



A COMPARATIVE STUDY FOR SUSTAINABLE FUSION ENERGY PRODUCTION USING PROXIMITY-BASED POTENTIALS

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ABSTRACT:We analyze the fusion probabilities for Proton-Proton (${}^1_1H + {}^1_1H$), Deuterium-Deuterium (${}^2_1H + {}^2_1H$) and Deuterium-Tritium (${}^2_1H + {}^3_1H$) nuclear reactions using proximity-based potentials. The current research work for a sustainable fusion energy resource at ITER (International Thermonuclear Experimental Reactor), Francemotivates us to examine the feasibility of Deuterium-Tritium (D-T) fusion as a sustainable energy resource. We find that the Deuterium-Tritium (D-T) reaction has the highest fusion probability as compared to that for Proton-Proton (P-P) and Deuterium-Deuterium (D-D) reactions and hence our study suggests it as the most suitable reaction for sustainable fusion energy.

Keywords–fusion reactor, fusion probability, stellar energy, proximity potential, fusion barrier

1. INTRODUCTION

With the growing worldwide population and the accelerating energy consumption due to rapid technological growth, the demand for cleaner, economical, large-scale and more sustainable energy resources have increased many-fold. Conventional energy resources such as coal, oil and natural gas are trailing in maintaining pace with this demand and also contribute heavily to pollution and carbon emissions. Although nuclear fission reactors contribute significantly to current energy needs, but suffer from serious limitations due to insufficient nuclear fuel resources, high expense involved and health threats due to radioactive waste generation. In this scenario, nuclear fusion presents a potential solution, as it mimics the process that produces stellar energy, especially solar energy. Moreover, nuclear fusion provides access to abundant natural fuel, eliminates radiation hazards and reduces environmental impact by generating immense amounts of clean energy.

A fusion reactor utilizes controlled fusion reactions for enormous energy production. A fusion reaction involves fusing lighter nuclei into a heavier nucleus with the release of a large amount of energy due to the mass defect. To fuse two nuclei, sufficient energy is required to overcome the Coulomb barrier formed due to repulsive as well as long-range Coulomb forces and attractive as well as short-range nuclear forces. This can be done either by classically overcoming the fusion barrier or by quantum mechanical tunnelling. The thermal energy of the colliding nuclei can be increased by heating them to overcome the Coulomb barrier classically.

Many studies have revealed that a fusion reaction, which is a source of energy in stars, can also be carried out on Earth. This idea led to the development of fusion reactors worldwide [Yao, 2023]. In this direction,

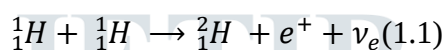
TOKAMAK (Toroidal Chamber with Magnetic Coils), which is based on magnetic confinement, has emerged as the most preferred fusion device. The world's largest fusion experiment ITER (International Thermonuclear Experimental Reactor) in France, is the most advanced application of the TOKAMAK concept [Karakoc, 2025]. Its design includes superconducting magnetic technology, which can withstand extreme heat conditions by generating powerful magnetic fields capable of confining the plasma without direct contact with the walls of the fusion reactor. The ITER aims to depict the feasibility of Deuterium-Tritium (D-T) fusion as a sustainable energy resource and to attain very high energy gain [Zhou, 2024].

Motivated by this current research work, we aim to study the fusion probabilities for nuclear reactions (${}^1_1\text{H} + {}^1_1\text{H}$) taking place in stars as well as fusion reactions (${}^2_1\text{H} + {}^2_1\text{H}$ & ${}^2_1\text{H} + {}^3_1\text{H}$) proposed by ITER. Firstly, we will understand stellar fusion, followed by fusion reactor dynamics.

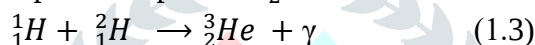
1.1 Stellar Fusion

Proton-Proton (P-P) fusion cycle as suggested by Gamow and Weizsacker in 1937 and further developed by Bethe in 1939 is the main energy source in stellar heavier than the Sun. It contributes to 99% of stellar energy.

