Abelian magnetic instability for the 4-7th gluons[32] and the 8th gluon is corresponding in color space to the diagonal generator[33]. In the magnetic symmetry of the SU(3) gauge principle, the confined generic potential as well as the gauge field strength are given by equations (11) and (12), respectively. In this theory, the space-time-independent octet λ_3 and λ_8 are represented by formulas (10). Equation (17) is the expression that describes the RCD Lagrangian of the SU(3) concept when there are no quarks or colored objects present. This Lagrangian results in color confinement, dyonic condensation, and dual superconductivity in SU(3) concept. Additionally, the existence of dual vector modes and dual scalar modes is an outcome of the existence of dual magnetic octets in the RCD of SU(3) concept. These octets are three-like and eight-like. Equations (24) and (25) respectively provide the masses of the two aforementioned scalars and two vector modes. The coherence lengths ε and ε' and the penetration length λ and λ'

are given by equations (26) and (27) respectively. The masses of scalar modes M_ϕ and $M_{\phi'}$ determine the speed at which the perturbative vacuum over the color source arrives at condensation, and the masses M_D and $M_{D'}$ of vector modes determine for the colored flux its penetration lengths.

The Lagrangian, represented by equation (17) for RCD in magnetic gauge in the lack of presence of any colored objects or quarks, establishing an analogy among superconductivity and the dynamic rupture of magnetic symmetry that is incorporating the restriction aspect in RCD vacuum where the feasible potential $V(\phi^*\phi, \phi'^*\phi')$, represented by equation (19), inducing the vacuum's dyonic condensation. A dynamic supercurrent is produced as a consequence of this. The electric component of the given current, also known as its real part, is responsible for screening the electric flux and containing the magnetic charges as a consequence of the typical Meissner effect. On the other hand, the magnetic component of this current, represented by its imaginary part, is responsible for screening the magnetic flux and containing the color iso-charges as a consequence of the dual Meissner function. As a consequence, the dynamical violation of the magnetic symmetry in the given framework finally results in the induction of the generalised Meissner effect, which has the electric constituent as the typical Meissner effect and its magnetic constituent as the dual Meissner effect. Within the framework of the current theory, the process governs how the magnetic and electric fluxes associated with dyonic quarks, are confined. The spontaneous breakdown of magnetic symmetry is responsible for the confinement of color. This spontaneous breaking of magnetic symmetry results in the formation of a non-vanishing magnetically charged Higgs condensate. The ruptured magnetic group is selected through an Abelianization process, as shown by equations (12) to (16). It demonstrates that Abelian degrees of freedom are the most important factor in the dyonic condensation mechanism of restriction in RCD structure. Boykov et al. [19] have provided evidence that demonstrates the existence of this Abelian dominance about monopole condensation. This finding has also been found in a dual superconductivity framework [20], which positions the Y-M vacuum nearest to the boundary among Type I and Type II superconductors and significantly on the Type II side.

Thus, these findings provide a coherent theoretical foundation that integrates analytical modeling, lattice-based results, and topological field theory to explain the nature of confinement. By recognizing the interplay of dyonic condensation, flux quantization, and monopole dynamics, this body of work establishes a consistent explanation for why quarks remain bound within hadrons and why the confinement phase persists under non-perturbative QCD conditions.

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