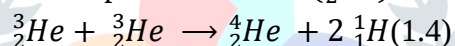
In Proton-Proton (P-P) fusion cycle [Redchuk, 2019], the two protons fuse to give Deuterium (D) by weak interaction



The deuterium fuses with one more proton to produce ${}^3_2\text{He}$



The ${}^3_2\text{He}$ will fuse with other similar nuclei to produce helium (${}^4_2\text{He}$) nucleus

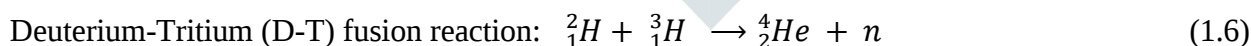


In this way, protons finally fuse into helium (${}^4_2\text{He}$) and release energy. This is how; the stellar energy is produced.

1.2 Fusion Reactor Dynamics

Proton-Proton (P-P) fusion reaction has very slow reaction rate due to very strong Coulomb repulsion between protons. Many studies have suggested that Proton-Proton (P-P) reaction has very low fusion probability and therefore, use of Deuterium and Tritium is suggested in a fusion reactor.

Deuterium-Deuterium (D-D) fusion reaction [Onofrio, 2018]:



In the present study, we attempt to calculate fusion probabilities for Proton-Proton (${}^1_1\text{H} + {}^1_1\text{H}$), Deuterium-Deuterium (${}^2_1\text{H} + {}^2_1\text{H}$) and Deuterium-Tritium (${}^2_1\text{H} + {}^3_1\text{H}$) nuclear reactions using proximity-based potentials. This paper is systematically organized as follows. The next section describes the research methodology, followed by the results and summary.

2. RESEARCH METHODOLOGY

The present study is carried out using proximity-based potentials [Kumari, 2013, Kumari, 2014, Kumari et al. 2015, Kaur et al. 2015, Jain et al. 2013 and Gharaei, 2018] like Bass potential (Bass 80) and Modified Denisov potential (Denisov DP). Both these nuclear potentials $V_N(R)$ are parameterized in the proximity fashion, where the nucleus-nucleus potential is given by product of a geometrical factor and a universal function. The geometrical factor depends on the masses of the colliding nuclei, whereas the universal

function is independent of the features of the colliding nuclei and instead, it depends on the separation distance between two colliding nuclei (s).

2.1 Total Interaction Potential (V_T)

The total interaction potential $V_T(R)$ is computed by adding Coulomb potential $V_C(R)$ to the nuclear interaction potential $V_N(R)$.

$$V_T(R) = V_N(R) + V_C(R), \quad (1.7)$$

here

$V_C(R) = \frac{1.44 \times Z_1 \times Z_2}{R}$ and Z_1 and Z_2 are the atomic numbers of the two colliding nuclei. In this study, the nuclear potential due to Bass and Denisov are used.

2.1.1 Bass Potential (Bass 80)

According to Bass [Bass, 1974], the nuclear part of nucleus-nucleus potential is written as:

$$V_N^{Bass}(R) = -4\pi\gamma \frac{R_1 R_2}{R_1 + R_2} f(s) = -\frac{R_1 R_2}{R_1 + R_2} \Phi(s) \text{ MeV}, \quad (1.8)$$

with γ being nuclear surface energy coefficient and

$$\frac{df}{ds} = -1, \text{ for } s = 0. \quad (1.9)$$

Here, $f(s)$ and $\Phi(s)$ are the universal functions.

The nuclear radii of the colliding nuclei have the form:

$$R_i = 1.16 A_i^{1/3} - 1.39 A_i^{-1/3} \text{ fm}, \quad (i = 1, 2) \quad (1.10)$$

The universal function $\Phi(s)$ was taken to be:

$$\Phi(s) = \left[0.033 \exp\left(\frac{s}{3.5}\right) + 0.007 \exp\left(\frac{s}{0.65}\right) \right]^{-1} \quad (1.11)$$

The expression for universal function $\Phi(s)$ was obtained by fitting the experimental data for fusion at incident energies above the Coulomb barrier.

2.1.2 Modified Denisov Potential (Denisov DP)

According to Denisov [Denisov, 2002], the nuclear part of nucleus-nucleus potential is written as:

$$V_N^{Denisov}(R) = -1.989843 \Omega \times \frac{R_1 R_2}{R_1 + R_2} \Phi(s) \text{ MeV}, \quad (1.12)$$

here

$$\Omega = 1 + 0.00325139 \left(\frac{A_1}{A_2} + \frac{A_2}{A_1} \right)^{3/2} - 0.4113263 (I_1 + I_2), \quad (1.13)$$

and

$$I_i = \frac{N_i - Z_i}{N_i + Z_i}, \quad (i = 1, 2). \quad (1.14)$$

The universal function $\Phi(s)$ was taken to be:

$$\begin{aligned} \Phi(s) = & \left\{ 1 - \frac{s}{0.7881663} + 1.229218 s^2 - 0.2234277 s^3 - 0.1038769 s^4 - \frac{R_1 R_2}{R_1 + R_2} (0.1844935 s^2 + \right. \\ & \left. 0.07570101 s^3) + (I_1 + I_2) (0.04470645 s^2 + 0.03346870 s^3) \right\} \quad \text{for } -5.65 \leq s \leq 0, \& \\ \Phi(s) = & \left\{ 1 - s^2 \left[0.05410106 \frac{R_1 R_2}{R_1 + R_2} \exp\left(-\frac{s}{1.760580}\right) - 0.5395420 (I_1 + I_2) \exp\left(-\frac{s}{2.424408}\right) \right] \right\} \times \\ & \exp\left(-\frac{s}{0.7881663}\right), \quad \text{for } s \geq 0 \end{aligned} \quad (1.15)$$

The radii form given by Royer and Rousseau [Royer et al. 2009] was used which is given by

$$R_i = 1.2332 A_i^{1/3} \left\{ 1 + \frac{2.348443}{A_i} - 0.151541 \left(\frac{A_i - 2Z_i}{A_i} \right) \right\} \text{ fm}, \quad (i = 1, 2). \quad (1.16)$$

This modified form of Denisov potential was labelled as Denisov DP.

2.2 Wong Formalism for Fusion Probability/Cross-section

In Wong formalism[Wong, 1973 and Wong, 1972], the fusion cross-section $\sigma_{fus}(E_{c.m.})$ is given by

$$\sigma_{fus}(E_{c.m.}) = \frac{\pi}{k^2} \sum_{l=0}^{l_{max}} (2l+1) T_l(E_{c.m.}), \quad (1.17)$$

where $k = \sqrt{\frac{2\mu E_{c.m.}}{\hbar^2}}$ and l_{max} represents the largest partial wave corresponding to the pocket in the total interaction potential and $T_l(E_{c.m.})$ corresponds to the energy-dependent barrier penetration factor.

For $E_{c.m.}$ (incident energy) $\gg V_B$ (barrier height), the above equation becomes

$$\sigma_{fus}(mb) = 10\pi R_B^2 \left(1 - \frac{V_B}{E_{c.m.}}\right). \quad (1.18)$$

Here, R_B represents the position of a potential barrier.

For $E_{c.m.} \ll V_B$, the above equation becomes

$$\sigma_{fus}(mb) = 10 \frac{R_B^2 \hbar \omega_0}{2E_{c.m.}} \exp \left[\frac{2\pi}{\hbar \omega_0} (E_{c.m.} - V_B) \right]. \quad (1.19)$$

3. RESULTS AND DISCUSSION

For this comparative study, we have considered the reactions of Proton-Proton (${}^1_1\text{H} + {}^1_1\text{H}$), Deuterium-Deuterium (${}^2_1\text{H} + {}^2_1\text{H}$) and Deuterium-Tritium (${}^2_1\text{H} + {}^3_1\text{H}$) nuclei.

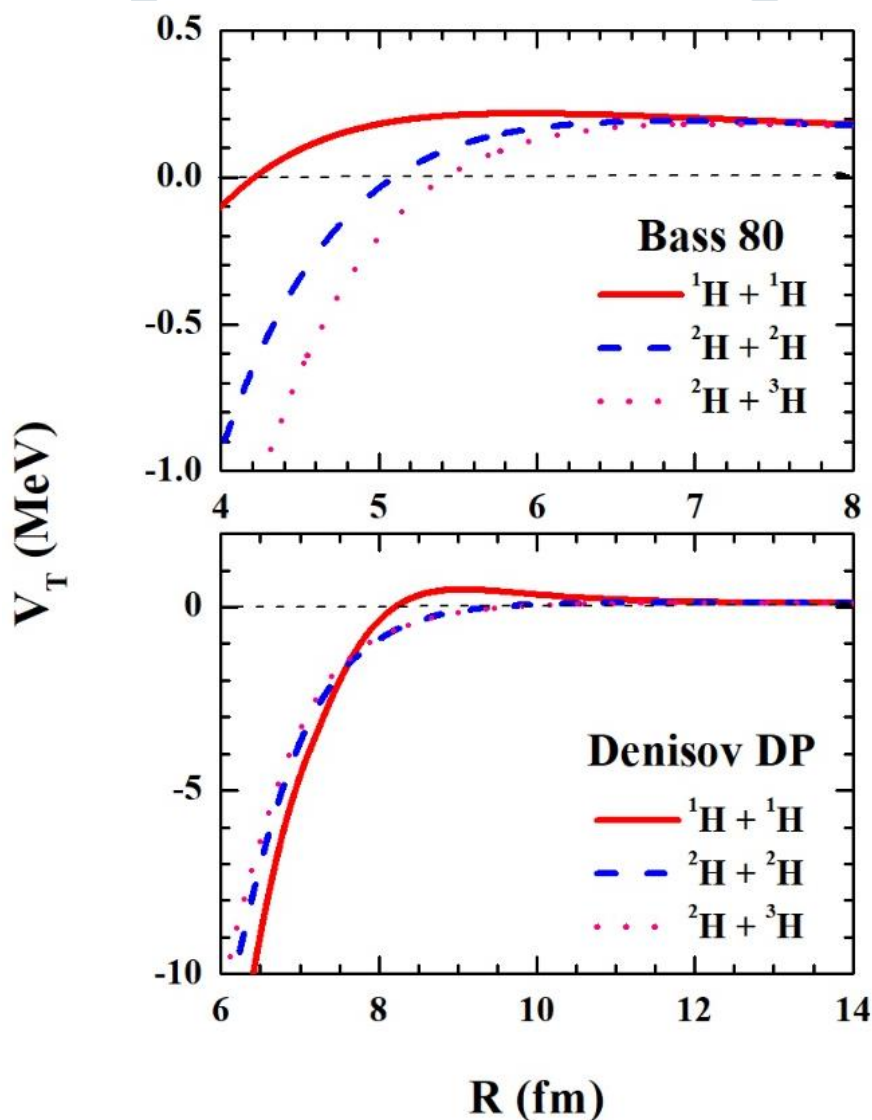


Figure 1. The total interaction potential $V_T(\text{MeV})$ is presented as a function of inter nuclear distance R (fm) for the reactions of Proton-Proton (${}^1_1\text{H} + {}^1_1\text{H}$), Deuterium-Deuterium (${}^2_1\text{H} + {}^2_1\text{H}$) and Deuterium-Tritium (${}^2_1\text{H} + {}^3_1\text{H}$). Further details are given in the text.

In Fig. 1, we present the total interaction potential $V_T(\text{MeV})$ versus inter-nuclear distance R (fm). The upper panel displays the total interaction potential calculated using Bass 80 nuclear potential. The lower panel

displays the total interaction potential calculated using Denisov DP nuclear potential. Here, the red solid, blue dashed and magenta dotted lines represent the reactions of Proton-Proton (${}^1_1\text{H} + {}^1_1\text{H}$), Deuterium-Deuterium (${}^2_1\text{H} + {}^2_1\text{H}$) and Deuterium-Tritium (${}^2_1\text{H} + {}^3_1\text{H}$), respectively.

From Fig. 1, we find that the height of fusion barrier [Kumari et al. 2015] with Bass 80 nuclear potential is maximum for ${}^1_1\text{H} + {}^1_1\text{H}$ fusion reaction and minimum for ${}^2_1\text{H} + {}^3_1\text{H}$ fusion reaction. Similar trend is observed while using nuclear potential due to Denisov DP. Here also, the fusion barrier is found to be highest for ${}^1_1\text{H} + {}^1_1\text{H}$ reaction as compared to ${}^2_1\text{H} + {}^2_1\text{H}$ and ${}^2_1\text{H} + {}^3_1\text{H}$ reactions. This implies that the Proton-Proton (${}^1_1\text{H} + {}^1_1\text{H}$) fusion reaction, which is the usual fusion reaction in stars, is the less probable reaction in actual practice due to very high potential barrier. However, the Deuterium-Deuterium (${}^2_1\text{H} + {}^2_1\text{H}$) and Deuterium-Tritium (${}^2_1\text{H} + {}^3_1\text{H}$) reactions have relatively lower fusion barriers indicating that the energy required to overcome the barrier in these reactions are less comparatively.

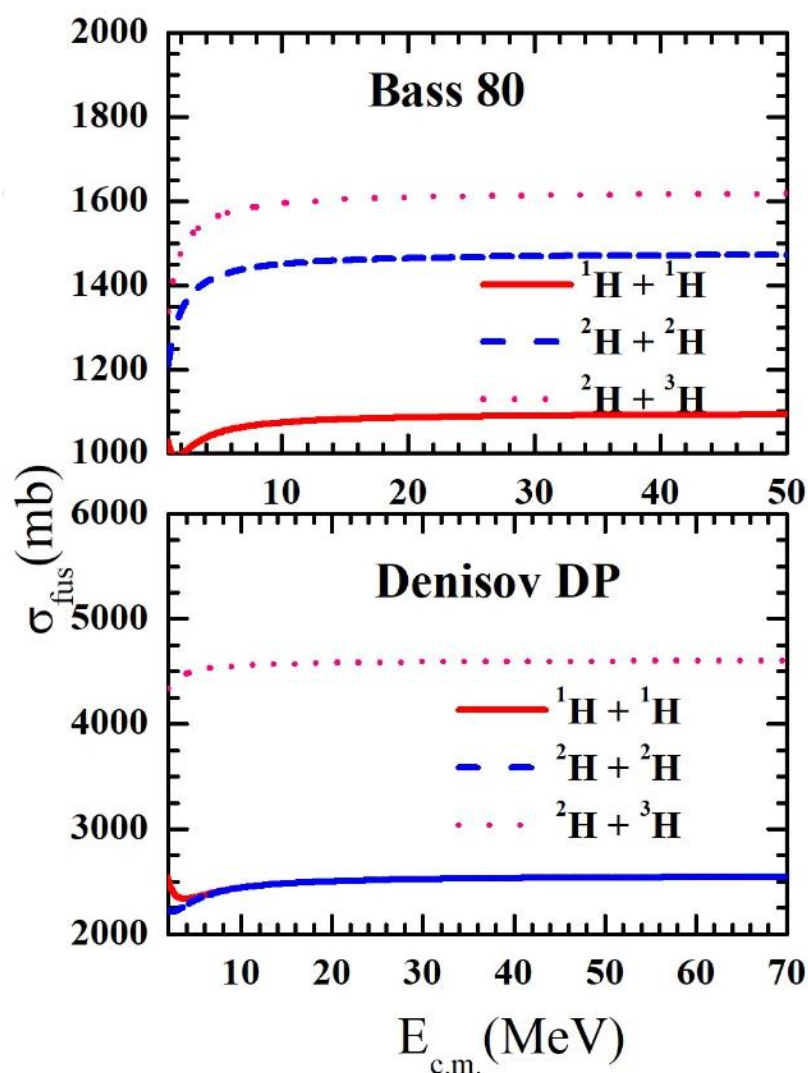


Figure 2. The fusion cross-section σ_{fus} (mb) is displayed as a function of the centre of mass energy $E_{c.m.}$ (MeV) for the reactions of Proton-Proton (${}^1_1\text{H} + {}^1_1\text{H}$), Deuterium-Deuterium (${}^2_1\text{H} + {}^2_1\text{H}$) and Deuterium-Tritium (${}^2_1\text{H} + {}^3_1\text{H}$) due to Bass 80 in upper panel and due to Denisov DP in lower panel. Further details are given in the text.

In Fig. 2, we present the fusion cross-sections σ_{fus} (mb) as a function of the centre of mass energy $E_{c.m.}$ (MeV). The upper panel displays the fusion cross-sections calculated using Bass 80 nuclear potential. The lower panel displays fusion cross-sections calculated using Denisov DP nuclear potential. Here, the red solid, blue dashed and magenta dotted lines represent the reactions of Proton-Proton (${}^1_1\text{H} + {}^1_1\text{H}$), Deuterium-Deuterium (${}^2_1\text{H} + {}^2_1\text{H}$) and Deuterium-Tritium (${}^2_1\text{H} + {}^3_1\text{H}$), respectively.

The upper panel of Fig. 2 reveals the highest fusion probabilities for Deuterium-Tritium (${}^2_1\text{H} + {}^3_1\text{H}$) reaction, which is comparatively lower for Deuterium-Deuterium (${}^2_1\text{H} + {}^2_1\text{H}$) reaction and lowest for Proton-Proton (${}^1_1\text{H} + {}^1_1\text{H}$) reaction. The lower panel of Fig. 2 also predicts the highest fusion probabilities for Deuterium-Tritium (${}^2_1\text{H} + {}^3_1\text{H}$) as compared to Deuterium-Deuterium (${}^2_1\text{H} + {}^2_1\text{H}$) reaction and Proton-Proton (${}^1_1\text{H} + {}^1_1\text{H}$) reaction.

4. SUMMARY

We have presented a comparative study for fusion probabilities for Proton-Proton (${}^1_1\text{H} + {}^1_1\text{H}$), Deuterium-Deuterium (${}^2_1\text{H} + {}^2_1\text{H}$) and Deuterium-Tritium (${}^2_1\text{H} + {}^3_1\text{H}$) nuclear reactions using proximity-based potentials. Our study revealed that fusion of deuterium and tritium nuclei encounter lowest potential barrier as compared to fusion of two deuterium nuclei and fusion of two protons. The fusion probabilities for Deuterium-Tritium nuclear reactions are found to be highest at all the incident energies as compared to for Proton-Proton and Deuterium-Deuterium reactions. Since tritium is very rare and radio-active isotope so the production of tritium in a self-sufficient manner in the fusion reactor and its timely consumption is very crucial for sustainable fusion. In this direction, extensive research is being carried on to maximize tritium breeding in fusion reactor envelopes. Our study depicted that the feasibility of Deuterium-Tritium (${}^2_1\text{H} + {}^3_1\text{H}$) fusion as a sustainable energy resource is quite high and using it, we can hope to attain high energy gain in ITER in the near future.

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